Heuristic Grasp Planning with Three Frictional Contacts on Two or Three Faces of a Polyhedron

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Abstract

This paper presents a heuristic approach to the synthesis of force closure grasps of polyhedral objects using three contact points with friction. The approach is valid for sets of three faces as well as for sets of two faces (i.e. two contact points in the same object face). First, the sets of two and three object faces whose relative orientations and positions allow force closure grasps are determined. Second, these sets are evaluated with a quality function and the best one is selected for the grasp. Finally, the grasp contact points that generate a force closure grasp are determined on the selected set of faces. The method uses simple geometric reasoning on the projections of the faces on two orthogonal planes.

1. Introduction

A force-closure grasp is able to reject external forces and torques applied on the grasped object by means of the forces applied by the fingers at the contact points. The theory regarding force-closure grasps has been deeply studied, and different techniques have been proposed depending on: the orientation of the faces to be contacted (parallel or non parallel), the number of fingers, the type the contact (hard or soft fingertips), and the object shape (concave or convex) [1]-[5].

Previous approaches to the problem of determining force closure grasps of polyhedral objects with more than two friction contact points assume that each contact is located on a different object face [4]-[6], and therefore grasps with, for instance, three fingers on two object faces one not considered.

Regarding the stability and robustness of the grasp, it was shown that a set of non-parallel grasping forces produce more stable and robust grasps than a set of parallel ones [7] [8]. Based on this idea, several algorithms were proposed for the determination of force closure grasps for non-parallel faces [1][5][6][8], but they are not applicable for parallel faces. Nevertheless, parallel faces are quite frequent in real objects and sometimes the constraints impose by the task or by the objects themselves force the use of parallel faces for the grasp.

In this paper we present a heuristic approach to the synthesis of force closure grasps of polyhedral objects using three contact points with friction, valid for sets of three faces as well as for sets of two faces (i.e. two contact points in the same object face), being them either parallel or non-parallel. The proposed approach uses heuristics to avoid iterative searching procedures. First, the sets of two and three object faces whose relative orientations and positions allow force closure grasps are determined. Second, these sets are evaluated with a quality function and the best one is selected for the grasp. Finally, the grasp contact points that generate a force closure grasp are determined on the selected set of faces.

2. Assumptions and basic nomenclature

The following assumptions are considered in this work:

- The objects are polyhedrons.
- The grasp is done using three fingers.
- Only the fingertips will contact with the object surface and the contact is a point (then for stability reasons, the contact points cannot be on an object edge).
- The friction coefficient μ is constant.

The following basic nomenclature will be used:

P_i: contact point on the object surface (i=1,2,3).
A_i: contacted face of the object (i=1,2 or i=1,2,3 depending on the number of contacted faces).
$\boldsymbol{n}_i$: object inward unitary vector normal to A_i.
$\alpha=\mathrm{tg}^{-1}\mu$: half-angle of the friction cone ($\alpha<\pi/2$).
C_{fi}: friction cone with half-angle, axis parallel to $\boldsymbol{n}_i$ and vertex at P_i.
C_i: friction cone with half-angle α, axis parallel to $\boldsymbol{n}_i$ and vertex at the origin of the reference system.
$\boldsymbol{f}_i$: contact force applied at contact P_i ($\boldsymbol{f}_i \subset C_{fi}$).
Π_p: grasp plane defined by the three contact points P_i.

3. Force-closure Grasps and Previous Considerations

A force-closure grasp (FCG) must satisfy [3]:

$$\sum_i^n \boldsymbol{f}_i = \boldsymbol{F}_{ex} \qquad \sum_i^n \boldsymbol{r}_i \times \boldsymbol{f}_i = \boldsymbol{M}_{ex} \tag{1}$$

where n is the number of contact points, $\boldsymbol{r}_i$ is the vector from the object center of mass to the contact point P_i, and $\boldsymbol{F}_{ex}$ and $\boldsymbol{M}_{ex}$ are, respectively, any external arbitrary force and torque applied on the object.

