ASSEMBLY CONTACT FORCE DOMAINS IN THE PRESENCE OF UNCERTAINTY

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Abstract. A key aspect of the automatic planning and execution of assembly tasks with robots is the prediction of the contact forces between the objects. The paper addresses the off-line computation of the sets of reaction force directions that can be expected in the presence of uncertainty for any contact situation during an assembly task. The approach, developed for polyhedral objects moved in a plane, makes use of the dual representation methodology, considering friction and different sources of geometric and sensory uncertainties. Theoretical concepts are illustrated through a simple example: positioning a block in a corner.

Key Words. Robots, Assembling, Reaction Forces, Uncertainty, Dual Force Representation.

1. INTRODUCTION

Assembly with robots in the presence of uncertainty is an important application field with significant aspects still without a general solution. Looking forward for the fully automatic programming of these tasks the problem of fine-motion planning has to be addressed. This problem takes special relevance when the uncertainty in the task parameters and variables is significant and the plan to perform the assembly needs to take into account sensory information.

Two approaches to deal with this uncertainty has been proposed: building a robust plan including uncertainty inside it and developing recovery strategies complementing a nominal plan built without uncertainty consideration. Several works have been done in the two lines, all of them having a common key point: the need of determining when a given movement must change in accordance to the on-line information.

This necessity coincides, for instance, with that of determining some *termination condition* in the approach of Lozano-Perez, Taylor and Mason (1984) and several subsequent works in the same line, e.g. Erdmann (1984) and Buckley (1987), and also with the problem of identifying the current *state* in the approaches which represent the task as a set of states, e.g. Laugier (1989), Xiao and Volz (1989) and Suárez and Basañez (1991).

For these problems the prediction of the reaction forces that can appear in any possible contact situation is a basic step. This prediction can be specially useful in identifying the current contact situation during the task execution, as it is described in Desai and Volz (1989), Basañez and Suárez (1992), Xiao (1993) and Spreng (1993), each one using a different approach.

This paper deals with the off-line determination of the sets of reaction forces that can appear in a given contact situation considering all the related uncertainty sources.

2. SOME CONSIDERATIONS AND ASSUMPTIONS

In this work, the assembly of polygonal rigid objects in a plane, i.e. with two translational and one rotational degrees of freedom, are considered. The results can be applied to the assembly of polyhedral objects when: a) the movement of the manipulated object takes place in a plane, and b) the edges of this object, or the edges of every static object involved in the assembly, are parallel or perpendicular to the plane of the movement. It is assumed that the robot has an impedance position/force controller and that the movements are slow enough to make inertias negligible.

The concepts discussed in the paper are illustrated by means of the task of positioning a block in a corner (figure 1). The identification number of the involved vertices and edges is shown in figure 3, and the basic contacts in the examples below are numbered as follows:

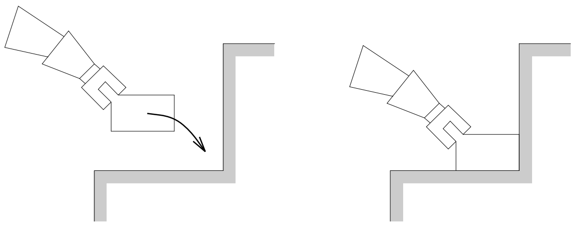

Fig. 1. *Positioning of a block in a corner.*

contact #	block	corner
1	vertex 1	edge 2
2	vertex 2	edge 1
3	vertex 2	edge 2
4	vertex 3	edge 1
5	edge 2	vertex 1

It is assumed a wrist force/torque sensor with the reference origin on the gripper symmetry axis and at the end of the parallel fingers, and the f_x-axis along the gripper symmetry axis.

