HEURISTIC APPROACH TO CONSTRUCT 3-FINGER FORCE-CLOSURE GRASPS FOR POLYHEDRAL OBJECTS*

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Abstract: In this paper a heuristic approach for the generation of force closure grasps with a three-finger mechanical hand is presented. First, the sets of three object faces that allow a force closure grasp are determined considering their relative orientations and positions; second, these sets are evaluated using a quality function; and third, the contact points of the fingers on the object faces with the best quality are determined using a heuristic that allows a fast computation and ensures a force-closure grasp.

Keywords: Robotics, Mechanical Hands, Grasping, Fixturing.

1. INTRODUCTION

A force-closure grasp has the property to reject external forces and torques exerted on the object using the forces applied by the fingers. The theory regarding force closure grasps has been deeply studied, and different techniques have been proposed depending on the orientation of the faces to be contact -parallel or not-parallel-, the number of fingers, the type of contact -hard or soft finger-, the object shape -concave or convex- (Nguyen, 1989; Ponce et al. 1993; Liu, Din and Wang 1999; Bicchi and Kumar, 2001). Nevertheless, frequently the geometry and kinematics of the hand are not considered, and the selected grasping points can not always be reached by a particular hand. Miller and Allen (1999) have proposed an interesting system for the evaluation of the quality of a grasp, including different concepts and considering the physical and kinematics constraints of a particular hand, but the computational cost is very high and the final hand-object position is strongly conditioned by the initial one. Borst, Fischer and Hirzinger (1999) have used some heuristics to determine the grasping contact points that are then evaluated with the method proposed by Ferrary and Canny (1994), they also proposed a linearization of the friction cone in order to improve the computation time.

The purpose of this work is the generation of force-closure grasps using heuristics to avoid iterative procedures. A good general configuration of the hand in order to obtain a reasonable behaviour in aspects such as different size of the object, range of forces to be applied, possibility of fine movements and the manipulation capability, is shown in Figure 1a (Iberral, 1997). The approach presented in this paper produces grasps considering this initial configuration.

The proposed heuristic method generates a force-closure grasp on any set of faces whose relative orientation and positions allow a force-closure grasp, either being parallel, nonparallel, convex, or concave.

2. ASSUMPTIONS AND BASIC NOMENCLATURE

The following assumptions are considered:

- Three fingers are used.
- Only the fingertips will contact with the object surface and the contact is a point.

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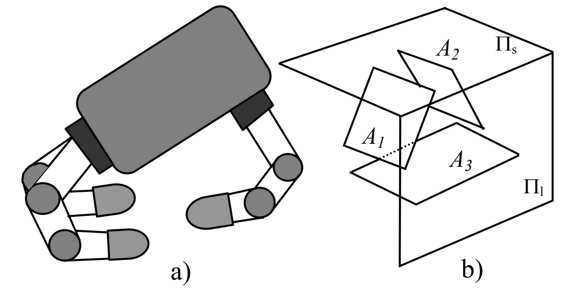


Fig. 1. a) Initial pose of the hand; b) Example of faces to be contacted (A_1, A_2 and A_3) and auxiliary planes ($\prod_s$ and $\prod_l$).

- Each finger contacts with a different object face.
- The friction coefficient μ is constant.

The following basic nomenclature will be used:

A_i: face contacted by finger i, i=1,2,3 (A_3 is the face to be contacted by the thumb, it will be the face whose normal does not form the smallest angle between any two face normals).

$\boldsymbol{n}_i$: unitary normal to A_i pointing inside the object.

Π_s: auxiliary plane parallel to A_3 (Figure 1b).

$\boldsymbol{n}^s_i$: projection of $\boldsymbol{n}_i$ on Π_s, i=1,2.

$\boldsymbol{n}^s_b$: bisecting vector ofthe angle between $\boldsymbol{n}^s_1$ and $-\boldsymbol{n}^s_2$.

Π_l: auxiliary plane orthogonal to Π_s and containing the vector $\boldsymbol{n}^s_b$.

$\boldsymbol{n}^l_i$: projection of $\boldsymbol{n}_i$ on Π_l.

P_i: contact point of finger i on A_i.

c_m: center of mass of the object.

c^s_m: projection of c_m on Π_s.