The proposed approach determines the three contact points P_i that allow a FCG based on the following proposition:

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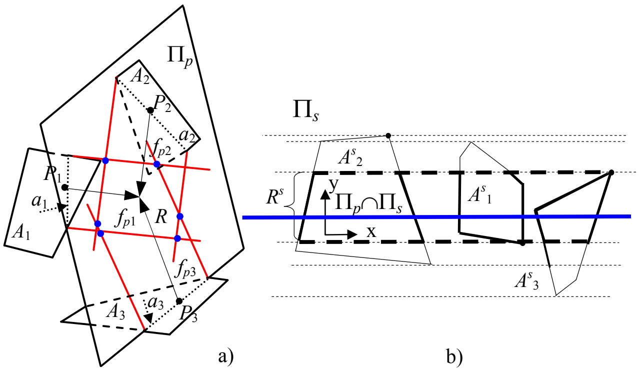
a) b)

Figure 1. a) Determination of R; b) Determination of R^s.

Proposición 1. Three non-colinear contact points P_i, i=1,2,3, allow a FCG if and only if the following three conditions are satisfied:

C1. $\Pi_p \cap (C_{f1} \cap C_{f2} \cap C_{f3}) \neq \varnothing$.

C2. The components $\boldsymbol{f}_{pi}$ of $\boldsymbol{f}_i$ over Π_p i=1,2,3, satisfy at least one of the following two cases:

1. $\boldsymbol{f}_{p1}$, $\boldsymbol{f}_{p2}$ and $\boldsymbol{f}_{p3}$ span Π_p and their supporting lines intersect in a point.
2. $\boldsymbol{f}_{p1}$, $\boldsymbol{f}_{p2}$ and $\boldsymbol{f}_{p3}$ are parallel and that in the middle of the other two has different sense.

C3. The components $\boldsymbol{f}_{gi}$ of $\boldsymbol{f}_i$ orthogonal to Π_p i=1,2,3, have the same sense.

Proof.

Sufficient Condition:

$\boldsymbol{F}_{ex}$ and $\boldsymbol{M}_{ex}$ can be decomposed into two components, one over Π_p and another one orthogonal to Π_p.

The condition *C1* allow the application of contact forces $\boldsymbol{f}_i$ with non-null components $\boldsymbol{f}_{pi}$ satisfying $\boldsymbol{f}_{pi} \subset C_{fi}$ and components $\boldsymbol{f}_{gi}$ with any sense.

The condition *C2* implies that a positive linear combination of the components $\boldsymbol{f}_{pi}$ can balance the component of $\boldsymbol{F}_{ex}$ over Π_p and the component of $\boldsymbol{M}_{ex}$ orthogonal to Π_p.

The condition *C3* implies that a positive linear combination of the components $\boldsymbol{f}_{gi}$ can balance the component of $\boldsymbol{F}_{ex}$ orthogonal to Π_p and the component of $\boldsymbol{M}_{ex}$ over Π_p.

The existence of a proper solution of the two previous positive linear combinations under the constraint $\boldsymbol{f}_i \subset C_{fi}$ can be guarantied just by applying forces $\boldsymbol{f}_i$ with components $\boldsymbol{f}_{pi}$ large enough.

Necessary Condition:

A necessary condition for the existence of a FCG is that equation (1) must be satisfied when $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0 [1][5][11]. In the case of a FCG with three contact points P_i it means that the three applied contact forces $\boldsymbol{f}_i$ must be coplanar [5] and, since they pass though the contact points P_i, this is possible only on the plane Π_p; as a consequence condition *C1* is necessary for the existence of a FCG.

The proof of case 1 of *C2* as necessary condition for the existence of a FCG can be found in [8], and the proof for case 2 of *C2* can be found in [7].

If the three components $\boldsymbol{f}_{gi}$ do not have the same sense then the component $\boldsymbol{f}_{gi}$ with different sense produce, with each of the others, two torques on Π_p with different directions (since the contact points P_i are not collinear these torques cannot be parallel), then any positive linear combination of the components $\boldsymbol{f}_{gi}$ produces a non-null torque over Π_p, and since the components $\boldsymbol{f}_{pi}$ do not produce torques over Π_p the resultant torque of the forces $\boldsymbol{f}_i$ will be always non null, and the grasp will not be a FCG. As a consequence condition *C3* is necessary for the existence of a FCG. ■

4. Selection of the Set of Faces that Allow Force-Closure Grasps

The selection of the object faces that allow a FCG is done in two phases:

1. Selection of faces according to their orientations.
2. Selection of faces according to their positions (from those passing the first phase).