3. DUAL FORCE REPRESENTATION

Given a force $\vec{f} = [f_x \ \ f_y]^T$ acting in the plane and producing a moment τ with respect to a reference origin O, the corresponding generalized force will be $\vec{g} = [f_x \ \ f_y \ \ \tau]^T$. The coordinates of the intersection point of the line of a generalized force with the unitary moment plane, and the sign of τ can be used to represent the generalized force direction. The representation on that plane coincides, with a simple $\pi/2$ rotation, with the dual representation of pure forces acting in a plane (Brost and Mason, 1989).

A force $\vec{f}$ acting along the line $ax + by + c = 0$ is represented by a dual point f' with coordinates $[a/c \ \ b/c]^T$. Geometrically, this is equivalent to placing the dual point on the normal to the force line through the origin O and at a distance $1/d$, being d the distance between the line and O (figure 2). The sense of $\vec{f}$ is included by adding to f' the sign of the torque produced by $\vec{f}$ around O. So, the direction of the generalized force $\vec{g}$ associated with $\vec{f}$ is represented by f' and the corresponding sign.

Properties of the dual force representation:

1. A line of force maps to a point.
2. The lines of forces passing through a point map to points of a line called the dual line of the point.
3. The lines of forces lying inside a cone $\widehat{r_1 r_2}$ map to points inside a cone $\widehat{r_1^\perp r_2^\perp}$ with vertex on the origin.
4. The lines of forces passing through a segment $\overline{AB}$ of a line l, map to points inside a cone $\widehat{A'B'}$ with vertex on l', the dual point of line l, A' and B' being the dual lines of points A and B respectively.
5. The lines of forces passing through a circumference map to a set of points bounded by a conic curve. The type of conic depends on the relative position of the origin and the circumference.

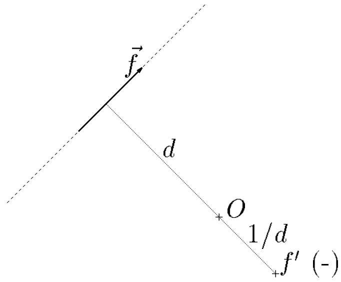

Fig. 2. *Mapping a line of force to a point.*

4. REACTION FORCES

The reaction force, measured by a force/torque sensor, can be used to perform compliant motions as well as to identify current contacts in order to determine the next robot action. For identification purpose only the reaction force direction is used, the module being not relevant. Factors affecting the force direction are discussed in the following subsections.

4.1. *Friction*

Friction force is modelled by $f_f = \mu f_n$ where f_n is the component of the normal force and μ the friction coefficient. This well known model defines a friction cone with angle $2 \arctan \mu$. The friction coefficient μ depends on the relative velocity of the objects in contact. This dependence is sometimes simplified by considering that μ can take only two values: a *static value* for static situations and a smaller *dynamic value* for any non-static situation. The actual value of μ for any relative velocity of the objects is smaller than the static value, which therefore generates the greater friction cone. Since the objective is computing all the possible reaction forces for any situation, μ will be taken constant and equal to its static value, covering in this way all the possible friction forces that can be generated in a given contact. In the examples below we will consider $\mu = 0.1$, i.e. a friction cone of $\sim 12^\circ$.

4.2. *Uncertainty*

Geometric Uncertainty. The direction normal to the contact edge cannot be exactly known due to the geometric uncertainty generated by:

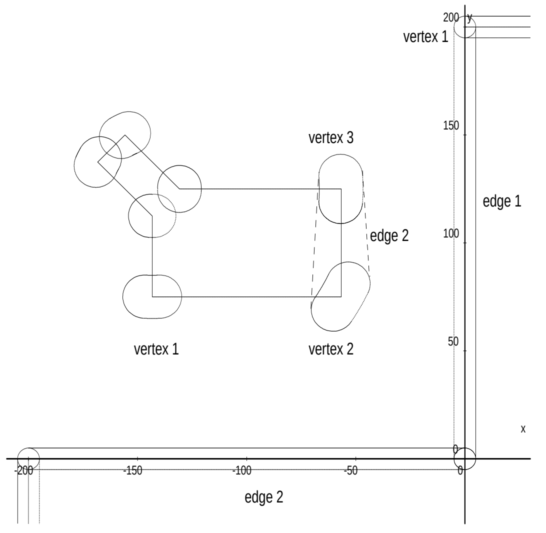

Fig. 3. *Resulting uncertainties affecting all the objects vertices in the block-in-the-corner problem.*

