# The Concept of "Difficulty" in Mating Operations *


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## Abstract

*In order to optimize the assembly operations, an optimization index is necessary. In this line, the paper presents a new definition of difficulty for mating operations. This definition considers the geometry of the objects and the strategy used to mate the parts, so that both aspects can be evaluated for the ease of the task. Uncertainties can be considered in a natural way, just taking into account the statistical distribution of the parameters. The application of the definition is illustrated with an example of the cylindrical peg-into-hole assembly task.*


## 1 Introduction

The optimization of any operation requires some indicators that have to be maximized (or minimized) in some sense. This is not the exception in assembly operations, where the main index to be optimized is the "economical cost". But this cost depends on different factors like, for instance, the time needed to perform the assembly, the cost of the necessary equipment, or the quality of the assembly in terms of the reaction forces between the parts during the assembly (i.e. avoiding large interaction forces that can damage the parts).

The common variables that affect the cost of an assembly can be grouped into two types:
a) The cost due to the parts themselves: design (e.g. inclusion of chamfers), physical properties (e.g. the friction coefficient), manufacturing (e.g. tolerances);
b) The cost due to the assembly operation: hardware (e.g. the robot), software (e.g. the strategy), the mating action itself (e.g. time, energy).

These variables have to be combined and balanced to reduce the global cost of the assembly, while keeping a desired rate of success in the assemblability. One of the problems is that they are not independent of each other (for instance, the strategy may be different if the objects to be assembled have chamfers or not), and they are also related to other aspects (for instance, besides both the functionability and the assemblability, the design may take into account the transportation and storage of the parts).

Therefore, the optimization of the global cost of an assembly is quite complex and it is usually done by considering only some of the variables, and assuming that the others have fixed values or are constrained to be in a fixed range. Finally, the real economical cost is also a function of the number of products to be assembled and the amortization of the whole system.

Within this context it is the mating action itself, i.e. the action that joins the parts such that they can be considered successfully assembled, which is the key part of the assembly process. Analyzing the economical cost of the mating action should not be done unless the whole context explained above is considered, so another criterion must be used to optimize (in some sense) the mating operation, whether it is performed with a robot, a special device or just by hand.

For this purpose, there is an intuitive concept that is the "difficulty" of a task. Everybody will have a clear idea if someone else says that one task is more difficult than another one. The problem is: how can the difficulty of a task be quantified? If possible, the difficulty of a task could be used as an optimization index for the mating operation without computing the global cost of the whole assembly process (which should be ultimately done at another level).

Previous works in this line have started with the analysis of manipulability of the assembly device (usually focused on the kinematics of the robot) to determine an index for the dexterity of the device (e.g. [8] [4]). The influence of the geometry of the task was also analyzed (e.g. [9] [2] [5]). Explicit merging of the end effector dexterity and the task constraints was then presented (e.g. [6]). More recently, another index of difficulty of assembly task was presented considering the geometry of the mating objects and the gripper, regardless of the kinematics of the robot [1]; this index is based on a geometric analysis of the tool access to


*During this work Raúl Suárez was at the University of Southern California as a post-doctoral visiting scholar; his stay was supported by the Government of Catalonia.

the target point, the necessary reorientation of the tool and the linear clearance of the gripper.

In the same line, the aim of this work is to introduce a formal definition of the concept of "difficulty" for mating operations, so that it is general enough to be applied in the presence of different sources of uncertainty and based on the geometry of the task and the strategy followed to perform it. The proposed approach was derived from a previous work by the authors [7] dealing with a general cost function for assembly tasks. The relation with the common criteria (for instance, the mating time) used to optimize the economical cost will be discussed later.