The selection procedures for the sets of three and two faces are described in the following subsections, where the following conditions and auxiliary regions are used.

If $\boldsymbol{f}_{pi}$, i=1,2,3, are non-parallel, their supporting lines must intersect in a point (Proposition 1, condition *C2*). This point belongs to each of the regions bounded each one by two straight lines parallel to each $\boldsymbol{f}_{pi}$ passing through the extremes of the segments $a_i = \Pi_p \cap A_i$ (Figure 1a). Then, the intersection, R, of these regions always satisfies

$$R \neq \varnothing \tag{2}$$

Let A^p_i be the projection of A_i over Π_p. Since $a_i = \Pi_p \cap A_i \Rightarrow a_i \subseteq A^p_i$. Then, the intersection, R^p, of the regions on Π_p bounded each one by two straight lines parallel to each $\boldsymbol{f}_{pi}$ passing through the extremes of A^p_i satisfies $R \subseteq R^p$, and therefore

$$R^p \neq \varnothing \tag{3}$$

In the case of parallel $\boldsymbol{f}_{pi}$ it is necessary to distinguish between the case of a grasp using two faces and the grasp using three faces; for two faces all the previous reasoning is valid and R and R^p must be non null for any FCG, but in the case of parallel $\boldsymbol{f}_{pi}$ applied on three faces R may be null, even for a FCG, and the same may happen with R^p (note that the regions whose intersection determine R are parallel strips and the two lateral ones, corresponding to forces with the same sense, may not intersect each other even for a FCG).

Let Π_s be an arbitrary plane orthogonal to Π_p and A^s_i the projection of A_i over Π_s. The projection of A^p_i over Π_s give a segment on the line $\Pi_p \cap \Pi_s$ that belong to A^s_i (this segment is always non-null, but it can degenerate into a point). Then, it is possible to define a region $R^s \neq \varnothing$ on Π_s bounded by two lines parallel to $\Pi_p \cap \Pi_s$ (Figure 1b), such that:

$$R^s \cap A^s_i \neq \varnothing \tag{4}$$

4.1 Selection of the sets of three faces

Selection of faces according to their orientations.

Let:

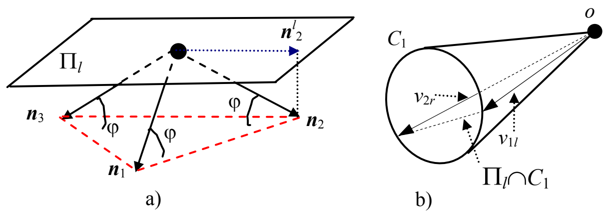

Figure 2. a) Determination of Π_l; b) determination of v_{2r} and v_{1l}.

Π_l be the plane parallel to the one defined by the extremes of $\boldsymbol{n}_1$, $\boldsymbol{n}_2$ and $\boldsymbol{n}_3$ and passing through the origin (Figure 2a).

φ be the angle between $\boldsymbol{n}_i$ and Π_l, i=1,2,3 (note that φ is the same for any i).

$\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ be the two unitary vectors that indicate the two boundary directions of $\Pi_l \cap C_i$ (Figure 2b) respectively (note that $\boldsymbol{v}_{il}=\boldsymbol{v}_{ir}$ when C_i is tangent to Π_l).

***Proposition* 2**. Any vector of $\Re^3$ can be obtained as a linear combination of three vectors, one from each friction cone C_i, if:

1. $\varphi < \alpha$.
2. $\mathbf{0} \in ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r})$.

Proof. If $\varphi \geq \alpha$ then all three cones C_i lie in one of the half-spaces of $\Re^3$ defined by Π_l and therefore vectors in the other half-space can not be obtained as a linear combination of any three vectors from the cones C_i (note that $\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ do not exist for $\varphi > \alpha$ and that $\boldsymbol{v}_{il}=\boldsymbol{v}_{ir}$ for the limit case $\varphi = \alpha$).