θ_i: angle between $\boldsymbol{n}_i$ (i=1,2) and $-\boldsymbol{n}_3$

A^J_i: projection of A_i on Π_J, J=s,l.

φ_i: angle between $\boldsymbol{n}_i$ and the plane defined by the extremes of normals of $\boldsymbol{n}_1$, $\boldsymbol{n}_2$, and $\boldsymbol{n}_3$.

3. SELECTION OF THE SET OF CONTACT FACES

The selection of the set of faces to be contacted by the fingers is done in the following three phases:

- Selection of the possible sets of three faces according to their orientations.
- From the resulting sets, selection of the possible sets of faces according to their relative positions.
- Estimation of the quality of the resulting sets of faces and selection of the most appropriate one.

3.1 Selection of faces according to their orientations

In this phase, only the sets of three faces whose orientations allow a force closure grasp are selected, i.e the sets of faces that satisfies the following condition: $\varphi_i<\alpha=tg^{-1}\mu$ (Figure 2a). Assuming the kinematics of an anthropomorphic hand, only the sets of faces with A_1 and A_2 (or part of them) lying in the semi-space defined by the plane containing A_3 where $\boldsymbol{n}_3$ is located are considered, and the cases where $A^s_1\subset A^s_2$ or $A^s_1\supset A^s_2$ are not considered.

Two faces i and j will be called *opposite-parallel* if the angle between $\boldsymbol{n}_i$ and $-\boldsymbol{n}_j$ is smaller than α. Then,

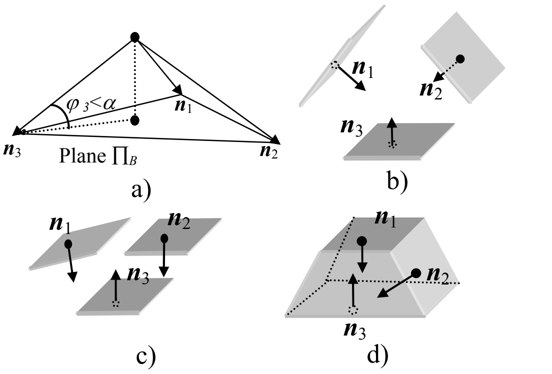


Fig. 2. a) Necessary condition for a force closure grasp; b) sets of *non-parallel* faces; c) sets of *double opposite-parallel* faces; d) sets of *simple opposite-parallel* faces.

the selected sets of three faces whose orientations allow force-closure grasps are included in any of the following types:

1. Sets of *non-parallel* faces. Sets of faces without any opposite-parallel faces satisfying that the projection of the origin on the plane defined by the end points of $\boldsymbol{n}_1$, $\boldsymbol{n}_2$ and $\boldsymbol{n}_3$ lies inside the triangle defined by these end points (Figure 2a and 2b).
2. Sets of *double opposite-parallel* faces. Sets of faces with two pairs of opposite-parallel faces (Figure 2c).
3. Sets of *simple opposite-parallel* faces. Sets of faces with only one pair of opposite-parallel faces (Figure 2d).

3.2 Selection of faces according to their position

In order to obtain force-closure grasps with three fingers it is necessary that the grasp forces applied by the fingers intersect in a point (Ponce et. al, 1993). In this phase, the sets of faces (from those selected in the previous phase) with relative positions that allow the intersection of grasp forces in a point are selected. The process depends on the type of set. In the procedures below, the *projection* with direction d of one polygon A on another polygon B in the same plane is considered to be the part of B swept by A when A is displaced in the direction d.

For a set of non-parallel faces

1. On the plane Π_s:
   1.1. Compute the intersection, $'I^s_{123}$, of the three regions limited by the three pairs of straight lines such that each pair (Figure 3a):
   - Have the direction of $\boldsymbol{n}^s_b$.
   - Pass through the vertices of each A^s_i i=1,2,3, whose components y are the maximum and the minimum (with respect to a reference system {x,y} with x-axis parallel to $\boldsymbol{n}^s_b$) respectively.

   If $'I^s_{123}=\phi$ then the set of faces is rejected (End of process).

