GRASP PLANNING WITH FOUR FRICTIONAL CONTACTS ON POLYHEDRAL OBJECTS

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Abstract: This paper addresses the problem of computing force-closure grasps with four non-coplanar contact points. The proposed approach in this work to determine force-closure grasp, is valid for sets of four faces with coplanar or non-coplanar normal, and is based on geometric operations. First, the sets of four object faces that due to their relative orientations and positions allow *concurrent*, *flat-pencil* and *regulus* types of grasp are determined; second, these sets are evaluated with a quality function and the best one is selected for the grasp; Finally, on the selected faces, according to the possible types of non-planar grasp, four contact points assuring a force-closure grasp are determined. *Copyright*© 2006 *IFAC*

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1. INTRODUCTION

A grasp of an object is force-closure if and only if the fingers can apply, through the contact points, forces that generate an arbitrary force and moment on the object. The theory regarding force-closure grasps has been deeply studied, and different techniques have been proposed for different cases (Nguyen, 1988, Liu *et al*., 1999; Xiangyang and Ding, 2004; Prado and Suarez, 2003-2005a).
The force-closure grasps with four non-coplanar contact forces are classified into three categories (Ponce *et al*., 1997): *concurrent* grasp, in this case the lines of action of the four contact forces intersect in a point (Figure 1a); *flat-pencil* grasp, here the lines of action of two contact forces intersect in a point and those of the other two forces intersect in another point, with these two points laying on the intersection of the planes defined by each pair of lines of action (Figure 1b); and *regulus* grasp where the distance between two lines of action is the same as the distance between the other two lines of action and the projections of each pair of lines of actions on the plane parallel and equidistant to them must form a *concurrent* or a *flat-pencil* (Figure 1c). Also if a set of four faces allows a *concurrent* grasp with the contact points in the interior of the face (i.e. the contacts do not belong to the face boundary) then it is always possible to determine *flat-pencil* and *regulus* grasps on the same set of faces; moreover, the different types of grasp can be reached using the same directions of force by changing only the contact points. In the same way, if a set of four faces allows a *flat-pencil* grasp (but not necessarily a *concurrent* grasp) then it is always possible to determine a *regulus* grasp on the same set of faces.
Ponce *et al*. (1997) developed an approach to determine *concurrent* grasps, but only on sets of four faces whose relative orientations satisfy a sufficient condition and their relative positions allow this type of grasp. However, the method does not work for *flat-pencil* and *regulus* grasps. Sundang and Ponce (1995) proposed a method for the construction of the three types of non-planar grasps over four faces whose relative orientations satisfy a sufficient condition but assuming that the relative positions of the faces to be contacted by the fingers allow *flat-pencil* and *regulus* grasps. Prado and Suárez (2005b) developed an approach to determine non-coplanar grasps on sets of object faces whose relative orientations satisfy a necessary and sufficient condition and their relative positions allow three (*concurrent*, *flat-pencil* and *regulus*) or two (*flat-pencil* and *regulus*) types of non-coplanar grasps. However, the approach does not determine *regulus*

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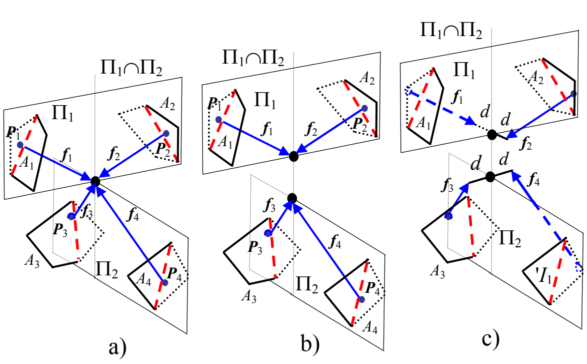


Fig. 1. Types of grasps: a) *Concurrent*; b) *flat-pencil*; c) *regulus*.

grasps over sets of faces that only allow this type of non-coplanar grasp.

In this paper, an approach to build the three types of non-coplanar grasps is proposed, and the main contribution is the determination of *regulus* grasps over sets of faces that only allow this type of grasp, a case that was not solved in any previous works. Same of the procedures used in this general approach were already used in Prado and Suárez (2005b) to solve particular cases, this will be pointed out when corresponds in the paper.

Assumptions. The following assumptions are considered in this work: (1) The objects are polyhedrons; (2) The grasp is done using four fingers and each finger contacts with a different face of the object; (3) Only the fingertips will contact with the object surface and the contact is a point; and (4) the friction coefficient μ is constant.