If $\varphi < \alpha$ and $\mathbf{0} \notin ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r})$ then the plane Π_l can not be spanned by a linear combination of the components on Π_l of any three vectors from the cones C_i, and therefore some vectors of $\Re^3$ cannot be obtained.

If $\varphi < \alpha$ and $\mathbf{0} \in ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r})$ then the plane Π_l can be spanned by a linear combination of the components on Π_l of three vectors from the friction cones C_i and, at the same time, there are forces in C_i with components orthogonal to Π_l pointing in both senses; as a consequence any vector of $\Re^3$ can be obtained as a linear combination of three vectors, one from each cone C_i. ■

Only sets of three object faces that satisfy the two conditions in Proposition 2 are selected.

Selection of faces according to their positions.

A set of three object faces will be considered as *parallel* for grasping purposes if they allow a FCG that in absence of external perturbations (i.e. satisfy equation (1) for $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0) reaches the equilibrium using parallel forces. This condition is possible if the contact friction cones satisfy $C_i \cap C_j \cap (-C_k) \neq \varnothing$, with $(-C_k)$ representing the negated of C_k, $\{i,j,k\}=\{1,2,3\}$ and the axis of C_k is the normal that does not form the smallest angle between any two normals. The selection of faces according to their positions is done in a different way for parallel and non-parallel sets of faces.

For a set of non-parallel faces. In absence of external perturbations each grasp force must lie on the grasp plane Π_p as well as in the corresponding friction cone C_{fi}, therefore it is interesting to maximize $\Pi_p \cap C_{fi}$, i=1,2,3.

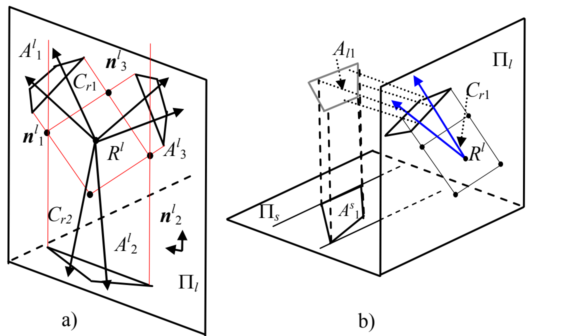

Figure 3. a) Selection on Π_l; b) determination of A_{l1}.

The plane Π_p that maximizes the minimum $\Pi_p \cap C_{fi}$ is parallel to Π_l, and makes $\Pi_p \cap C_{fi}$ to have the same size for any i. For this reason, the condition $\Pi_p // \Pi_l$ is imposed in the grasp search, and it is considered in this selection of object faces. Let the plane Π_s (remember that Π_s is orthogonal to Π_p and therefore also to Π_l) be orthogonal to the projection, $\boldsymbol{n}^l_i$, of any of the three vectors $\boldsymbol{n}_i$ over Π_l (without loss of generality from now on in this subsection it is assumed that $\Pi_s \perp \boldsymbol{n}^l_1$). Now, given a set of three non-parallel faces A_i, i=1,2,3, the procedure to test if this set is valid to produce a force closure grasp is the following:

1. On the plane Π_l (Figure 3a):
   1.1. Compute the projection, A^l_i, of A_i over Π_l $\forall i$.
   1.2. Compute the intersection, R^l, of three regions limited each one by two parallel lines such that for i=1,2,3:
      - The lines are parallel to $\boldsymbol{n}^l_i$.
      - The lines are tangent to A^l_i.

      If $R^l=\varnothing$ (i.e. $R^p=\varnothing$, in equation (3)) then Return (***Invalid***).
   1.3. Compute the projection, A^l_i, of A_i over Π_l $\forall i$.
   1.4. Trace three planar cones, C_{ri}, on Π_l with the origin at the centroid of R^l, axis with the directions of $\boldsymbol{n}^l_i$ (i=1,2,3) and half-angles α-φ.
   1.5. Compute $A^l_i \cap C_{ri}$ (by construction $A^l_i \cap C_{ri} \neq \varnothing$).
   1.6. Compute the portion, A_{li}, of each face A_i whose projection on Π_l gives $A^l_i \cap C_{ri}$.
2. On the plane Π_s (Figure 3b):
   2.1. Compute the projection, A^s_i, of A_{li} over Π_s $\forall i$.
   2.2. Compute the intersection, R^s, of three regions limited each one by two parallel lines such that for i=1,2,3:
      - The lines are parallel to $\Pi_l \cap \Pi_s$.
      - The lines are tangent to A^s_i.

