CONTACT IDENTIFICATION FOR ROBOTIC ASSEMBLY TASKS WITH UNCERTAINTY¹

Jan Rosell* Luis Basañez* Raúl Suárez*

* Institut d'Organització i Control de Sistemes Industrials (UPC) Diagonal 647 Planta 11, 08028 Barcelona, SPAIN Phone: +34 (93) 4016653, Fax: +34 (93) 4016605 e-mails: rosell@ioc.upc.es, basanez@ioc.upc.es, suarez@ioc.upc.es

Abstract: In this paper the effect of modeling and sensing uncertainties in contact situations during a robotic assembly task is analyzed. The paper is focused in the contact identification problem, and it is particularized for planar assembly tasks with two degrees of freedom of translation and one of rotation. The algorithms make use of the configuration sensory information from the robot joints and of the geometric description of the assembly task. *Copyright*[©] 2000 IFAC

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1. INTRODUCTION

Different approaches to the automation of assembly tasks with robots have been proposed. However, nearly all of them face the same problem: the need of identifying the current contact situation from sensory data. This is a difficult problem due, mainly, to the uncertainties that affect the task. The problem is still unsolved for 6 d.o.f. tasks and even there are not many approaches that deal with it in a general way for planar assembly tasks.

This paper presents a procedure to solve this problem for planar assembly tasks using the knowledge of the robot configuration and taking into account modeling and sensing uncertainties. This procedure jointly with a complementary procedure based on force information (Basañez *et al.*, 1996) is part of a two-phase fine-motion planner developed by the authors (Rosell, 1998; Rosell *et al.*, 1999).

2. RELATED WORKS

The representation of a contact state between two polyhedral objects is usually done in terms of the involved geometric elements, i.e. faces, edges and vertices. In this sense, Lozano-Perez (1983) presents the contact states as a set of contact primitives that are defined as vertexedge contacts between 2-D objects, and vertexface and edge-edge contacts between 3-D objects. Desai and Volz (1989) define contact primitives (they call them elemental contacts) as pairs of geometric elements, and contact states as sets of elemental contacts called contact formations. Contact analysis is simpler with these primitives because less primitives are required to describe a contact state. Further on, Xiao (1993) introduces the concept of principal contacts as those elemental contacts necessary for characterizing motion freedom, and the contact formations as sets of principal contacts.

Besides configuration information, force information is also used for contact identification. Hirai and Iwata (1992) and Hirai and Asada (1993) deal with the estimation of contact states from

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force information by using state classifiers based on geometric models of the objects; the approach uses the theory of polyhedral convex cones. Brost and Mason (1989) present the dual representation of forces, which is a method to analyze planar contact problems that represents planar motions and forces by acceleration centers. This graphical method allows the determination of feasible contact motions and the set of forces consistent with them. Other approaches use force information to estimate the contact position when the geometry of the manipulated object is assumed to be unknown (Kitagaki *et al.*, 1993; Hashimoto, 1995).

In the presence of uncertainties Contact identification is even a more complex issue, since several contact states may be compatible with the sensed information.

Desai and Volz (1989) present an algorithm to verify termination conditions of compliant guarded motions which has an static phase and an active phase based on an hypothesis-and-test scheme. In a similar way, Spreng (1995) uses test motions to verify contact hypothesis in terms of motion freedoms.

In another direction, Suárez *et al.* (1995*a*) present a method for planar assembly tasks that computes, for each task state, the set of all the configurations and generalized forces that can be measured when the state occurs, taking into account all the uncertainties affecting the task geometry. The contact identification algorithm also uses the dual representation of forces and includes the uncertainty in the force measurements besides the geometric uncertainties of the task (Basañez *et al.*, 1996). A comparison of this analytical method with some learning methods applied to contact identification can be found in (Suárez *et al.*, 1995*b*).

In order to avoid the complexity of determining the possible contacts in 6 d.o.f. assembly tasks, Xiao and Zhang (1996) and Xiao and Zhang (1997) introduce a method for growing a polyhedral object by its location uncertainties in physical space, and implement an algorithm for finding all principal contacts possibly established between their features.