Notation. The following notation will be used:

P_i: contact point on the object surface (i=1,2,3,4).

A_i: contacted face of the object (i=1,2,3,4).

$\boldsymbol{n}_i$: unitary vector with object inward direction normal to A_i.

$\alpha=\mathrm{tg}^{-1}\mu$: half-angle of the friction cone ($\alpha<\pi/2$).

C_{fi}: friction cone with half-angle α, axis parallel to $\boldsymbol{n}_i$ and vertex at P_i.

C_i: friction cone with half-angle α, axis parallel to $\boldsymbol{n}_i$ and vertex at the origin of the reference system (representation of C_{fi} in the force space).

$\boldsymbol{f}_i$: contact force applied at contact P_i ($\boldsymbol{f}_i\subset C_{fi}$).

$\boldsymbol{F}_{ex}$: external arbitrary force applied on the object.

$\boldsymbol{M}_{ex}$: external arbitrary moment applied on the object.

c_m: object center of mass.

2. APPROACH OVERVIEW

The approach proposed in this work first selects the sets of faces that allow at least one type of non-coplanar grasp. Then, from these sets, the one that maximizes a quality function is selected and, finally, on the selected faces four contact points assuring a force-closure grasp (FCG) are determined.

The selection of the sets of four object faces that allow non-planar grasps is done in two phases:

1) *Selection of faces according to their orientations.* In this phase the sets of four faces whose relative orientations allow the application of forces $\boldsymbol{f}_i\in C_i$ i=1,2,3,4 that span $\Re^3$ are selected. Then, for each of these sets of faces subsets $^*C_i$ of the friction cones C_i are determined (Figure 2) such that, if $\boldsymbol{f}_i\in {}^*C_i$ i=1,2,3,4, then the four $\boldsymbol{f}_i$ span $\Re^3$ with independence of the contact point; $^*C_i$ is approximated by the largest cone included in it. This phase was already solved in Prado and Suárez (2005b).

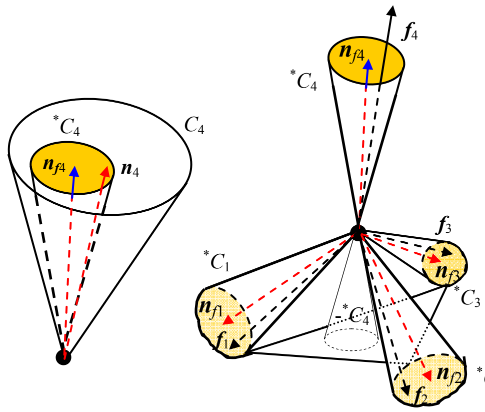


Fig. 2. $\boldsymbol{f}_i\in {}^*C_i$ i=1,2,3,4, then the four $\boldsymbol{f}_i$ span $\Re^3$ with independence of P_i.

2) *Selection of faces according to their positions.* In order to have the largest range of variation of the directions of $\boldsymbol{f}_i$ to keep a FCG when $\boldsymbol{F}_{ex}\neq 0$ and $\boldsymbol{M}_{ex}\neq 0$, it is desirable the direction of $\boldsymbol{f}_i$ to be aligned with the axis, $\boldsymbol{n}_{fi}$, of $^*C_i$ when $\boldsymbol{F}_{ex}=0$ and $\boldsymbol{M}_{ex}=0$, and therefore it is considered as a constraint in this selection of faces. In this phase a procedure is proposed to determine all the sets four faces that allow any type of non-coplanar grasps (including those that only allow *regulus* grasp) considering $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$. This procedure is composed of three modules independent from each other. The first module determines whether a set of faces allows the three types of non-coplanar grasps (*concurrent*, *flat-pencil* and *regulus*), the second module determines whether a set of faces allows two types of non-coplanar grasps (*flat-pencil* and *regulus*) and the last module determines whether the set of faces allows only *regulus* grasp.

All the sets of faces that allow at least one type of non-coplanar grasps are evaluated according to a quality measure that considers:

- The relative positions of the faces and their locations with respect to the object center of mass.
- The forces $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ should have similar modules in absence of external perturbations (i.e. for $\boldsymbol{F}_{ex}=0$ and $\boldsymbol{M}_{ex}=0$).

Then the set of faces that maximizes the quality function is selected for the grasp.

The positions of P_i, i=1,2,3,4, on the selected faces are determined such that the centroid of the tetrahedron that they define is close to the object center of mass c_m.