   1.2. Compute the projection, A_{is}, of $A^s_i\cap{}'I^s_{123}$ on A_i with direction normal to Π_s (Figure 3b).

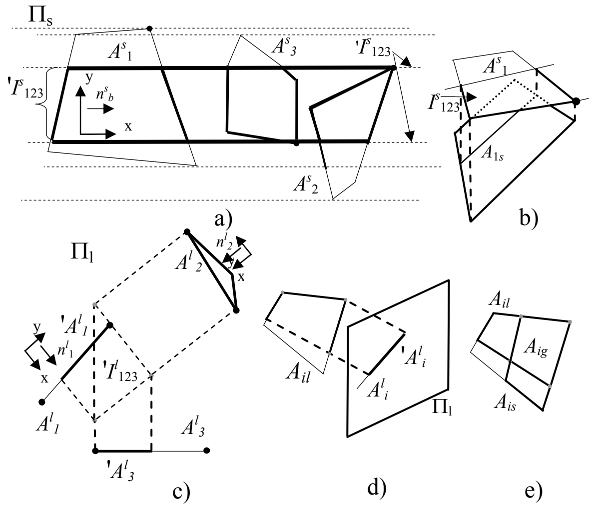


Fig. 3. Selection of faces according to their relative position.

2. On the plane Π_l (Figure 3c):
   2.1 Compute the region $'\Gamma^l_{123}$ in the same way as $'\Gamma^s_{123}$ (step 1.1) but replacing $\boldsymbol{n}^s_b$ by $\boldsymbol{n}^l_i$ (note that now each pair of lines has different direction) and A^s_i by A^l_i. If $'\Gamma^l_{123}=\phi$ then the set of faces is rejected (End of process).
   2.2 Compute the projection, $'A^l_i$, of $'\Gamma^l_{123}$ on A^l_i.
   2.3 Compute the projection, A_{il}, of $'A^l_i$ on A_i with direction normal to Π_l (Figure 3d).
3. Compute $A_{ig}=A_{is}\cap A_{il}$ (Figure 3e). If $A_{ig}=\phi$ then the set is rejected (End of process).
4. Compute the projection, A^s_{ig}, of A_{ig} on Π_s.
5. Compute the region $'\Gamma^s_{ig}$ in the same way as $'\Gamma^s_{123}$ (step 1.1) but replacing A^s_i by A^s_{ig}. If $'\Gamma^s_{ig}=\phi$ then the set of faces is rejected (End of process).
6. The set is accepted as candidate to perform force-closure grasps.

The estimation of the quality of each set of faces, presented below in Section 3.3, will require an auxiliary region Γ^s; for a set of *non-parallel* faces this region is computed as follows:

- Compute a region A^l_{ig} by, first, projecting $A^s_{ig}\cap\Gamma^s_{ig}$ on A_i with direction normal to Π_s and, then, projecting the resulting region on Π_l.
- Compute the region $'\Gamma^l_{ig}$ in the same way as $'\Gamma^l_{123}$ (step 2.1 above) but replacing A^l_i by A^l_{ig}.
- Let c^l_{ig} be the centroid of $'\Gamma^l_{ig}$. Compute the intersection, s^l_i, of A^l_{ig} with a ray from c^l_{ig} with direction of $\boldsymbol{n}^l_i$.
- Compute the projection, s_i, of s^l_i on A_{ig} with direction of the normal to Π_l.
- Compute the projection, s^s_i, of s_i on Π_s.
- Compute the region $'\Gamma^s$ in the same way as $'\Gamma^s_{123}$ (step 1.1 above) but replacing A^s_i by s^s_i.
- Compute the region Γ^s as el convex hull of the regions of A^s_i, i=1,2,3, included in $'\Gamma^s$.

For a set of double opposite-parallel faces

On the plane Π_s:

1. Construct the convex hull U of A^s_1 and A^s_2.

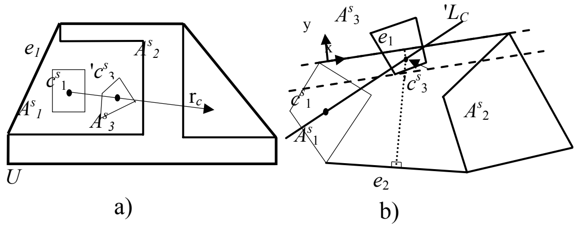


Fig. 4. a) $A^s_1\subset A^s_2$, but the relative positions of A^s_1, A^s_2 and A^s_3 allow force-closure grasps; b) Set of faces in which $'L_c\cap A^s_2=\phi$.