- Manufacturing tolerances
- Observation of the static object position
- Robot gripper position and orientation
- Slipping of the object in the gripper

Basañez and Suárez (1991) have analyzed and modelled all these uncertainty sources, merging them both in physical and configuration space. They have shown that, due to geometric uncertainty, the actual position of each vertex of the manipulated object is somewhere within a domain Uv bounded by four arcs of circumference, while each of the vertices of the static objects in the work environment must be inside a domain Uv bounded by a circumference centered on its nominal position. Of course, the domain Uv of a vertex of the object in the robot gripper depends on the robot configuration. Figure 3 illustrates the domains Uv of the vertices of the objects involved in the block-in-the-corner problem. Figure 4 shows the arcs of circumference (circumferences Cr_M, Cr_m, Cg_M and Cg_m) bounding the domain Uv of a manipulated object vertex.

The angle covered by the arc of the circumference Cr_M is quiet small, therefore it can be approximated by two straight segments tangent to Cr_M in the arc end points (figure 4), the amount of added uncertainty being negligible.

Since each vertex can be anywhere inside the corresponding domain Uv, the direction normal to each object edge can not be exactly determined, but it should be inside a given range.

Force Uncertainty. Friction and geometric uncertainties affect the direction of the reaction forces that can really appear. When these forces are measured the uncertainty of the force/torque sensor must also be included.

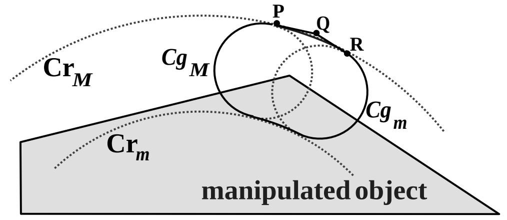

Fig. 4. *Approximation of the arc of the circumference Cr_M by two segments $\overline{PQ}$ and $\overline{QR}$.*

Typical force/torque sensors give forces and torques as independent values with a specified resolution. So, each measured component f_i (force or torque) is supposed to really be anywhere in the range $[f_i - \epsilon_{f_i}, f_i + \epsilon_{f_i}]$, being ϵ_{f_i} the sensor resolution for f_i. So, for planar movements the force uncertainty model is a rectangular parallelepiped, U_g, of sides ϵ_{f_x}, ϵ_{f_y} and ϵ_τ, centered in the generalized force measured value $[f_{xo}\ \ f_{yo}\ \ \tau_o]^T$.

5. DOMAINS OF POSSIBLE REACTION FORCES

Due to uncertainty, the contact points and the direction of the reaction force cannot be exactly determined, but using the geometric uncertainty and the friction models it is possible to compute the domain of possible reaction forces, DGr, for any contact situation of an assembly task. The dual representation of DGr will also be referred as DGr when the meaning is clear.

The following subsections describe how this domain is computed for one, two, three, and more than three type-1 basic contacts (vertex of the manipulated object against edge of a static object), and for only one type-2 basic contact (edge of the manipulated object against vertex of a static object). As it will be shown, a type-2 basic contact can be considered, for this purpose, as two type-1 basic contacts. From now on, when not specified, any basic contact is assumed to be type-1.

For one basic contact, the following nomenclature will be used in the next subsections (figure 5).