3. SELECTION OF THE SETS OF FOUR FACES THAT ALLOW FCG

The proposed procedure for the selection of the sets of four faces that allow FCG is described in the following two subsections.

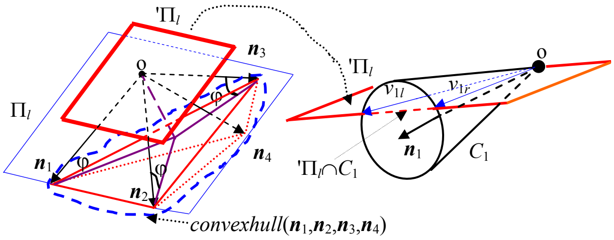


Fig. 3. Set of faces with $\mathbf{0} \notin ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$.

3.1 Selection of faces according to their orientations

Let:

Π_l be the closest plane to the origin that contains a face of $ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$ (Figure 3).

φ be the angle between Π_l and any of the three $\boldsymbol{n}_i$ that determines Π_l (note that φ is the same for any i).

$'\Pi_l$ be the plane parallel to the Π_l that contains the origin.

$\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ be the two unitary vectors that indicate the two boundary directions of $'\Pi_l \cap C_i$, i=1,2,3,4, respectively ($\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ don not exist when $'\Pi_l \cap C_i = \varnothing$ and $\boldsymbol{v}_{il} = \boldsymbol{v}_{ir}$ when C_i is tangent to $'\Pi_l$).

The sets of four faces that satisfy any of the two conditions below allow applied forces $\boldsymbol{f}_i \in C_i$ i=1,2,3,4 that span $\Re^3$, therefore they are selected as candidates for a FCG:

1) $\mathbf{0} \in ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$.

2) $\varphi < \alpha$ and $\mathbf{0} \in ConvexHull(\boldsymbol{v}_{il}, \boldsymbol{v}_{ir}, i=1,2,3,4)$.

After selecting the sets of faces that allow FCG, the largest cone $^*C_i$, included in each friction cone C_i, with directions that assure the FCG are determined. The procedure to determine $^*C_i$ is based on geometric operations with elements obtained from the friction cone C_i and the $ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$ respectively. The two conditions previous described and the determination of $^*C_i$ were developed and justified by Prado and Suárez 2005b.

3.2 Selection of faces according to their positions

The proposed procedure determines all the sets of four faces that allow to apply $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ i=1,2,3,4, ($\boldsymbol{n}_{fi}$ being the axis of $^*C_i$) whose lines of action determine *concurrent*, *flat-pencil* or *regulus* grasps.

Let D_i be the volume swept by the face A_i when A_i is displaced in the direction of $\boldsymbol{n}_{fi}$, i=1,2,3,4.

A set of four faces A_i, i=1,2,3,4, is valid to produce a FCG if it satisfies any of the following three cases:

A) Sets of faces that allow concurrent, flat-pencil and regulus grasps (Figure 4a)

If $D_1 \cap D_2 \cap D_3 \cap D_4 \neq \varnothing$ then the set of faces is ***valid*** for the three types of non-coplanar grasps

In this case the projection on A_i with directions of $\boldsymbol{n}_{fi}$, i=1,2,3,4, of any point belonging to $D_1 \cap D_2 \cap D_3 \cap D_4$ always determines a *concurrent* grasp (remind that if a set of faces allows a *concurrent* grasp also allows *flat-pencil* and *regulus* grasps).

If $D_1 \cap D_2 \cap D_3 \cap D_4 = \varnothing$ then the set of faces does not allow *concurrent* grasps.