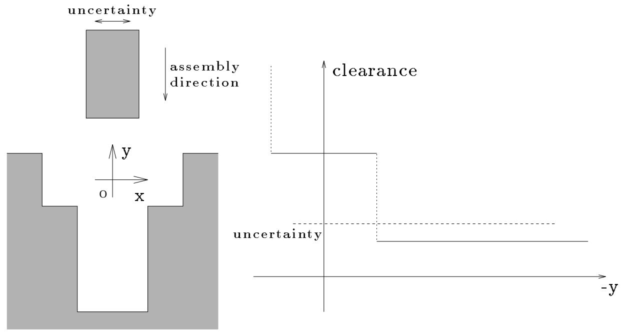

Figure 1: *Relation between clearance and precision: the second reduction is critical for the precision of the positioning system.*

## 2 The Concept of "Difficulty"

Let us first review from an intuitive point of view when an assembly task can be considered to be a "difficult one". There two main sources of difficulty:

1. It is not known which features should be mated.
2. The relative position of the mating features is not precisely known.

The first point is a difficulty when there is no information about the way the parts have to be put together to assemble the final product (like a puzzle). This can be really a problem when there is a large number of complex parts. Nevertheless, in general, this is not a problem for industrial assemblies, since the objects are designed to mate in some particular way that is known when they have to be assembled.

The second point is a difficulty that arises when the uncertainty in the relative position of the mating features is too large to guarantee that the parts can be put together (assembled) without any contact during the assembly operation, i.e. a *clean* assembly is unlikely. The possible problem, due to the contact between the parts, is that they can be jammed before reaching the final position when they are put together following a fixed movement. This is the real source of difficulty in the mating operation itself.

If there is no uncertainty in the position of the mating features, a nominal path can always be followed without any difficulty (note that factors like the length or the shape of the path themselves do not affect the difficulty).

If there is uncertainty in the position of the mating features, it can be compensated for by the clearance between the objects, i.e. if the clearance is large enough, the assembly can be performed with null difficulty. Nevertheless, in practice, uncertainties are always present and the clearances cannot be arbitrarily enlarged due to, for instance, functionality constraints and the fact that the effect of large clearances can be a source of position uncertainty in a chain of parts [7][3].

Usually, problems appear when the clearance is suddenly reduced to a level below the precision of the position knowledge during the assembly movement, as it is illustrated in figure 1. One approach to minimize this effect was found in the progressive decrease in the clearance during the assembly such that the physical constraints to the movement work as guides to properly proceed with the assembly. This progressive reduction of the clearance can be done in two different ways: by properly changing the shape of the parts (e.g. using chamfers), and by looking for suitable assembly strategies (e.g. tilting the peg to "find" the hole at the beginning of the insertion)

The progressive decrease in the clearance allows the use of the reaction forces to adjust the relative position of the objects using passive or active compliance. At this level, the difficulty is related to the ability to use the geometrical constraints as guides for the movement, which includes the suitable use of reaction forces and torques. Friction must be considered at this point because it affects the direction of the reaction forces.

Therefore, the difficulty is a function of how much one object becomes an obstacle for the trajectory of the other object when they are moved to be assembled. Thus, the following aspects determine the difficulty in performing a mating operation:

- Uncertainty in the position of the mating features
  - Precision of the positioning devices
  - Object tolerances
  - Clearances between objects already assembled
- The nominal shape of the parts
- The strategy to perform the assembly

There is another important point related to the difficulty of a task. When the difficulty is evaluated (even in an intuitive way), it is implicitly assumed that the task is going to be successfully performed. If a rate of success smaller than 100% is acceptable, which is the variation in the difficulty of the task? It is clear that the difficulty is related to the desired percentage