2. Compute $'A^s_3=U\cap A^s_3$. If $'A^s_3=\phi$ then the set of faces is rejected (End of process).
3. If the convex hull of A^s_1 is not included in the convex hull of A^s_2 (or vice versa) or $A^s_2\cap 'A^s_3\neq\phi$ (or respectively $A^s_1\cap 'A^s_3\neq\phi$) then the set of faces is approved (End of process).
4. Consider without lost of generality that the convex hull of A^s_1 is included in the convex hull of A^s_2 or $A^s_2\cap 'A^s_3=\phi$ (Figure 4a). Trace a ray r_c with origin in the centroid, c^s_1, of A^s_1 passing through the centroid, $'c^s_3$, of $'A^s_3$, then
   - If $r_c\cap A^s_2=\phi$ then the set is rejected.
   - If r_c intersects with A^s_2 before passing through $'c^s_3$ then the set is rejected.
   - If r_c intersects with A^s_2 after passing through $'c^s_3$ then the set is accepted.

In the sets of *double opposite-parallel* faces it is not necessary to analyze the projection of the faces on the plane Π_l since the data obtained in the plane Π_s are sufficient to determine if the relative positions of the faces allow force-closure grasps. For the sets of *double opposite-parallel* faces the region Γ^s used below in the quality estimation is computed as:

- Select the region A^s_1 or A^s_2 with smaller area (without lost of generality here it is assumed that $A^s_1\leq A^s_2$).
- Trace the straight line $'L_c$ that passes through the centroids c^s_1 and $'c^s_3$ respectively.
  - If $'L_c\cap A^s_2=\phi$ (Figure 4b) then
    - Select the edge e of U closest to $'c^s_3$ whose extremes are vertices of A^s_1 and A^s_2 respectively
    - Compute $'\Gamma^s$ in the same way as $'\Gamma^s_{123}$ for *non-parallel* faces (step 1.1) but replacing $\boldsymbol{n}^s_b$ by e.
  - If $'L_c\cap A^s_2\neq\phi$ then
    Compute $'\Gamma^s$ in the same way as $'\Gamma^s_{123}$ for *non-parallel* faces (step 1.1) but replacing $\boldsymbol{n}^s_b$ by $'L_c$.
- Compute Γ^s in the same way as in the case of *non-parallel* faces

For a set of simple opposite-parallel faces

Without lost of generality, here it is assumed that A_1 and A_3 form the pair of *opposite-parallel* faces.

On the plane Π_s:

1. Compute $A^s_{13}=A^s_1\cap A^s_3$. If $A^s_{13}=\phi$ then the set of faces is rejected (End of process).
2. Compute $'\Gamma^s_{123}$ in the same way as for *non-parallel* faces (step 1.1), but replacing $\boldsymbol{n}^s_b$ by $\boldsymbol{n}^s_2$

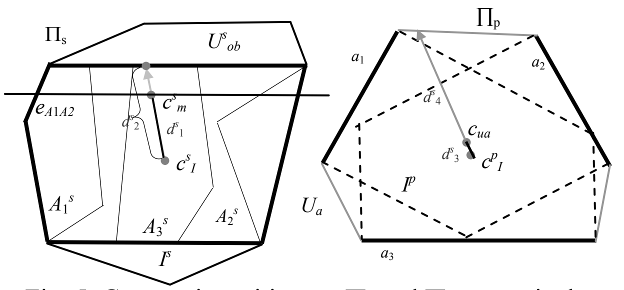


Fig. 5. Geometric entities, on Π_s and Π_p respectively, involved in the quality estimation of a set of faces.

and A^s_1 and A^s_3 by A^s_{13}. If $'I^s_{123}=\phi$ then the set of faces is rejected (End of process).

3. The set of faces is accepted as candidate to perform force-closure grasps.

The auxiliary region I^s is determined by the convex hull of $A^s_{13}\cap 'I^s_{123}$ and $A^s_2\cap 'I^s_{123}$.