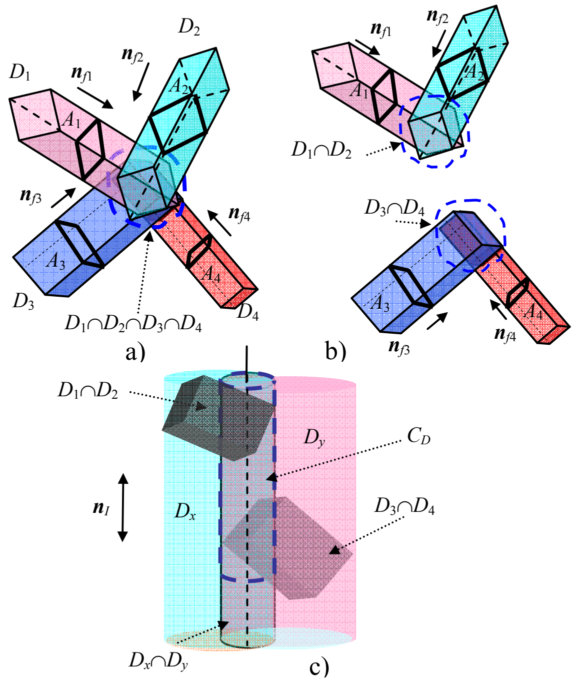


Fig. 4. Set of faces with: a) $D_1 \cap D_2 \cap D_3 \cap D_4 \neq \varnothing$; b) $D_1 \cap D_2 \neq \varnothing$ and $D_3 \cap D_4 \neq \varnothing$; c) $D_x \cap D_y \neq \varnothing$.

B) Sets of faces that allow flat-pencil and regulus grasps

If $\exists D_j \cap D_k \neq \varnothing$ $k,j \in \{1,2,3,4\}$ with $j \neq k$, then:

1. Determine $D_j \cap D_k$ and $D_r \cap D_h$, where at least $D_j \cap D_k \neq \varnothing$, with $\{j,k,r,h\}=\{1,2,3,4\}$ (Figure 4b).
2. Determine the volumes, D_x and D_y, swept by $D_j \cap D_k$ and $D_r \cap D_h$ respectively (Figure 4c), when they are displaced in the direction of $\boldsymbol{n}_I = (\boldsymbol{n}_{fj} \times \boldsymbol{n}_{fk}) \times (\boldsymbol{n}_{fr} \times \boldsymbol{n}_{fh})$. If $D_x \cap D_y \neq \varnothing$ then the set is ***valid*** for a *flat-pencil* and *regulus* grasps.

   In this case the projections on A_j and A_k of a point belonging to $D_x \cap D_y \cap D_j \cap D_k$ and the projections on A_r and A_h of another point belonging to $D_x \cap D_y \cap D_r \cap D_h$, with both points contained in a straight line parallel to $\boldsymbol{n}_I$, always determine a *flat-pencil* grasp.

   The straight line that contains the two projected points must be parallel to $\boldsymbol{n}_I$ to allow a resultant of null torque. Note that any straight line parallel to $\boldsymbol{n}_I$ and passing through a point belonging to $D_x \cap D_y \cap D_j \cap D_k$ always intersects with $D_x \cap D_y \cap D_r \cap D_h$.

If $D_j \cap D_k = \varnothing$ $\forall k,j$ then the set of faces does not allow *concurrent* and *flat-pencil* grasps.

C) Sets of faces that only allow regulus grasps

Let:

L_i be the line of action of $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ i=1,2,3,4.

Π_x and Π_y be the planes parallel and equidistant to L_j and L_k, and to L_r and L_h respectively, $\{j,k,r,h\}=\{1,2,3,4\}$.

p_x be the intersection point of the projections of L_j and L_k on Π_x.

p_y be the intersection point of the projections of L_r and L_h on Π_y.

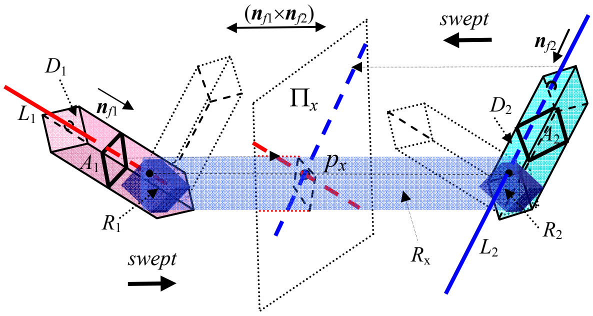


Fig. 5. a) Determination of R_1 and R_2.

s_x be the segment whose extremes are contained in L_j and L_k respectively, parallel to $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ and containing to p_x.

s_y be the segment whose extremes are contained in L_r and L_h respectively, parallel to $\boldsymbol{n}_{fr}\times\boldsymbol{n}_{fh}$ and containing to p_y.

Note that p_x and p_y are the midpoints of s_x and s_y respectively, therefore $p_x=s_x\cap\Pi_x\cap\Pi_y$ and $p_y=s_y\cap\Pi_x\cap\Pi_y$.