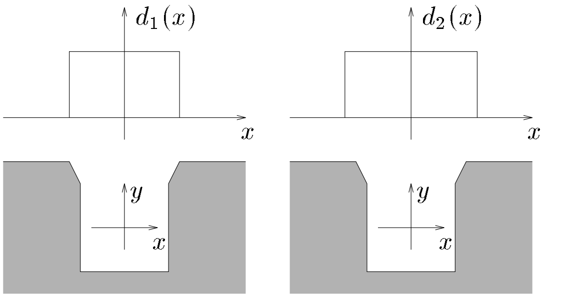


Figure 2: *Same task with different distribution of probability.*

of success in the execution of the task. Let us illustrate the idea with a simple example. Consider the assembly of a peg into a hole with two translational degrees of freedom shown in figure 2. The probability distribution of the position of the peg along $x$-axis is uniform, with two different ranges. In the first case, the range is coincident with the outer width of the chamfer and in the second case it is 20% larger. When using a compliant device, the first case will always be successfully assembled while in the second case the rate of success will be 80%. The difficulty in the second case is larger than in the first one if 100% of success is desired (additional actions have to be taken), but if a rate of success of 80% is accepted, then the difficulty in both cases will be the same.

# 3 Definition of "Difficulty" for mating operations

In this section the intuitive discussion of section 2 is formalized in order to quantify the difficulty of a mating operation.

Let $a$ be a feature of a manipulated object $\mathcal{A}$ that is going to be assembled with a feature $b$ of a static object $\mathcal{B}$. Let $\mathbf{p}_o$ be the initial configuration of $\mathcal{A}$, and $T$ the path followed in Configuration Space from $\mathbf{p}_o$ to a final configuration $\mathbf{p}_f$, in which the assembly is considered to have been successfully completed.

Let $\hat{\mathbf{t}}(\mathbf{p})$ be the direction of the commanded movement of $\mathcal{A}$ at the configuration $\mathbf{p} \in T$, and $\hat{\mathbf{n}}_i(\mathbf{p})$ be the external normal to the C-surface$_i$ if $\mathbf{p}$ is a contact configuration, or the null vector otherwise. Then:

**Definition:** *The difficulty $D$ of the assembly of $\mathcal{A}$ with $\mathcal{B}$ is given by*

$$D = -\int_{along\ T} \left[ \sum_i \hat{\mathbf{t}}(\mathbf{p}) \cdot \hat{\mathbf{n}}_i(\mathbf{p}) \right] \ d\mathbf{p}$$

Note that $\hat{\mathbf{n}}_i(\mathbf{p})$ depends on the geometry of the parts and $\hat{\mathbf{t}}(\mathbf{p})$ on the assembly strategy, i.e. $\hat{\mathbf{t}}(\mathbf{p})$ determines the path $T$, including the effect of friction in the contact configurations. When there is no uncertainty, the difficulty evaluates how well the assembly strategy fits the geometry of the parts. In the absence of uncertainty and without external constraints null difficulty should be achieved.

In the presence of uncertainty not all the variables involved in the definition can be exactly known, but just their statistical distribution of probability; thus, the difficulty must be computed statistically, weighting the difficulty $D$ of each possible solution with the probability of occurrence of that solution. Traditional statistical procedures can be used to compute $D$ in the presence of different sources of uncertainty with different distributions of probability.

Note that the difficulty tends to zero when the strategy allows to arrive always at the goal following a nominal trajectory, the length of the trajectory being irrelevant. On the contrary, the difficulty tends to infinity when hundred per cent of success is desired and the task is unlikely to be solved.

# 4 Relation between difficulty and cost

As it was pointed out in the introduction, the difficulty is not a direct measure of the economical cost of performing a task, which can involve factors other than those strictly related with the mating operation.

Difficulty $D$ is an indicator of the adjustments that are necessary in order to perform the mating operation under certain conditions and with a given strategy. The greater $D$, the greater the time, energy, or interaction forces (typical parameters to be optimized in a mating operation) will be, and therefore it is an indirect measure of the "cost" of the task in terms of these parameters; nevertheless, it is possible, for instance, to solve two tasks with different $D$ in the same time, but increasing then the reaction forces during the operation. As an example, consider the assembly of figure 2. If the strategy is to move the peg straight down with constant velocity and the precision of the system allows the peg to arrive always at the chamfer of the hole, the use of a passive device (RCC) may solve the task in the same time, despite the angle of the chamfer (within some limits), but the interaction force will be higher for chamfers with smaller slopes, which is coincident with the notion that the smaller the slope the larger the difficulty. If a constraint in the module of the reaction force is imposed, then the most difficult task will take more time.