Other approaches model the assembly tasks as discrete event dynamic systems and focus on the recognition of the contact events. McCarragher *et al.* (1997) use a process monitor based on Hidden Markov Models for this purpose.

A different approach to the estimation of the geometric uncertainties models the point contacts by means of virtual contact manipulators (Dutré *et al.*, 1997), incorporating the geometric uncertainties into the corresponding kinematic model.

3. APPROACH FEATURES

The main characteristics of the approach to contact identification presented in this paper are the following:

- Both modeling and sensing uncertainties are considered.
- The procedure uses the nominal (with no uncertainty) Configuration Space.
- All uncertainties are mapped into a configuration domain associated to the measured configuration for each basic contact.
- The local and global effects of the sources of uncertainty on the geometric elements of the involved polygonal objects are considered for multiple-contact situations.
- Complementary contact situations (those that cannot occur with the nominal geometry) are handled as the nominal ones.

4. SOURCES OF UNCERTAINTY

Manufacturing, manipulation and sensing uncertainties affect the assembly task planning and execution. Manufacturing uncertainties include deviations in the shape and size of the objects and manipulation uncertainties deviations in their positioning. Sensing uncertainties refer to the deviations in the sensory information when the configuration is observed. This section presents the uncertainty models and a study of the contact uncertainty dependences.

4.1 Uncertainty Models

a) Manufacturing tolerances of object shape and size. Each object vertex is supposed to be constrained inside a circle of radius ϵ_t centered on its nominal position. Let (v_{x0}, v_{y0}) and (v_x, v_y) be, respectively, the nominal and the actual vertex position in the object reference system. Then $||(v_x, v_y) - (v_{x0}, v_{y0})|| \leq \epsilon_t$, where ϵ_t will be expressed as ϵ_{t_m} and ϵ_{t_s} to distinguish between the tolerances of the manipulated object and those of the static objects. The effect of manufacturing tolerances can change the shape and size of the object, allowing complementary contact situations.

b) Imprecision in the position of the static objects. It depends on how the objects are positioned in the work environment. It is assumed that part feeders are able to position any static object within tolerances in such a way that the actual position (a_x, a_y) of each vertex lies inside a circle of radius ϵ_s centered on its nominal position (a_{x0}, a_{y0}) , i.e. $||(a_x, a_y) - (a_{x0}, a_{y0})|| \le \epsilon_s$. Therefore, part feeder must have a maximum error $\epsilon_f = \epsilon_s - \epsilon_{t_s}$ and ϵ_{t_s} must satisfy $\epsilon_{t_s} < \epsilon_s$. This source of uncertainty can also give rise to complementary contact situations when there is more than one static object in the work environment.

c) Imprecision in the position and orientation of the robot. Let (x_o, y_o, ϕ_o) and (x_r, y_r, ϕ_r) be the observed and the actual configuration of the robot, i.e. the position and orientation of the gripper reference system with respect to the world reference system. It is assumed that the actual position is inside a circle of radius ϵ_{p_r} centered at the observed position, and that the actual orientation has a maximum deviation ϵ_{ϕ_r} with respect to the observed one, i.e. $||(x_r, y_r) - (x_o, y_o)|| \le \epsilon_{p_r}$ and $|\phi_r - \phi_o| \le \epsilon_{\phi_r}$. The orientation constraint can be rewritten as $\phi_r \in [\phi_{o_m}, \phi_{o_M}]$, where $\phi_{o_m} = \phi_o - \epsilon_{\phi_r}$ and $\phi_{o_M} = \phi_o + \epsilon_{\phi_r}$.

d) Imprecision in the position of the object in the robot gripper. The position of the vertices of the manipulated object depends on the uncertainties from sources (a), (b), (c) and on undesired slippings of the object in the gripper; nevertheless it can be regarded as a source itself since the grasping operation can reduce these uncertainties. It is assumed that any static object within tolerances can be grasped in such a way that the actual position (h_x, h_y) of each vertex lies inside a circle of radius ϵ_m centered on its nominal position (h_{x0}, h_{y0}) , i.e. $||(h_x, h_y) - (h_{x0}, h_{y0})|| \leq \epsilon_m$. Therefore, the grasping operation must have a maximum error $\epsilon_g = \epsilon_m - \epsilon_{t_m}$ and ϵ_{t_m} must satisfy $\epsilon_{t_m} < \epsilon_m$.