3.3. Estimation of the quality of the sets of faces

In this section, a proposal is given to evaluate the quality of the sets of faces that have passed the previous selection process. The quality is estimated considering the relative position between the faces and their location with respect to the center of mass of the object.

The function used for the estimation of the quality of a set of faces uses elements obtained through the following steps (Figure 5):

- Compute an auxiliary straight line e_{A1A2} that
  - passes through c^s_m if $c^s_m \in I^s$, or, passes through the centroid of I^s otherwise.
  - with direction
    - for sets of non-parallel faces: $e_{A1A2} // \boldsymbol{n}^s_b$
    - for sets of *double opposite-parallel* faces:

      $e_{A1A2} // e$ if $'L_c \cap A^s_2 = \phi$

      $e_{A1A2} // 'L_c$ otherwise
    - for sets of *simple opposite-parallel* faces:

      $e_{A1A2} // \boldsymbol{n}^s_2$ if $\boldsymbol{n}^s_1 // \boldsymbol{n}^s_3$

      $e_{A1A2} // \boldsymbol{n}^s_1$ if $\boldsymbol{n}^s_2 // \boldsymbol{n}^s_3$
- Compute the plane Π_p orthogonal to A_3 that contains the straight line e_{A1A2}.
- Compute the projection, $\boldsymbol{n}^p_i$, of $\boldsymbol{n}_i$ on Π_p.
- Compute the segments $a_i = \Pi_p \cap A_i$, i=1,2,3.
- Compute the convex hull, U_a, of a_1, a_2 and a_3.
- Compute the centroid, c_{ua}, of U_a.
- Compute an auxiliary region I^p as,
  - For sets of *double opposite-parallel* faces I^p is the intersection of U_a with the region limited by the two straight lines that pass through the end points of a_3 with the direction of $\boldsymbol{n}^p_3$.
  - For sets of *simple opposite-parallel* faces or *non-parallel* faces I^p is computed in the same way as $'I^s_{123}$ (section 3.2, step 1.1) but replacing A^s_i by a_i and $\boldsymbol{n}^s_b$ by $\boldsymbol{n}^p_i$.
- Compute the centroid, c^p_I, of I^p.
- Construct the convex hull U^J_{ob} of the projection of the object on Π_J, $J=p,s$.
- Compute the projection, c^p_m, of the center of mass of the object on Π_p.
- Compute the centroid, c^s_I, of the auxiliary region I^s (defined in section 3.2).
- Compute $A^s_{ov}=A^s_1\cap A^s_2\cap I^s$.
- Compute the distances d^J_1 from c^J_m to c^J_I, $J=p,s$.
- Compute the distances d^J_2 from c^J_I to the intersection of the boundary of I^J with a ray from c^J_I that passes through c^J_m, $J=p,s$.
- Compute the distance d^p_3 from c^p_I to c_{ua}.
- Compute the distance d^p_4 as
  - For sets of *double opposite-parallel* faces d^p_4 is the distance from c^p_I to the intersection of the boundary of I^p with a ray from c^p_I that passes through c_{ua}.
  - For sets of *simple opposite-parallel* faces or *non-parallel* faces d^p_4 is the distance from c^p_I to the intersection of the boundary of U_a with a ray from c^p_I that passes through c_{ua}.

The function that indicates the quality of a set of faces returns a value in the range [0,1] (being 1 the optimal value) and is given by:

$$Q=\prod_{i=1}^{3} q_i \qquad (1)$$

where q_i is determined as:

1. q_1 indicates whether $c^J_m \in I^J$ ($J=p,s$) and in such a case how close is c^J_m to the centroid of I^J,

$$q_1=\begin{cases}\dfrac{1}{2}\sum_{J=p,s}\left\|\dfrac{d^J_2-d^J_1}{d^J_2}\right\| & \text{if } c^J_m \in I^J\\ 0 & \text{otherwise}\end{cases} \qquad (2)$$

2. q_2 indicates the relations between the areas of I^s with U^s_{ob} and I^p with U^p_{ob},

$$q_2=\frac{1}{2}\left\|\frac{I^s-A^s_{ov}}{U^s_{ob}}+\frac{I^p}{U^p_{ob}}\right\| \qquad (3)$$

3. q_3 indicates the distance from c^p_I to c_{ua} (for *double opposite-parallel* faces if $c_{ua}\notin I^p$ then q_3 is forced to be q_3=0).

$$q_3=\left\|\frac{d^p_4-d^p_3}{d^p_4}\right\| \qquad (4)$$

The set of faces that maximizes Q will be selected for the grasping.