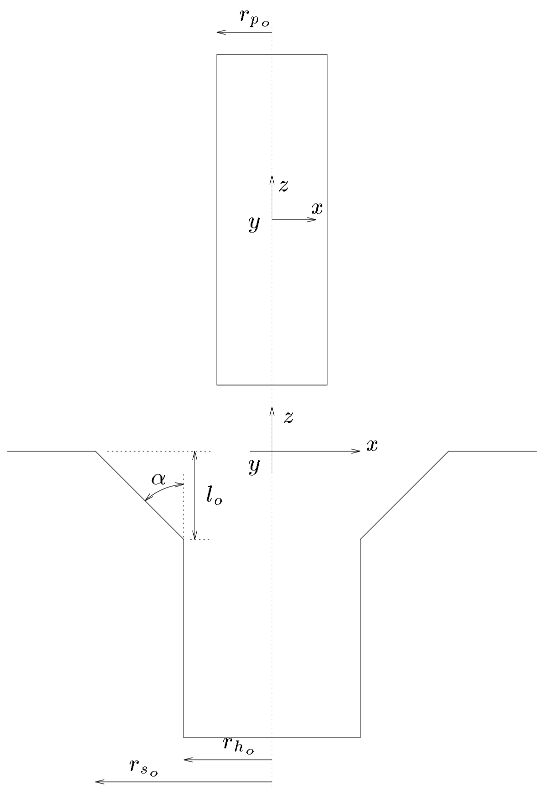


Figure 3: *Peg P into a hole H.*

# 5 Example: assembly of a peg into a hole in the presence of uncertainty

In order to illustrate the application of the proposed index, in this section the statistical difficulty of the assembly of a cylindrical peg $P$ into a cylindrical chamfered hole $H$, considering only translational degrees of freedom, is evaluated.

The nominal sections of the peg and the hole along the symmetry axis are shown in figure 3. According to the nominal models, the symmetry axis of the peg and the hole are initially aligned, and the assembly commanded movement is constant along the negative $z$-axis, i.e. $\hat{\mathbf{t}} = (0, 0, -1)$. The following uncertainties are initially considered:

*a) The position of the peg in the xy-plane.* Let us assume that the accumulation of uncertainty due to previous subassemblies and/or the local positioning of the peg with respect to the hole allows the peg to have configurations $\mathbf{p}_p = (p_x, p_y, p_z)$ with $p_x$ and $p_y$ inside a circle of radius $r_{p_{max}}$ centered on the nominal position $\mathbf{p}_{p_o} = (0, 0, p_{z_o})$, and with probability distribution $d(\mathbf{p}_p) = 3(1 - r/r_{p_{max}})/(\pi r_{p_{max}})$ where $r = \sqrt{x^2 + y^2}$ (figure 4).