4.2 Contact Uncertainty Dependence

Let C_S be a contact situation involving a set S of basic contacts that can simultaneously occur (a basic contact is a vertex-edge contact between two polygons).

The sources of uncertainty are *dependent* if they affect in the same way all the geometric elements involved in S. Given the deviation produced by a dependent source of uncertainty on a geometric element of S, the deviations produced by this source of uncertainty on all the other geometric elements are fixed. The following sources of uncertainty are dependent:

- a) The uncertainty in the position and orientation of the robot.
- b) The uncertainty in the grasping operation.
- c) The uncertainty in the part feeder if all the contacts of S involve the same static object.

The sources of uncertainty are *independent* if they affect each geometric element of S in a different way. Given the deviation produced by an independent source of uncertainty on a geometric element

of S, the deviation produced by this source of uncertainty on all the other geometric elements remains indefinite, although it may be constrained to a subset of all the possible deviations (e.g. the manufacturing tolerances when the involved geometric elements are contiguous). The following sources of uncertainty are independent:

- a) The manufacturing tolerances.
- b) The uncertainty in the part feeder if the contacts of S involve different static objects.

An independent source of uncertainty can give rise to complementary contact situations.

Let:

- ϵ_D : be the maximum deviation in the contact position due to the set of dependent sources of uncertainty affecting the basic contacts of S.
- ϵ_I : be the maximum deviation in the contact position due to the set of independent sources of uncertainty affecting the basic contacts of S.

Considering the above classification of sources of uncertainty and the corresponding models presented in Section 4.1, the values of ϵ_D and ϵ_I are:

a) If the basic contacts involve the same static object:

$$\epsilon_D = \epsilon_{p_r} + \epsilon_f + \epsilon_g \tag{1}$$

$$\epsilon_I = \epsilon_{t_m} + \epsilon_{t_s} \tag{2}$$

b) If the basic contacts involve different static objects:

$$\epsilon_D = \epsilon_{p_r} + \epsilon_g \tag{3}$$

$$\epsilon_I = \epsilon_{t_m} + \epsilon_{t_s} + \epsilon_f = \epsilon_{t_m} + \epsilon_s \qquad (4)$$

5. CONTACT IDENTIFICATION

Let us introduce the following nomenclature:

- $c_o = (x_o, y_o, \phi_o)$: the current observed configuration.
- $d(\phi)$: the distance from (x_o, y_o) to the nominal contact positions of the contact situation C_S for an orientation ϕ .
- $R^i_{\phi} = [\phi^i_m, \phi^i_M]$: the nominal range of contact orientations of the basic contact $i \in S$.
- R_{ϕ}^{S} : the nominal range of contact orientations of C_{S} .
- $f_i(\phi)$: the segment containing the contact positions of basic contact $i \in S$ for orientation ϕ .

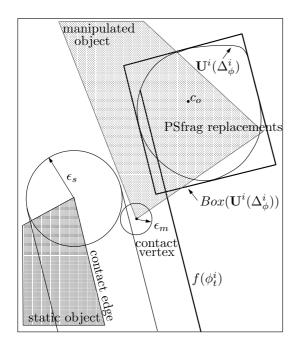


Fig. 1. Single-contact identification.

The following proposition holds (Rosell, 1998): **Proposition:**

 c_o is compatible with the occurrence of C_S iff C_S can take place at the orientation ϕ_o^S that minimizes $d(\phi)$ in $[\phi_{o_m}, \phi_{o_M}]$.

Corollary 1:

The contact identification algorithms need to consider only orientation ϕ_o^S from all the robot uncertainty range $[\phi_{o_m}, \phi_{o_M}]$.

Corollary 2:

Even if $\phi_o^S \notin R_{\phi}^i$, contact $i \in S$ may take place due to the uncertainty in the orientation of the involved edges (the contact edge and the edges adjacent to the contact vertex).