4. DETERMINATION OF THE CONTACT POINTS

The final position of P_i (i=1,2,3) on each face A_i will be chosen on the segment a_i ($a_i=\Pi_p\cap A_i$) that, by construction, is always non-null. Since $\varphi_i<\alpha$ then it is always possible to determine on Π_p a force from the friction cone at each P_i, such that the three forces are coplanar (Section 3.1). The determination of the contact points depends on the type of the selected set of object faces and is described in the following subsections. In order to avoid the contact points to lie on the border of the object faces, each a_i can be shortened a given desired security distance.

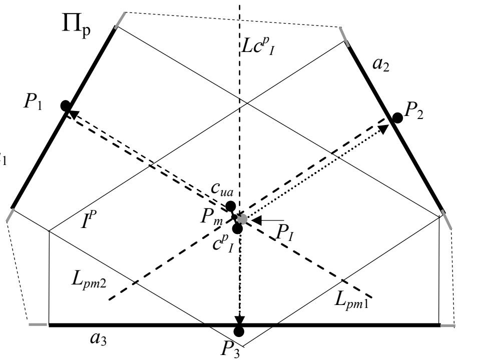


Fig. 6. Example of the determination of P_i for a set of *non-parallel* faces.

4.1 Determination of the contact points: case of non-parallel and simple opposite-parallel faces

The contact points are determined as follows:

1. Compute the midpoint, P_m, of the segment whose end points are c^P_I and c_{ua} (Figure 6).
2. Trace a straight line, Lc^P_I, orthogonal to a_3 through c^P_I.
3. Trace two straight lines, L_{Pm1} and L_{Pm2}, through P_m with directions of $\boldsymbol{n}^P_1$ and $\boldsymbol{n}^P_2$, respectively.
4. Compute $P_{m1}=Lc^P_I \cap L_{pm1}$ and $P_{m2}=Lc^P_I \cap L_{pm2}$.
5. Determine a point P_I as
   - If $P_{m1} \in I^P$ and $P_{m2} \in I^P$ then P_I is equal to P_{m1} or P_{m2}, the one closest to c^P_I .
   - If $P_{m1} \notin I^P$ and $P_{m2} \notin I^P$ then P_I is equal to the solutions of $Lc^P_I \cap I^P$ closest to a_3.
   - If $P_{m1} \in I^P$ and $P_{m2} \notin I^P$ then $P_I = P_{m1}$.
   - If $P_{m2} \in I^P$ and $P_{m1} \notin I^P$ then $P_I = P_{m2}$.
6. Trace three rays from P_I with the directions of $\boldsymbol{n}^P_1$, $\boldsymbol{n}^P_2$ and $\boldsymbol{n}^P_3$, the intersection points of these rays with a_1, a_2 and a_3 determine P_1, P_2 and P_3 respectively.

4.2 Determination of the contact points: case of double opposite-parallel faces

In this case, a sufficient condition to obtain force-closure grasps is that, in Π_p, P_1 and P_2 are separated by the straight line perpendicular to A_3 that passes through P_3 (Park and Starr, 1992; Chen, Walter and Cheatham, 1995). To fulfill this condition the contact points are determined as follows (Figure 7):

- Compute the segments $a'_i \subseteq a_i$ (i=1,2) whose projections on the straight line that contains a_3 do not intersect each other.
- Compute the intersections, Ra'_i (i=1,2), of U_a with the regions determined by the two straight lines that pass through the extremes of a'_i with the direction of $\boldsymbol{n}^P_i$.
- Let $c_{a'i}$ be the centroid of Ra'_i. Compute the projection, c_{a3}, of the centroid c_{ua} of U_a on the straight line that contains a_3
  - If $c_{a3} \in a_3$ then $P_3=c_{a3}$.
  - If $c_{a3} \notin a_3$ then P_3 is the extreme of a_3 closest to c_{a3}.
- Determine the straight line, L_{p3}, orthogonal to a_3 through P_3.