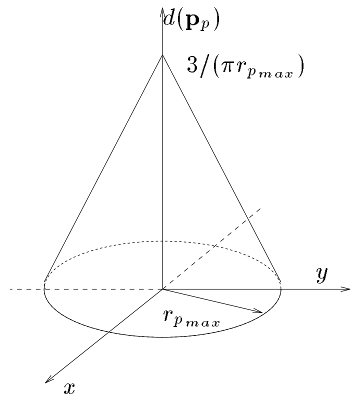


Figure 4: *Probability distribution of the pose of P with respect to H.*

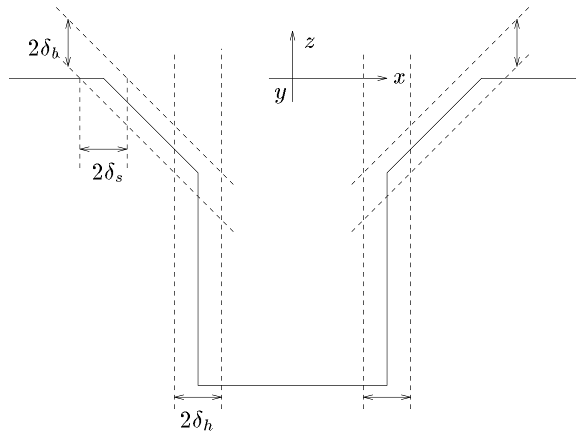


Figure 5: *Tolerances in the size of H.*

*b) The radius of the hole.* Let us assume that the tolerances in the object manufacturing allows the radius $r_h$ of the hole to uniformly vary between $\pm\delta_h$ from the nominal value $r_{h_o}$ (figure 5).

*c) The machining of the chamfer of the hole.* Let us assume that the bit used for the machining of the chamfer has a uniform positioning error along the $z$-axis in the range $\pm\delta_b$ with respect to the nominal position. This will keep the angle $\alpha$ of the chamfer constant and the radius $r_s$ of the top bound of the chamfer will be unknown, but it will have a deviation from the nominal value $r_{s_o}$ bounded by $\pm\delta_s$, being $\delta_s = \delta_b \tan(\alpha)$ (figure 5).

The geometric uncertainties are illustrated in the slice of Configuration Space shown in figure 6.

Since $\hat{\mathbf{t}} = (0, 0, -1)$ is constant, then $-(\hat{\mathbf{t}} \cdot \hat{\mathbf{n}}) = n_z$, $n_z$ being the component of $\hat{\mathbf{n}}$ along $z$-axis. From the definition of $\hat{\mathbf{n}}$ and due to the symmetry of the objects, $n_z$ can be expressed as a function of the distance $r$ from the nominal position as follows (refer to figure 6):

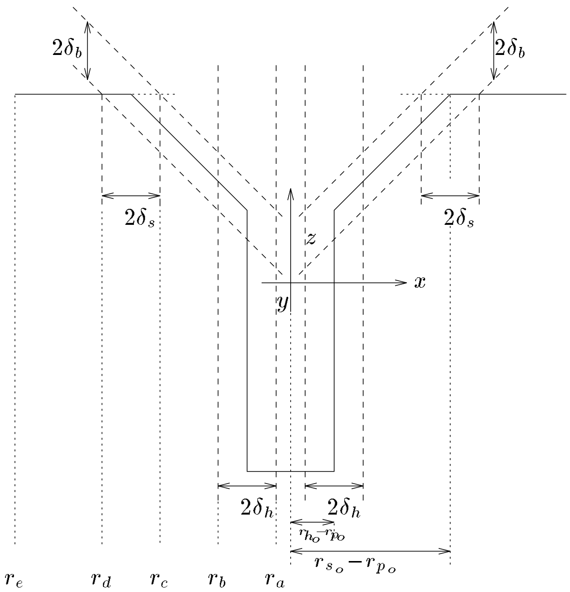


Figure 6: *Geometric uncertainties in Configuration Space.*

$$n_z(r) = \left\{ \begin{array}{lll} n_{z_0} = 0 & & 0 < r < r_a \\ n_{z_0} = 0 \text{ or } n_{z_1} = \sin\alpha & & r_a < r < r_b \\ n_{z_1} = \sin\alpha & \text{if} & r_b < r < r_c \\ n_{z_1} = \sin\alpha \text{ or } n_{z_2} = 1 & & r_c < r < r_d \\ n_{z_2} = 1 & & r_d < r < r_e \end{array} \right\}$$