Let Δ_{ϕ}^{i} be the signed distance from ϕ_{o}^{S} to R_{ϕ}^{i} :

$$\Delta_{\phi}^{i} = \begin{cases} 0 & \text{if } \phi_{o}^{S} \in R_{\phi}^{i} = [\phi_{m}^{i}, \phi_{M}^{i}] \\ \phi_{o}^{S} - \phi_{m}^{i} & \text{if } \phi_{o}^{S} < \phi_{m}^{i} \\ \phi_{o}^{S} - \phi_{M}^{i} & \text{if } \phi_{o}^{S} > \phi_{M}^{i} \end{cases}$$
(5)

and let ϕ_t^i be the orientation of R_{ϕ}^i closest to ϕ_o^S , i.e.:

$$\phi_t^i = \phi_o^S - \Delta_\phi^i \tag{6}$$

 Δ_{ϕ}^{i} and ϕ_{t}^{i} are used in the following subsections for the identification of single and multi-contact situations.

5.1 Single-Contact Identification

Let the Contact Position Domain $\mathbf{U}^{i}(\Delta_{\phi}^{i})$ be the set of positions associated to the measured position (x_{o}, y_{o}) , whose intersection with the nominal contact positions of i for ϕ_t^i is non-empty when the occurrence of C_S is compatible with c_o .

The shape of $\mathbf{U}^i(\Delta_{\phi}^i)$ is determined by the fact that it represents all the possible contact positions due to uncertainty associated to the contact at a given point of the contact edge.

The shape of $\mathbf{U}^{i}(\Delta_{\phi}^{i})$ depends on Δ_{ϕ}^{i} . If $\phi_{o}^{S} \in R_{i}$, then $\Delta_{\phi}^{i} = 0$ and $\mathbf{U}^{i}(\Delta_{\phi}^{i})$ is a circle of radius $\epsilon_{D} + \epsilon_{I}$ centered at (x_{o}, y_{o}) :

$$\mathbf{U}^{i}(0) = \mathbf{C}(x_{o}, y_{o}, \epsilon_{D} + \epsilon_{I})$$
(7)

Otherwise, $\mathbf{U}^{i}(\Delta_{\phi}^{i})$ has a more complex shape (Rosell, 1998), because the contact is only possible for some given range of deviations in the orientation of the involved edges. In this latter case, a bounding box, $Box(\mathbf{U}^{i}(\Delta_{\phi}^{i}))$, is computed to simplify the contact identification algorithms.

The Contact Identification algorithm using the bounding box is:

 $\begin{array}{l} \textbf{Single-Contact-Identification}(c_o, \, \textbf{U}^i(\Delta^i_\phi))\\ \textbf{IF} \ \phi^S_o \in R^i_\phi \ \textbf{THEN}\\ \textbf{IF} \ \textbf{C}(x_o, y_o, \epsilon_D + \epsilon_I) \cap f(\phi^i_t) \neq \emptyset \ \textbf{RETURN TRUE}\\ \textbf{ELSE RETURN FALSE}\\ \textbf{ELSE}\\ \textbf{IF} \ Box(\textbf{U}^i(\Delta^i_\phi)) \cap f_i(\phi^i_t) \neq \emptyset \ \textbf{RETURN TRUE}\\ \textbf{ELSE RETURN FALSE}\\ \textbf{END} \end{array}$

As an example, Figure 1 shows a situation where contact is possible since $Box(\mathbf{U}^i(\Delta^i_{\phi})) \cap f_i(\phi^i_t) \neq \emptyset$. Notice that this possibility is due to the uncertainty in the orientation of the involved edges, since $\phi^S_o \notin R^i_{\phi}$ (corollary 2).

5.2 Analysis of the Contact Uncertainty Dependence

For multi-contact situations the dependence or independence of the sources of uncertainty must be considered. First the Contact Position Domain considering the dependent sources of uncertainty (D) is computed for the set Sof basic contacts. Then, a Contact Position Domain considering the independent sources of uncertainty (I) is computed for each basic contact involved in S.