      (note that in this case the six lines are parallel).
      If $R^s=\varnothing$ (see equation (4)) then Return (***Invalid***).
3. Return (***Valid***).

For a set of parallel faces. Without loss of generality we will assume here that A_1 is the face with opposite direction to A_2 and A_3. In this case the procedure uses only the plane Π_s, as follows:

1. If $A^s_1 \cap A^s_2 \neq \varnothing$ or $A^s_1 \cap A^s_3 \neq \varnothing$ (Figure 4a) then Return(***Valid***).

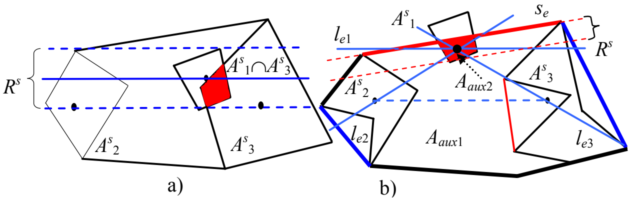

Figure 4. Set of faces with: a) $A^s{}_1 \cap A^s{}_3 \neq \emptyset$; b) $A^s{}_1 \cap A^s{}_3 = \emptyset$, $A^s{}_1 \cap A^s{}_2 = \emptyset$ and $A^s{}_1 \cap A_{aux1} \neq \emptyset$.

2. Compute the auxiliary region A_{aux1}=$ConvexHull(A^s{}_2, A^s{}_3)$-$A^s{}_2$-$A^s{}_3$ that has vertices of $A^s{}_2$ and vertices of $A^s{}_3$. If $A^s{}_1 \cap A^s{}_{aux1} \neq \emptyset$ (Figure 4b) then Return (***Valid***).
3. Return (***Invalid***).

This is a conservative approach because there may be sets of three faces that allow a FCG and they are not considered as valid, but in any case these sets would permit only extreme grasp configurations close to lose the force closure condition.

4.2 Selection of the sets of two faces

Selection of faces according to their orientations.

Let φ be the angle between $\boldsymbol{n}_i$ and the segment defined by $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ (note that φ is the same for i=1 and i=2). A set of two object faces is selected as valid candidate for a FCG according to their orientations if $\varphi < \alpha$.

Selection of faces according to their positions.

Let the plane Π_s (remember that Π_s is orthogonal to Π_p) be orthogonal to the segment defined by $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$, and $A^s{}_i$ be the projection of A_i over Π_s. A set of two object faces is selected as valid candidate for a FCG according to their positions if $ConvexHull(A^s{}_1) \cap ConvexHull(A^s{}_2) \neq \emptyset$.

This is a conservative approach because there may be sets of two faces that allow a FCG and they are not considered as valid, but in any case these sets would permit only extreme grasp configurations close to lose the force closure condition.

5. Determination of the Plane Π_p

The orientation of the plane Π_p and the region R^s (equation (4)) that fixes the position of Π_p assuring the existence of a FCG are determined as follows.

For a set of three faces:

- *For non-parallel faces*. The orientation of Π_p is selected such that $\Pi_p // \Pi_l$ and the region R^s is computed as in the Step 2.2 of Subsection 4.1.
- *For parallel faces*. Let:
  A_{aux2} be the intersection $A^s{}_1 \cap ConvexHull(A^s{}_2, A^s{}_3)$,
  l_{e1} be the straight line parallel to that defined by the centroids of $A^s{}_2$ and $A^s{}_3$ passing through the centroid of A_{aux2},
  l_{ej} be the straight line passing through the centroids of A_{aux2} and $A^s{}_j$, j=2,3.
  - If ($l_{ei} \cap A^s{}_2 \neq \emptyset$ and $l_{ei} \cap A^s{}_3 \neq \emptyset$) is satisfied for any i=1,2,3, then compute the largest segment $s_e = l_{ei} \cap ConvexHull(A^s{}_2, A^s{}_3)$ from those obtained for the values of i that satisfy ($l_{ei} \cap A^s{}_2 \neq \emptyset$ and $l_{ei} \cap A^s{}_3 \neq \emptyset$). Otherwise, compute the edge, s_e, of $ConvexHull(A^s{}_2, A^s{}_3)$ that has vertices of $A^s{}_2$ and vertices of $A^s{}_3$ and that is closer to the centroid of A_{aux2}.
  - Compute the intersection, R^s, of three regions limited each one by two parallel lines such that:
    - The lines are parallel to s_e.
    - The lines are tangent to $A^s{}_i$.