Then, the difficulty $D(r)$ of the assembly from a distance $r$ is given by:

| interval | difficulty $D(r)$ |
|---|---|
| $[r_a, r_b]$ | if $r_h > r$: $D_{(a,b)}(r) = 0$ |
| | if $r_h \leq r$: $D_{(a,b)}(r) = n_{z_1}(r - r_h)$ |
| $[r_b, r_c]$ | $D_{(b,c)}(r) = n_{z_1}(r - r_d) + D_{(a,b)}(r_b)$ |
| $[r_c, r_d]$ | if $r_s > r$: $D_{(c,d)}(r) = n_{z_1}(r - r_c) + D_{(b,c)}(r_c)$ |
| | if $r_s \leq r$: $\nexists$ solution |
| $[r_d, r_e]$ | $\nexists$ solution |

In some intervals the difficulty is also a function of $r_h$ and $r_s$, which are not known with precision due to uncertainty. In these cases, the probability distributions of $r_h$ and $r_s$ are used to compute the average difficulty and to obtain a difficulty function with $r$ as the only independent variable. Thus, if the assembly is possible,

$$\begin{aligned} D_{(a,b)}(r) &= \int_{r_a}^{r} n_{z_1} \frac{(r - r_h)}{(r_b - r_a)} \, \mathrm{d}r_h = \frac{n_{z_1}(r_a - r)^2}{2(r_b - r_a)} \\ D_{(b,c)}(r) &= n_{z_1}(r - r_b) + D_{(a,b)}(r_b) \\ D_{(c,d)}(r) &= n_{z_1}(r - r_c) + D_{(b,c)}(r_c) \end{aligned}$$

Then, the statistical total difficulty $D$ of the task is

$$\begin{aligned} D &= \frac{1}{(1-F)} \int_0^\infty 2\pi \, r \, d(r) \, D(r) \, \mathrm{d}r = \\ &= \frac{2\pi}{(1-F)} \int_{r_a}^{\min(r_{p_{max}}, r_d)} r \, \frac{3(r - r_{p_{max}})}{\pi r_{p_{max}}^3} \, D(r) \, \mathrm{d}r \end{aligned}$$

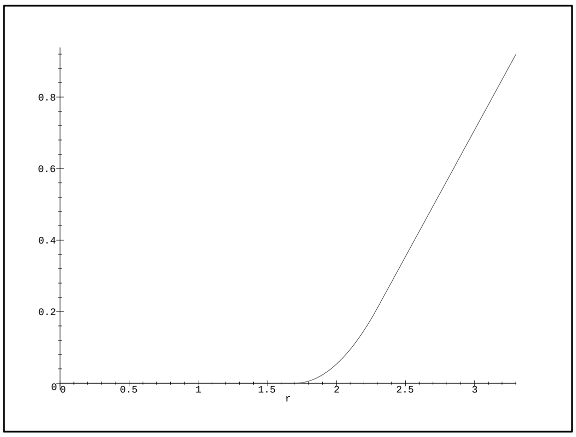


Figure 7: $D(r)$, *difficulty as a function of* $r$.

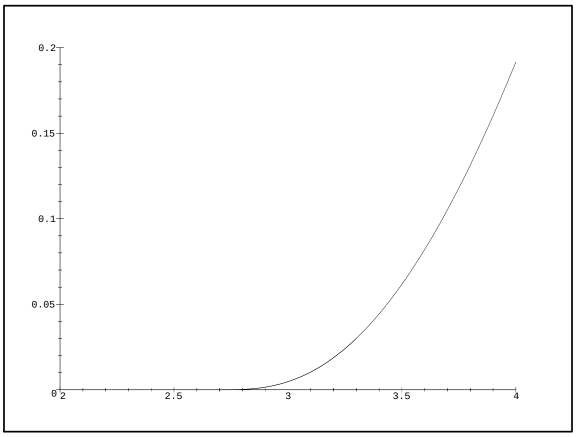


Figure 8: *Probability of failure as a function of* $r_{p_{max}}$.