- cs, cs^*: tangents to region Uv with orientation $\phi s = \psi_{na} + atan(\mu) + \epsilon_{\psi_{na}}$; where μ is the friction coefficient, ψ_{na} the orientation of the external normal to the contact edge and $\epsilon_{\psi_{na}}$ the uncertainty associated to ψ_{na}.
- ci, ci^*: tangents to region Uv with orientation $\phi i = \psi_{na} - atan(\mu) - \epsilon_{\psi_{na}}$.
- $cs^\perp$, $ci^\perp$: lines through the reference origin perpendicular to cs and ci respectively.
- V: region resulting from the union of cones $\widehat{ci\ cs}$ and $\widehat{ci^* cs^*}$.
- Vs: boundary of V generated by cs and ci^*.
- Vi: boundary of V generated by ci and cs^*.

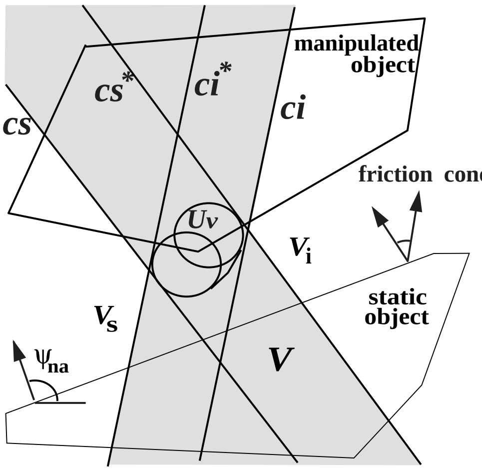

Fig. 5. *Nomenclature description.*

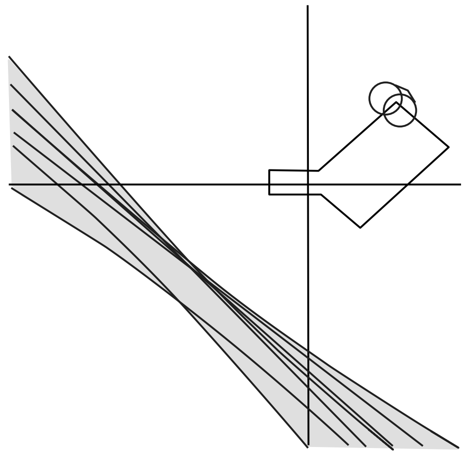

Fig. 6. *Dual region H satisfying the contact-point condition for one basic contact.*

For more than one basic contact the following notation is also introduced:

- $S^{\perp}$: $cs^{\perp}$ of the contact with maximum ϕs.
- $I^{\perp}$: $ci^{\perp}$ of the contact with minimum ϕi.
- h_i: dual line of the nominal position of vertex of contact i.

For any point of V, there exists at least one line of force through it that crosses region Uv and whose direction is included in the friction cone.

5.1. *One Basic Contact*

Let DGr_i be the domain of possible reaction forces for one basic contact i. Any force in DGr_i will satisfy the following two conditions:

1) Contact-point Condition: The line of the reaction force must intersect the region Uv of the contact vertex. The set of reaction forces whose lines intersect Uv can be generated by the union of the sets of forces whose lines intersect at least one of the circumferences C_{gM} and C_{gm}, or one of the segments which approximate the arc of the circumference Cr_M (figure 4). The dual region of forces through a circumference is bounded by a conic, and those through a straight segment by a cone (section 3). Therefore, the dual region, H, representing the set of reaction force directions satisfying the contact-point condition is given by the union of four regions: two bounded by conics and two by cones. Figure 6 shows an example in which the two conics are two hyperbolas.

2) Direction Condition: The reaction force direction must lie inside the contact friction cone enlarged with the orientation uncertainty of the contact edge. From section 3, the dual region representing the lines of forces whose directions are inside the friction cone $\widehat{ci\,cs}$ is bounded by a cone $\widehat{ci^{\perp}cs^{\perp}}$ with vertex on the origin.