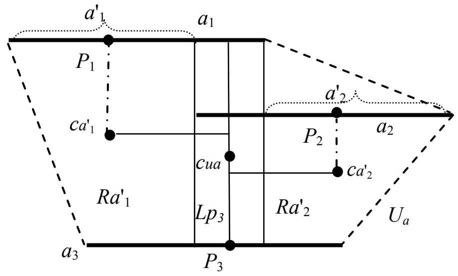


Fig. 7. Example of the determination of P_i for a set of *double opposite-parallel* faces.

- Select $c_{a'1}$ or $c_{a'2}$, the one more far away from L_{p3} (without lost of generality consider that $c_{a'1}$ is selected, being d_m the distance from $c_{a'1}$ to L_{p3}).
- Compute P_1 as the projection of $c_{a'1}$ on a_1.
- Compute the point, P_2^*,of the supporting line of a_2 at a distance d_m of L_{p3}.
  - If $P_2^* \in a_2$ then $P_2=P_2^*$.
  - If $P_2^* \notin a_2$ then P_2 is the extreme of a_2 more far away from L_{p3}.

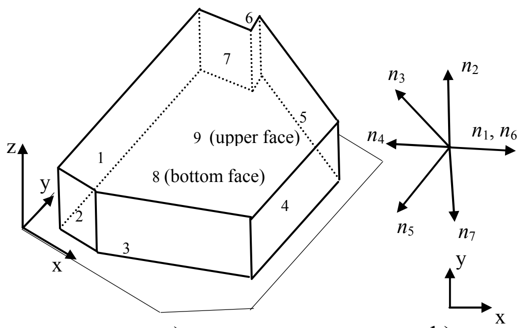
5. NUMERICAL EXAMPLE

In this section, a simple example of the application of the proposed method is given. A force-closure grasp of an object with 9 faces (enumerated as it is shown in Figure 8a) must be generated. A constant friction coefficient μ=0,36 is assumed. The procedure was implemented in Matlab; running on an INTEL Server Biprocessor Pentium III 1,4 GHz it took 7,6 s to determine the three contact points for this example.

Selection of the set of faces according to their orientation. Note that the vectors normal to faces #1 to #7 (Figure 8b) are coplanar and therefore φ_i=0. The object lies on face #8 so it cannot be contacted by the fingers, and for the considered value of μ face #9 does not fulfil the condition $\varphi_i<\alpha$ with any other two faces (section 3.1). Eliminated faces #8 and #9, 35 different sets of three faces can be formed with faces #1 to #7 and, according to the face normal directions, only 16 of these sets (those shown in Table 1) fulfil the requirements to allow the force-closure condition (Section 3.1).

Selection of the set of faces according to their position. The sets {2,4,6}, {3,4,6}, {4,5,6} and {3,5,6} do not have suitable relative positions and therefore they are eliminated, the first three because $U \cap A^s_3 = \phi$ and last one because the corresponding region $'I^I_{123}$ does not exist (Section 3.2). The resulting sets of faces that allow force-closure grasps are listed in Table 2.

Evaluation of the quality of the sets of faces. Table 2 shows the parameters used for the evaluation of each set of faces ($A^s_{ov}=\phi$ and U^P_{ob}=32cm^2 for all the sets so they are not included in the table). Table 3 shows the resulting estimation of the quality of each set of faces, being {1,3,5} the set with maximum quality according to equation (1).



a) b)

Fig. 8. a) Object of 9 faces; b) normal vectors to the object faces (coplanar faces #1 to #7).

Determination of the contact point on each face. {1,3,5} is a set of non-parallel faces, then the procedure described in Subsection 4.1 is applied. A point P_I is determined, rays with directions $\boldsymbol{n}^p{}_1$, $\boldsymbol{n}^p{}_2$ and $\boldsymbol{n}^p{}_3$ are traced from it, and the intersections with segments a_1, a_2 and a_3 produce the contact points, as it is shown in Figure 9 (at half the height of the object).