where $F$, the index of failure for this strategy, is

$$F = \int_{r_c}^{r_{aux}} \int_{r_h}^{r_{p_{max}}} \frac{2\pi r \, d(r)}{r_d - r_c} \, \mathrm{d}r \;\, \mathrm{d}r_h$$

$$\text{with } r_{aux} = \left\{ \begin{array}{lll} r_c & \text{if} & r_{p_{max}} \leq r_c \\ r_{p_{max}} & \text{if} & r_c < r_{p_{max}} \leq r_d \\ r_d & \text{otherwise} & \end{array} \right.$$

## Numerical examples

Being: $r_{p_o} = 1.5$, $r_{h_o} = 2$, $\alpha = \pi/4$, $\delta_h = 0.3$, $\delta_b = 0.3$, $l_o = 1$ and $r_{p_{max}} = 3.0$, then: $\delta_s = 0.3$, $r_{s_o} = 3$, $r_a = 1.7$, $r_b = 2.3$, $r_c = 2.7$, $r_d = 3.3$, $r_e = 5$, $n_{z_0} = 0$ and $n_{z_1} = 1/\sqrt{2}$ and $n_{z_2} = 1$. With these numerical values, the resulting difficulty function $D(r)$ is shown in figure 7, and the statistical total difficulty of the task is $D = .067$, which drops to $D = .016$ if $r_{p_{max}}$ is reduced from 3.0 to 2.5. Figure 8 shows the probability of failure as a function of $r_{p_{max}}$. Finally, figure 9 shows $D(r_{p_{max}})$ for different task conditions.

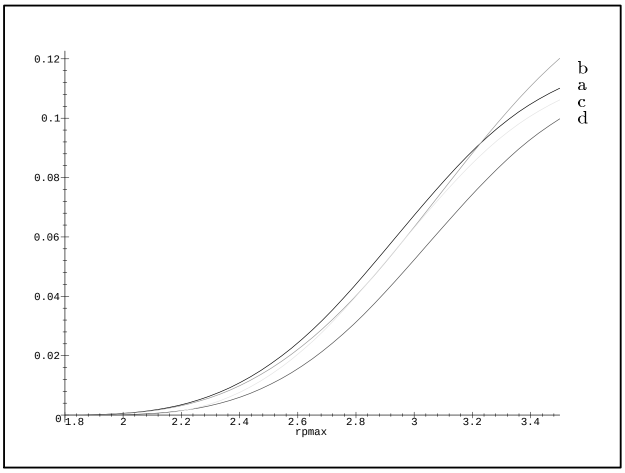


Figure 9: $D(r_{p_{max}})$ *for: a)* $\alpha = 45^o$, $\delta_h = 0.3$, $r_{h_o} = 2$; *b)* $\alpha = 50^o$, $\delta_h = 0.3$, $r_{h_o} = 2$; *c)* $\alpha = 45^o$, $\delta_h = 0.1$, $r_{h_o} = 2$; *and d)* $\alpha = 45^o$, $\delta_h = 0.3$, $r_{h_o} = 2.1$.

# 6 Conclusion

The intuitive concept of difficulty has been formalized for assembly operations. The proposed approach takes into account the factors that are inherent in the mating operation: the geometry of the parts and the strategy to perform the task. Uncertainty can be statistically taken into account.

The approach is based on a reviewed version of the cost measure introduced in [7] such that the movement to solve the task does not need to be on a straight line (constant direction) and the effect of friction is included in a completely different way: instead of affecting a local cost function, in the proposed definition of difficulty it affects the path that the manipulated object will follow in the Configuration Space under a certain assembly strategy.

The difficulty of a task is not a direct index of the economical cost of performing the assembly, but it provides an index to analyze the influence of the object shape, the assembly strategy and the effect of different sources of uncertainty on the optimization of mating operations.

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