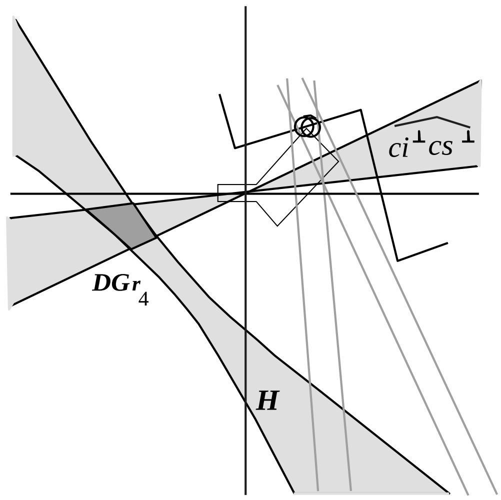

Fig. 7. *Domain DGr_4 (type-1 basic contact).*

The dual region of a domain DGr_i is then computed as the intersection of region H and cone $\widehat{ci^{\perp}cs^{\perp}}$. Note that the vertices of DGr_i are the dual points of lines ci, cs, ci^* and cs^*. As an example, figure 7 shows the domain DGr_4 for the basic contact 4 with a block orientation of -105°.

5.2. *Two Basic Contacts*

The domain of possible reaction forces in the presence of uncertainty for more than one basic contact is the set of all positive linear combinations of possible reaction forces for each basic contact.

The dual representation of the domain of possible reaction forces for two basic contacts i and j, DGr_{ij}, is determined as follows:

1) If $DGr_j \subset DGr_i$ (or $DGr_i \subset DGr_j$) then $DGr_{ij} = DGr_i$ (or $DGr_{ij} = DGr_j$) .

2) Otherwise, DGr_{ij} is equal to the union of DGr_i, DGr_j and the region bounded by them

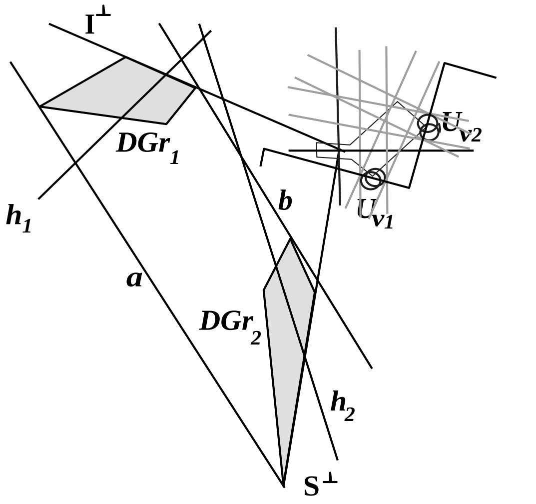

Fig. 8. *Domains* DGr_1 *and* DGr_2.

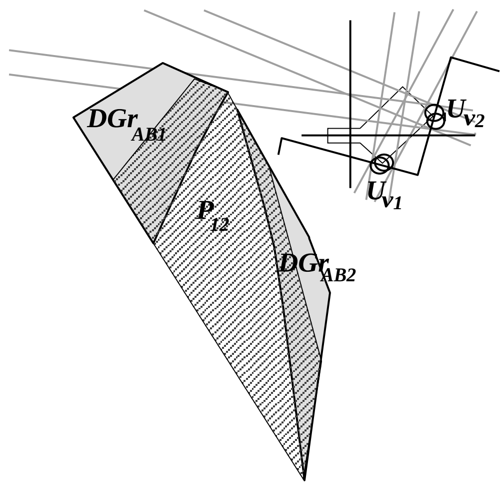

Fig. 9. *Domain* DGr_{12}.

and two of their common tangents. These two tangents define a dual cone, $\widehat{a\,b}_{(ij)}$, that contains region DGr_{ij}. The cone $\widehat{a\,b}_{(ij)}$ can be built by firstly determining the corresponding straight segment $\overline{AB}$ in real space and then finding the dual representation of forces through it. The points A and B are the intersection of Vi_i with Vi_j and of Vs_i with Vs_j.