Let first consider the simple case where $\phi_o^S \in R_{\phi}^S$:

- Taking only into account D, the Contact Position Domain of C_S , \mathbf{U}_D^S , is a circle of radius ϵ_D centered at (x_o, y_o) .
- Taking only into account I, the Contact Position Domain of each basic contact $i \in S$, \mathbf{U}_{I}^{i} , is a circle of radius ϵ_{I} centered at (x_{t}, y_{t}) , being $(x_{t}, y_{t}) \in \mathbf{U}_{D}^{S}$ the point such that

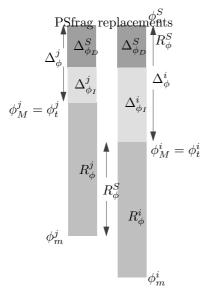


Fig. 2. Ranges of nominal contact orientations $(R^i_{\phi}, R^j_{\phi} \text{ and } R^S_{\phi})$ and signed distances Δ^i_{ϕ} and Δ^j_{ϕ} from ϕ^S_o to R^i_{ϕ} and R^j_{ϕ} , respectively which are due to both dependent and independent sources of uncertainty.

 $\mathbf{U}_{I}^{i}(x_{t}, y_{t})$ is closest to $f_{i}(\phi_{t}^{i}) \forall i \in S$ (Rosell, 1998).

Consider now the case where $\phi_o^S \notin R_{\phi}^S$ (Figure 2). In this case, the deviation that makes the contact situation possible is due to both dependent and independent sources of uncertainty, i.e. Δ_{ϕ}^i has a component due to the dependent sources of uncertainty $(\Delta_{\phi_D}^S)$, which is the same for all contacts of S, and another one due to the independent ones $(\Delta_{\phi_I}^i)$:

$$|\Delta_{\phi}^{i}| = |\Delta_{\phi_{D}}^{S}| + |\Delta_{\phi_{I}}^{i}| \tag{8}$$

Since any pair of values of $|\Delta_{\phi_D}^S|$ and $|\Delta_{\phi_I}^i|$ satisfying (8) is possible, the worst case is assumed, i.e. the values such that the uncertainty in the contact position is maximum (for each contact $i \in S$, the sum of the areas of the associated Contact Position Domains due to the dependent and independent sources of uncertainty is maximum). The algorithm to compute these values can be found in (Rosell, 1998).

Once $\Delta_{\phi_D}^S$ and $\Delta_{\phi_I}^i \quad \forall i \in S$ are computed, $\mathbf{U}_D^S(\Delta_{\phi_D}^S)$ is approximated by the intersection of the bounding boxes of the corresponding Contact Position Domains and each $\mathbf{U}_I^i(\Delta_{\phi_I}^i)$ is computed as any single-contact domain.

5.3 Multi-Contact Identification

 c_o is compatible with the occurrence of C_S iff:

$$\mathbf{U}_{I}^{i}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{i}) \cap f_{i}(\phi_{t}^{i}) \neq \emptyset \quad \forall i \in S \quad (9)$$

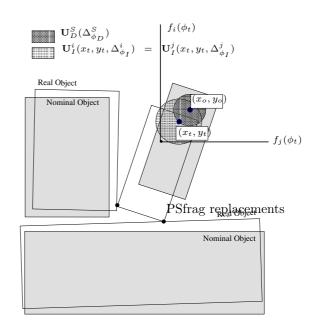


Fig. 3. Effect of dependent and independent sources of uncertainty: The current configuration of the manipulation object is at a possible contact configuration of a nominal two-basic contact situation, since $\mathbf{U}_{I}^{i}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{i}) \cap f'_{i}(\phi_{t}^{i}) \neq \emptyset$ and $\mathbf{U}_{I}^{j}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{i}) \cap f'_{j}(\phi_{t}^{j}) \neq \emptyset$.

where $\mathbf{U}_{I}^{i}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{i})$ are the Contact Position Domains built considering independent sources of uncertainty, and located with respect to $(x_{t}, y_{t}) \in \mathbf{U}_{D}^{S}$, the point such that \mathbf{U}_{I}^{i} is closest to $f_{i}(\phi_{t}^{i}) \forall i \in S$ (Rosell, 1998).