A *regulus* grasp of $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ i=1,2,3,4, implies that $\overline{p_x p_y}\subset\Pi_x\cap\Pi_y$ and $\|s_x\|=\|s_y\|$, if any of these two conditions is not satisfied then the resultant torque produced by the applied forces is always non-null. The procedure to determine if the set of faces allows *regulus* grasp is as follow:

1. Compute the region, R_j, of D_j swept by D_k when D_k is displaced in the direction $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ (Figure 5). In the same way, determine the region, R_k, of D_k swept by D_j when D_j is displaced in the direction $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$.
   Since the extremes of s_x belong to L_j and L_k respectively, and $L_j\subset D_j$ and $L_k\subset D_k$, then the extremes of s_x are always contained in R_j and R_k respectively.
   Let R_x be the region formed by all the segments parallel to $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ and whose extremes are contained in R_j and R_k respectively ($s_x\subset R_x$).
2. Repeat the step 1 replacing D_j and D_k by D_r and D_h and $\boldsymbol{n}_{fj}$ and $\boldsymbol{n}_{fk}$ by $\boldsymbol{n}_{fr}$ and $\boldsymbol{n}_{fh}$, respectively, and let R_r and R_h be the corresponding resulting regions.
   Let R_y be the region formed by all the segments parallel to $\boldsymbol{n}_{fr}\times\boldsymbol{n}_{fh}$ and whose extremes are contained in R_r and R_h respectively ($s_y\subset R_y$).
3. Determine the volumes, D_x and D_y, swept by R_x and R_y respectively, when they are displaced in the direction of $\boldsymbol{n}_I=(\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk})\times(\boldsymbol{n}_{fr}\times\boldsymbol{n}_{fh})$ (Figure 6). If $D_x\cap D_y=\varnothing$ then the set of faces is ***non-valid*** for a *regulus* grasps.
   Any straight line parallel to $\boldsymbol{n}_I$ that intersects R_x and R_y is always include in $D_x\cap D_y$, this implies that if $D_x\cap D_y=\varnothing$ then $\overline{p_x p_y}$ is not parallel to $\boldsymbol{n}_I$, therefore $\overline{p_x p_y}\not\subset\Pi_x\cap\Pi_y$ ($\Pi_x\cap\Pi_y // \boldsymbol{n}_I$). Also $\overline{p_x p_y}\subset\Pi_x\cap\Pi_y$ implies necessarily that $p_x\subset R_x\cap D_x\cap D_y$ and $p_y\subset R_y\cap D_x\cap D_y$.
4. Determine the largest and shortest segments, s^1_{mx} and s^1_{nx}, parallel to $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ whose extremes belong to R_j and $R_x\cap D_x\cap D_y$ respectively. In the same way

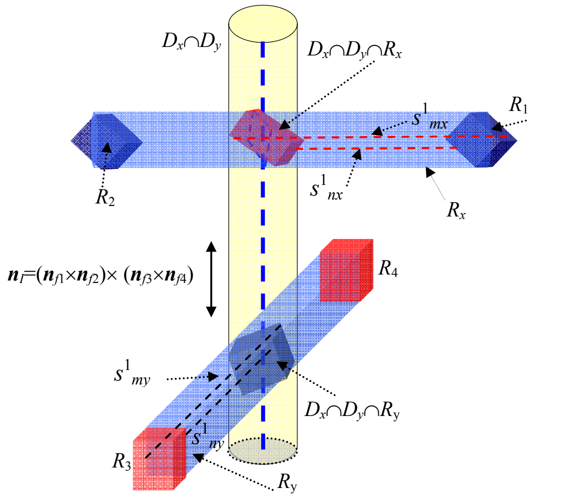


Fig. 6. Determination of a set of faces that allow only *regulus* grasps.