Then, the domain DGr_{ij} can be obtained by the union of three dual regions:

- DGr_{ABi}, resulting from the intersection of cone $\widehat{a\,b}_{(ij)}$ and cone $\widehat{I^\perp S^\perp}$ with H_i.
- DGr_{ABj}, idem with H_j.
- The region P_{ij} resulting from the intersection of three cones: $\widehat{a\,b}_{(ij)}$, $\widehat{I^\perp S^\perp}$, and $\widehat{h_i h_j}$. If Uv_i (or Uv_j) lies inside V_j (or V_i) the line h_i (or h_j) of cone $\widehat{h_i h_j}$ will be replaced by line a (or b).

Figure 8 shows the domains DGr_1 and DGr_2 for the basic contacts 1 and 2 with a block orientation of 15°, and figure 9 shows the corresponding domain DGr_{12}.

5.3. *Three Basic Contacts*

The dual representation of the domain of possible reaction forces for three basic contacts i, j and k, DGr_{ijk}, can be built as the union of four dual regions: DGr_{ij}, DGr_{jk}, DGr_{ik} and a polygonal region, P_{ijk}, bounded by the axes of the cones $\widehat{a\,b}_{(ij)}$, $\widehat{a\,b}_{(jk)}$ and $\widehat{a\,b}_{(ik)}$.

The region P_{ijk} can be determined by the intersection of any two of the three cones $\widehat{C}_i$, $\widehat{C}_j$ and $\widehat{C}_k$, cone $\widehat{C}_\alpha$ being the region bounded by the axes of $\widehat{a\,b}_{(\alpha\beta)}$ and $\widehat{a\,b}_{(\alpha\gamma)}$ that do not contain the vertex of $\widehat{a\,b}_{(\beta\gamma)}$.

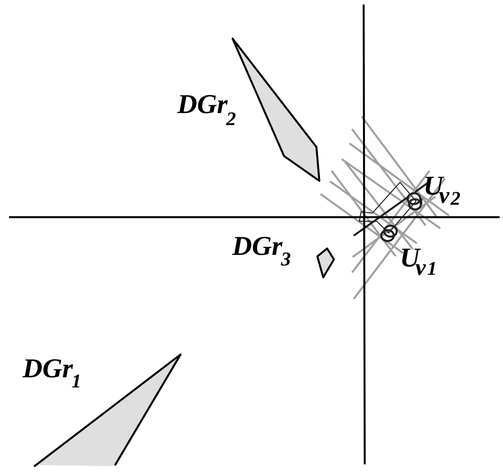

Fig. 10. *Domains* DGr_1, DGr_2 *and* DGr_3.

Figure 10 shows the three domains DGr_1, DGr_2 and DGr_3, for basic contacts 1, 2 and 3 respectively [1], with a block orientation of -45°; figure 11a shows the corresponding domains DGr_{12}, DGr_{23} and DGr_{13}; and figure 11b shows the cones $\widehat{C}_2$ and $\widehat{C}_3$ and the region P_{123} defined by them.

5.4. *More Than Three Basic Contacts*

Since planar movements have only three degrees of freedom, the dual representation of the domain of possible reaction forces for n basic contacts, with $n > 3$, can be built as the union of the $\binom{n}{3}$ dual regions representing the domains of possible reaction forces DGr_{ijk} for each possible subset of three contacts.

[1] Notice that according to the nominal models of the block and of the corner it is not possible to reach only three basic contacts, but due to uncertainty it is.