Table 1. Set of faces selected according to their orientation.

faces	not-parallel	doubles opposite-parallel	simple opposite-parallel	Chosen as A_3
{1,2,4}			✔	1
{1,2,5}	✔			5
{1,2,7}			✔	2
{1,3,4}			✔	1
{1,3,5}	✔			3
{1,3,7}	✔			3
{1,4,5}			✔	1
{1,4,6}		✔		4
{1,4,7}			✔	4
{2,3,7}			✔	7
{2,4,6}			✔	6
{2,4,7}			✔	7
{3,4,6}			✔	6
{3,5,6}	✔			6
{3,6,7}	✔			3
{4,5,6}			✔	6

Table 2. Set of faces selected according to their position, and parameters used for their evaluation.

faces	$d^s{}_1$	$d^p{}_1$	$d^s{}_2$	$d^p{}_2$	$d^p{}_3$	$d^p{}_4$	l^p	l^s	$U^s{}_{ob}$
{1,2,4}	2.5	2.7	8.7	9.2	4.0	10	7.5	21	26.25
{1,2,5}	3.4	3.6	5	7	2.5	8.8	12	15	21.75
{1,2,7}	-	-	-	-	1.5	3.8	10	4.8	16.80
{1,3,4}	1.4	2.5	6.8	9	3.3	8.0	28	20.40	26.25
{1,3,5}	0.4	0.42	7.0	7.8	1.5	8.1	30	26.25	26.25
{1,3,7}	4.0	3.0	4.2	4	0.8	5.0	9.5	15	25.50
{1,4,5}	2.6	2.6	6.8	6.5	1.8	6.5	28	19.5	26.25
{1,4,6}	0.15	0.5	8.8	7.0	1.2	6.6	25	25.95	25.95
{1,4,7}	2.1	2.2	5.8	6.4	3.8	8.7	8.0	15.75	25.95
{2,3,7}	1.9	2.6	6.2	4	4.2	9.5	9.5	15.75	19.35
{2,4,7}	0.3	0.9	7.2	7.2	4.4	10	7.5	20.75	20.75
{3,6,7}	0.1	1.5	2	7.7	6.8	15	4.0	5.7	24.3

Table 3. Quality estimation of the selected sets of faces.

faces	q_1	q_2	q_3	Q
{1,2,4}	0.7095	0.5172	0.6000	0.2202
{1,2,5}	0.4029	0.5323	0.7159	0.1535
{1,2,7}	0	0.3304	0.6053	0
{1,3,4}	0.7581	0.8261	0.5875	0.3679
{1,3,5}	**0.9446**	**0.9688**	**0.8148**	**0.7456**
{1,3,7}	0.1488	0.4426	0.8400	0.0552
{1,4,5}	0.6088	0.8089	0.7231	0.3561
{1,4,6}	0.9558	0.8906	0.8182	0.6965
{1,4,7}	0.6471	0.4285	0.5632	0.1562
{2,3,7}	0.5217	0.5554	0.5579	0.1617
{2,4,7}	0.9166	0.6172	0.5600	0.3168
{3,6,7}	0.8776	0.1798	0.5467	0.0863

6. CONCLUSION

The heuristic method presented in this paper selects a set of faces from all the sets of three faces (concave or convex) whose orientations and relative positions allow force-closure grasps, and then, on the selected faces, determines three contact points which assure that the contact forces applied by the three fingers intersect in a point, thus ensuring a force-closure grasp. The method is based on the use of two auxiliary planes perpendicular to each other where the set of faces to be analyzed are projected. Using simple 2D geometric reasoning it is determined: whether a set of faces is worth to generate force-closure grasps, an estimation of the best set of faces to be contacted by the fingers, and the contact point of each finger on the object.

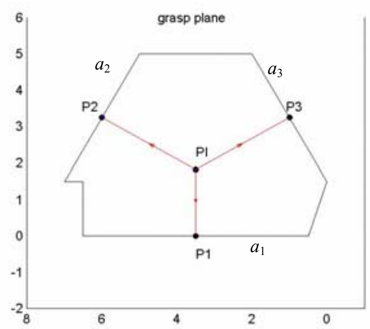


Fig. 9. Determination of P_i, in grasp plane.

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