determine the largest and shortest segments, s^2_{mx} and s^2_{nx}, parallel to $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ whose extremes belong to R_k and $R_x\cap D_x\cap D_y$ respectively (note that the four segments are parallel and are include in R_x). If shortest$\{s^1_{mx}, s^2_{mx}\}$< largest$\{s^1_{nx}, s^2_{nx}\}$ then the set of faces is ***non-valid*** for a *regulus* grasps.
   If shortest$\{s^1_{mx}, s^2_{mx}\}$<largest$\{s^1_{nx}, s^2_{nx}\}$ then does not exist a segment parallel to $\boldsymbol{n}_{fj}\times\boldsymbol{n}_{fk}$ that satisfies that its midpoint belong to $R_x\cap D_x\cap D_y$ and its extremes belong to R_j and R_k respectively. This implies that the set of faces do not allow determine s_x (remind that p_x is the midpoint of s_x and $p_x\subset R_x\cap D_x\cap D_y$).
5. Repeat the step 4 replacing D_j and D_k by D_r and D_h and $\boldsymbol{n}_{fj}$ and $\boldsymbol{n}_{fk}$ by $\boldsymbol{n}_{fr}$ and $\boldsymbol{n}_{fh}$, respectively. The resulting segments are identified with the same names but replacing x by y. If shortest$\{s^1_{my}, s^2_{my}\}$< largest$\{s^1_{ny}, s^2_{ny}\}$ then the set of faces is ***non-valid*** to determine *regulus* grasps.
   Let s^*_{my}=largest$\{s^1_{my}, s^2_{my}\}$ s^*_{ny}=shortest$\{s^1_{ny}, s^2_{ny}\}$.
6. If shortest$\{s^*_{mx}, s^*_{my}\}$<largest$\{s^*_{nx}, s^*_{ny}\}$ then the set of faces is ***non-valid*** for a *regulus* grasps. Otherwise it is ***valid***.
   If shortest$\{s^*_{mx}, s^*_{my}\}$<largest$\{s^*_{nx}, s^*_{ny}\}$ then $\|s_x\|=\|s_y\|$ this implies that the distance between L_j and L_k is always different from the distance between L_r and L_h, therefore the torque resultant produced by the applied forces is always non-null.

4. QUALITY OF THE SETS OF FACES

In order to select the set of faces to be contacted by the fingers, all the valid ones are evaluated according to a quality measure that considers:

- The tetrahedron defined by P_1, P_2, P_3 and P_4 should have the maximum possible volume and its centroid should be as close as possible to the object center of mass. This produces better results in front of gravitational forces and torques (Ponce *et al.*, 1997; Liu *et al.*, 1999).
- The forces $\boldsymbol{f}_i$ should have similar modules in absence of external perturbations (i.e. for $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0). This produces a larger range of variation of

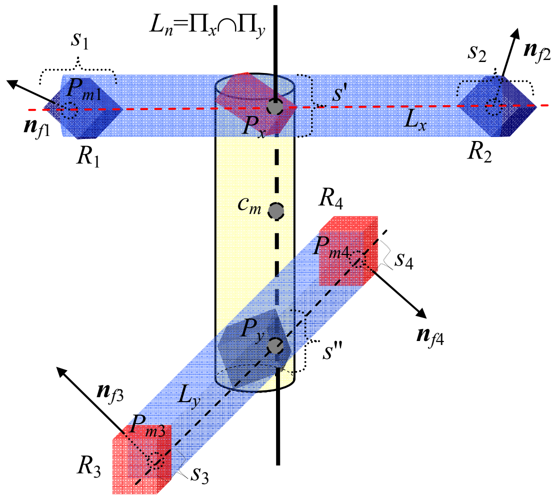


Fig. 7. Determination of a *regulus* grasps.

the applied forces to keep the FCG when external perturbations exist (Nakamura *et al.*, 1989).

The proposed quality function requires to determine, for each valid set of faces, the region C_D that contains all the non-coplanar grasps that allow to apply $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ that reaches the equilibrium. C_D is determined as follows.

- Determine the portion A^d_i, of each face A_i, i=1,2,3,4, as:
  - If the set of faces allows the three types of non-coplanar grasps then A^d_i, is the portion of A_i swept by $D_1 \cap D_2 \cap D_3 \cap D_4$ when it is displaced in the direction of $\boldsymbol{n}_{fi}$.
  - If the set of faces allows two types of non-coplanar grasps then A^d_j and A^d_k are the portions of A_j and A_k swept by $D_x \cap D_y \cap D_j \cap D_k$ when it is displaced in the direction of $\boldsymbol{n}_{fj}$ and $\boldsymbol{n}_{fk}$ respectively. In the same way A^d_r and A^d_h, are the portions of A_r and A_h swept by $D_x \cap D_y \cap D_r \cap D_h$ when it is displaced in the direction of $\boldsymbol{n}_{fr}$ and $\boldsymbol{n}_{fh}$ respectively.
  - If the set of faces only allows *regulus* grasps then A^d_i, is the portion of A_i swept by R_i when R_i is displaced in the direction of $\boldsymbol{n}_{fi}$, i=1,2,3,4.
- C_D=*convexhull*(A^d_1, A^d_2, A^d_3, A^d_4).

Since C_D contains all the non-coplanar grasps that allow to apply $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ that reaches the equilibrium, then the volume of C_D has a direct relationship with the volume of the tetrahedron defined by four contact points of any non-coplanar grasp contained in C_D.