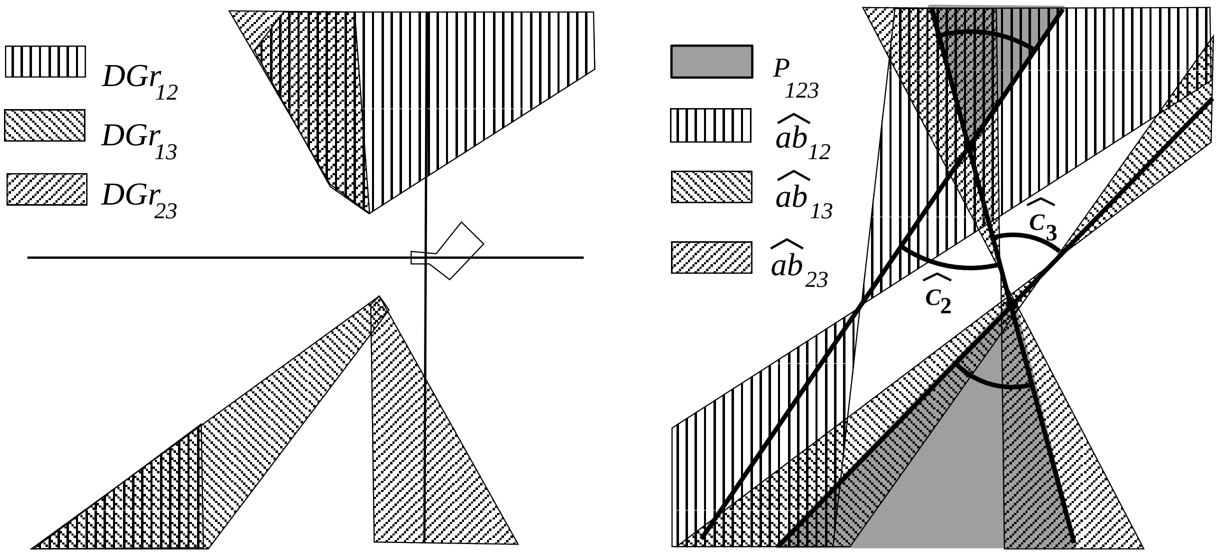

Fig. 11. *a) Domains DGr_{12}, DGr_{23} and DGr_{13}; b) Cones $\widehat{C}_2$ and $\widehat{C}_3$ defining polygonal region P_{123}.*

5.5. *One Type-2 Basic Contact*

Any force of the domain DGr of one type-2 basic contact will satisfy two conditions similar to those described for the case of one type-1 basic contact (subsection 5.1):

1) Contact-point Condition: The line of the reaction force must intersect the uncertainty geometric region of the contact edge. This region is built by linking the uncertainty regions Uv of the two vertices of the edge by two tangents (figure 3) (Basañez and Suárez, 1991). Forces through any point of the contact edge are equivalent to a positive linear combination of two forces, each one through one of the edge vertices. The same effect takes place in the presence of uncertainty; so, forces satisfying the contact-point condition can be obtained as a positive linear combination of forces through Uv of the two vertices.

2) Direction Condition: The reaction force direction must lie inside the contact friction cone enlarged by the orientation uncertainty of the contact edge. The difference with type-1 basic contacts is that now the orientation of the dual representation of the friction cone is invariant for any orientation of the block because the contact edge belongs to it.

Satisfying these conditions means that DGr coincides with the positive linear combination of two forces with directions inside cone $\widehat{ci\,cs}$, each one intersecting the region Uv of each vertex. Therefore, a situation with an unique type-2 basic contact can be considered as a situation with two type-1 basic contacts with the same friction cone. As a consequence, the segment $\overline{AB}$ lies at infinity and cone $\widehat{a\,b}$ coincides with cone $\widehat{ci^{\perp}cs^{\perp}}$.

As an example, consider the basic contact 5. Figure 12a shows the dual regions H_2 and H_3, representing the possible forces through the vertices 2 and 3 of the block, and the cone $\widehat{ci^{\perp}cs^{\perp}}$; and figure 12b shows the corresponding domain DGr_5 for this type-2 basic contact.