Note that, due to the independent sources of uncertainty, a contact situations C_S may ocurr even if the contact is not possible in the abscence of uncertainty (i.e. then C_S is a complementary contact situation), since equation (9) is independently verified for every contact $i \in S$.

As an example, Figure 3 shows a contact situation with two basic contacts i and j involving different static objects. Since $\phi_o^S \in R_{\phi}^{ij}$ then:

$$\begin{split} \Delta^{i}_{\phi_{I}} &= \Delta^{j}_{\phi_{I}} = \Delta^{S}_{\phi_{D}} = 0\\ \phi^{i}_{t} &= \phi^{j}_{t} = \phi^{S}_{o}\\ \mathbf{U}^{S}_{D}(\Delta^{S}_{\phi_{D}}) \text{ is a circle of radius } \epsilon_{D} \text{ centered at the observed position } (x_{o}, y_{o}).\\ \mathbf{U}^{i}_{I}(x_{t}, y_{t}, \Delta^{i}_{\phi_{I}}) \text{ and } \mathbf{U}^{j}_{I}(x_{t}, y_{t}, \Delta^{j}_{\phi_{I}}) \text{ are circles of radius } \epsilon_{I} \text{ centered at } (x_{t}, y_{t}). \end{split}$$

In this example, the current configuration of the manipulated object may correspond to a contact configuration, since $\mathbf{U}_{I}^{i}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{i}) \cap f_{i}(\phi_{t}^{i}) \neq \emptyset$ and $\mathbf{U}_{I}^{j}(x_{t}, y_{t}, \Delta_{\phi_{I}}^{j}) \cap f_{j}(\phi_{t}^{j}) \neq \emptyset$.

The algorithm to identify the possible occurrence of a given contact situation C_S is the following. It uses the algorithm for the single-contact case for each of the involved contacts in C_S .

$$\begin{split} & \textbf{Multi-Contact-Identification}(c_o, \ C_S) \\ & \textbf{IF} \ C_S \ \text{involves only one basic contact} \ i \ \textbf{THEN} \\ & r = \texttt{Contact-Identification}(c_o, \ \mathbf{U}^i(\Delta_{\phi}^i)) \\ & \textbf{RETURN} \ r \\ & \textbf{ELSE} \\ & \phi_o^S = \texttt{Find-orientation}(c_o, C_S) \\ & \{\Delta_{\phi_D}^S, \Delta_{\phi_I}^i\} = \texttt{Uncertainty-balance}(\phi_o^S) \\ & \{X_t, y_t\} = \texttt{Determine-position}(c_o, \Delta_{\phi_D}^S, \Delta_{\phi_I}^i) \\ & \textbf{FOR} \ i = 1 \ \texttt{TO} \ S \\ & r = \\ & \texttt{Single-Contact-Identification}(c_o, \ \mathbf{U}_I^i(x_t, y_t, \Delta_{\phi_I}^i)) \\ & \textbf{IF} \ r = \texttt{FALSE THEN RETURN FALSE} \\ & \textbf{RETURN TRUE} \\ & \textbf{END} \end{split}$$

6. CONCLUSIONS

The proposed contact-identification procedure has the following main features which make it a good alternative to the other existing methods: capacity to cope with both modeling and sensing uncertainties, consideration of the global and local effects of the uncertainty sources in multi-contact situations, ease to deal with complementary contact situations (i.e. those that cannot occurr with the nominal geometry), and use of the nominal Configuration Space through the definition of Configuration Domains associated to the measured configuration and for each basic contact.

The proposed algorithm has been implemented in C on a Silicon Graphics workstation and graphically tested for different contact situations using different uncertainty values. The accurate results due to the thorough analysis of uncertainty makes it suitable for off-line use during the planning phase of an assembly task to analyze the feasibility of proposed solutions. In order to allow its efficient on-line use during the execution of the assembly tasks, some Configuration Domains have been approximated by bounding boxes.

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