The quality function uses the centroid c_d and the volume V_c of C_D, as well as the distance d_c from c_d to the object center of mass c_m.

The function that returns the quality of a set of faces as a value in the range [0,1] (being 1 the highest quality) is

$$Q = \prod_{i=1}^{3} q_i \tag{1}$$

with:

$$q_1 = \left| \frac{d_{c\max} - d_c}{d_{c\max}} \right| \tag{2}$$

where $d_{c\max}$ is the maximum value of d_c from all the valid sets of faces (q_1 indicates how close is c_m from c_d);

$$q_2 = \left| \frac{V_c}{V_{c\max}} \right| \tag{3}$$

where V_{cmax} is the maximum value of V_c from all the valid sets of faces that allow a FCG (q_2 indicates how close is V_c from V_{cmax});

$$q_3 = \left| \frac{V_e}{V_{e\max}} \right| \tag{4}$$

where V_e is the radius of the largest sphere centered at the origin and included in *convexhull*($\boldsymbol{n}_{f1}$,$\boldsymbol{n}_{f2}$,$\boldsymbol{n}_{f3}$,$\boldsymbol{n}_{f4}$), and V_{emax}=0.333 is the maximum possible value of V_e, and it is obtained in the particular case where the angle between any two $\boldsymbol{n}_{fi}$ is 109°.47'. If q_3=1 then the forces $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$, i=1,2,3,4, have the same modules.

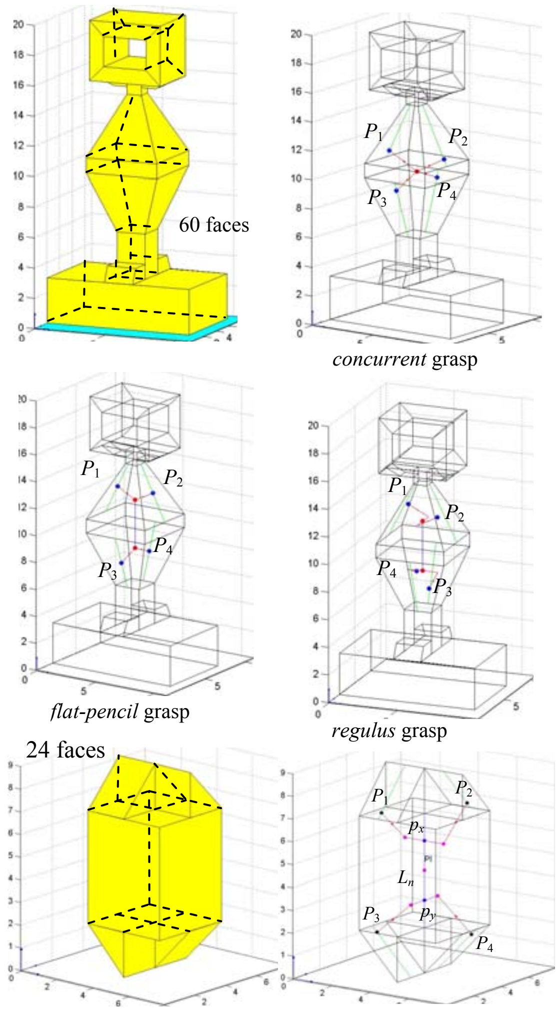
The set of faces with the largest Q is selected for the grasp.

5. DETERMINATION OF THE CONTACT POINTS

The procedure to determine the contact points depends on the type of non-coplanar grasp. The constructions of *concurrent* and *flat-pencil* grasps were already solved in Prado and Suárez (2005b). In that work a *concurrent* grasp is determined such that the intersection point of the lines L_i, i=1,2,3,4, is close to c_m and a *flat-pencil* grasp is determine such that the straight line that contains $L_j \cap L_k$ and $L_r \cap L_h$ with {j,k,r,h}={1,2,3,4} is close to c_m. The procedure described here solves the remaining case determining *regulus* grasps such that $\Pi_x \cap \Pi_y$ is close to c_m.