Similarly, the set DGr of a contact situation with, for instance, one type-2 and one type-1 basic contacts can be determined in a similar way as for a three type-1 basic contacts situation.

6. OBSERVED REACTION FORCES

A sensed generalized force may indicate a real reaction force belonging to a given domain DGr if any of the forces contained in the corresponding uncertainty parallelepiped U_g (subsection 4.2) belongs to DGr. In such a case the reaction force is considered to belong to the domain.

This condition can be tested by analyzing the relative position of the dual points describing the generalized forces given by the vertices of U_g and the domain DGr. The algorithms to perform this test for any number and type of basic contacts are described by Suárez, Basañez and Rosell (1994).

7. CONCLUSIONS

The dual-force representation gives rise to a convenient methodology for determining the sets of reaction force that can appear in the presence of uncertainty, for all possible contact situation of an assembly task of polygonal objects. This methodology can be also applied to the assembly of polyhedral objects moved in a plane.

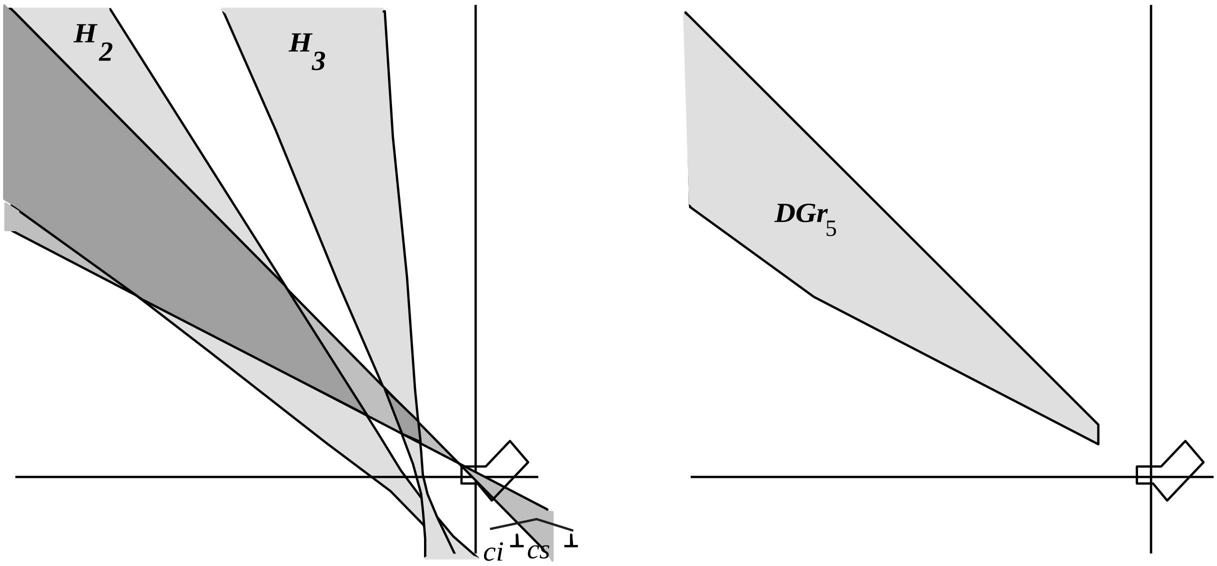

Fig. 12. *a) regions H_2, H_3 and cone $\widehat{ci^{\perp}cs^{\perp}}$, b) Domain DGr_5 (type-2 basic contact).*

The proposed approach is specially relevant since it takes into consideration friction and geometric uncertainties. In the same way, uncertainty in the measurement of the generalized reaction force during task execution is easily taken into account.

The method has been implemented on a Silicon Graphics workstation (Crimson/ELAN). The time to decide if a generalized force belongs to a given domain DGr of the block-in-the-corner problem is, in the worst case, 300 μs.

The proposed methodology forms part of an automatic system for planning and execution of assembly tasks with robots.

Acknowledgements

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