Let L_n be the straight line closest to c_m, parallel to $\boldsymbol{n}_I$ and intersect with $R_x \cap D_x \cap D_y$ and $R_y \cap D_x \cap D_y$ (Section 3, step 3). The process is as follow:

1. Determine s'=$L_n \cap (R_x \cap D_x \cap D_y)$ and s''=$L_n \cap (R_y \cap D_x \cap D_y)$, with {$j,k,r,h$}={1,2,3,4}. As L_n is close to c_m then is imposed that L_n=$\Pi_x \cap \Pi_y$ and p_x and p_y are the midpoints of s' and s'', respectively. Note that $\overline{p_x p_y} \subset \Pi_x \cap \Pi_y$.
2. Trace two straight lines, L_x and L_y, through p_x and p_y and parallel to $\boldsymbol{n}_{fj} \times \boldsymbol{n}_{fk}$ and to $\boldsymbol{n}_{fr} \times \boldsymbol{n}_{fh}$, respectively. Since $L_x // \boldsymbol{n}_{fj} \times \boldsymbol{n}_{fk}$, $L_y // \boldsymbol{n}_{fr} \times \boldsymbol{n}_{fh}$, $p_x \in R_x$ and $p_y \in R_y$ then $s_j = L_x \cap R_j \neq \varnothing$, $s_k = L_x \cap R_k \neq \varnothing$, $s_r = L_y \cap R_r \neq \varnothing$ and $s_h = L_y \cap R_h \neq \varnothing$.
   Let P_{mi} be the midpoint of s_i, i=1,2,3,4.
3. Compute d_m=*min*(d_{m1},d_{m2},d_{m3},d_{m4}) with $d_{mj}=\left|\overline{P_{mj}P_x}\right|$, $d_{mk}=\left|\overline{P_{mk}P_x}\right|$, $d_{mr}=\left|\overline{P_{mr}P_y}\right|$ and $d_{mh}=\left|\overline{P_{mh}P_y}\right|$.
4. Trace the segment s_x parallel to $\boldsymbol{n}_{fj} \times \boldsymbol{n}_{fk}$, with length $2d_m$ such that p_x is its midpoint, and another segment s_y parallel to $\boldsymbol{n}_{fr} \times \boldsymbol{n}_{fh}$ with length $2d_m$ and p_y as its midpoint.



regulus grasp

Fig. 8. Two examples of FCG obtained with the proposed approach.

Note that the extremes of s_x lay inside $R_j \subset D_j$ and $R_k \subset D_k$ respectively. In the same way the extremes of s_y lay inside $R_r \subset D_r$ and $R_h \subset D_h$ respectively.

5. Trace a straight line through each extreme of s_x and s_y with the direction of $\boldsymbol{n}_{f1}$, $\boldsymbol{n}_{f2}$, $\boldsymbol{n}_{f3}$ and $\boldsymbol{n}_{f4}$ respectively. The intersection points of these straight lines with A_1, A_2, A_3, and A_4 determine P_1, P_2, P_3 and P_4, respectively.

The straight line used to determine P_i, i=1,2,3,4, (step 5) is the line of action of $\boldsymbol{f}_i \in C_{fi}$ with direction of $\boldsymbol{n}_{fi}$, applied in P_i. These forces can be applied to reach the equilibrium in absence of external perturbations, nevertheless, a positive linear combination of these forces may not necessarily to balance any $\boldsymbol{F}_{ex} \neq \emptyset$ and $\boldsymbol{M}_{ex} \neq \emptyset$, but due to friction it can be assured that P_i allow to apply the necessary $\boldsymbol{f}_i \in C_{fi}$ (possibly with directions different from $\boldsymbol{n}_{fi}$) to balance any $\boldsymbol{F}_{ex} \neq \emptyset$ and $\boldsymbol{M}_{ex} \neq \emptyset$.

6. EXAMPLES

Two examples are shown to illustrate the proposed approach. In all the cases it is assume a constant friction coefficient μ=0,36.

Figure 8 shows two objects with the resulting grasping points according to the possible types of non-planar grasp, the *concurrent* and *flat-pencil* grasps are determined according to the approach described in Prado and Suárez (2005b) and the *regulus* grasps according the approach described in this work.

7. CONCLUSION

The approach presented in this paper computes force-closure grasps for polyhedral objects using four contact points frictional on sets of four faces with coplanar or non-coplanar normal. First all the sets of four faces whose relative orientations and positions allow *regulus* grasps are determined. Then from these sets of faces, the one that maximizes a quality function is selected and finally, on the faces of the selected set, four contact points assuring a force-closure grasp are determined for the case of a *regulus* grasps. The approach is based on geometric operations. The time used in the selection of the best one clearly increases with the number of faces, but on the other hand, once the contact faces were selected, the determination of the contact points in not time consuming, with the presented methodology it is possible to compute any type of non-coplanar grasps (*concurrent*, *flat-pencil* and *regulus*).

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