  Then, the orientation of Π_p is selected such that $\Pi_p \perp \Pi_s$ and $\Pi_p // s_e$.

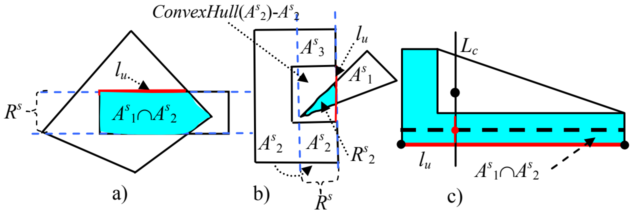

Figure 5. Set of faces with: a) $R^s{}_{max\mathrm{T}} = A^s{}_{12}$; b) $R^s{}_{max\mathrm{T}} = R^s{}_2$; and c) $R^s{}_{max\mathrm{T}} = A^s{}_{12}$ and $c^s{}_m \notin A^s{}_{12}$.

For a set of two faces:

- Compute the intersections:
  - $A^s{}_{12} = A^s{}_1 \cap A^s{}_2$ (Figure 5a).
  - $R^s{}_1$= $(ConvexHull(A^s{}_1)$-$A^s{}_1) \cap A^s{}_2$.
  - $R^s{}_2$= $(ConvexHull(A^s{}_2)$-$A^s{}_2) \cap A^s{}_1$.

  By construction at least one out of $A^s{}_{12}$, $R^s{}_1$ and $R^s{}_2$ is always not null.
- Select from $A^s{}_{12}$, $R^s{}_1$ and $R^s{}_2$ the one with larger area, and call it $R^s{}_{max\mathrm{T}}$.
  - If $R^s{}_{max\mathrm{T}} = A^s{}_{12}$ then let l_u be the largest edge of $A^s{}_{12}$.
  - If $R^s{}_{max\mathrm{T}} = R^s{}_i$, i=1,2, then let l_u be the edge of the region $ConvexHull(A^s{}_i)$-$A^s{}_i$ that contains $R^s{}_i$, and whose extremes are not continuous vertices of $A^s{}_i$.
- Compute R^s as the intersection of two regions limited each one by two parallel lines such that:
  - The lines are parallel to l_u (Figure 5b).
  - The lines are tangent to $A^s{}_i$, i=1,2,3.

Then, the orientation of Π_p is selected such that $\Pi_p \perp \Pi_s$ and $\Pi_p // l_u$. If $R^s{}_{max\mathrm{T}} = R^s{}_i$ for any i=1,2, then the two portions of $A^s{}_i$ that are included in R^s will be considered as corresponding to two different faces, and therefore if we call them $A^s{}_i$ and $A^s{}_3$, respectively. These can be subsequently processed as the sets of three faces. For the same reason, if $R^s{}_{max\mathrm{T}} \neq R^s{}_i$ i=1,2, then $A^s{}_i = A^s{}_3$.

Previous works [5]-[10] have shown that it is a desirable condition that the grasping plane Π_p contains the object center of mass c_m. Then, considering this condition, the position of Π_p is determined such that it contains a specific point for the different cases. Let $c^s{}_m$ be the projection of c_m on Π_s.

For any set of three faces and for sets of two faces with $R^s{}_{max\mathrm{T}} = R^s{}_i$

- If $c^s{}_m \in R^s \cap ConvexHull(A^s{}_1, A^s{}_2, A^s{}_3)$ then Π_p is fixed to contain c_m.
- Otherwise, Π_p is fixed to contain the centroid, $c^s{}_I$, of $R^s \cap ConvexHull(A^s{}_1, A^s{}_2, A^s{}_3)$.

For sets of two faces with $R^s_{max\mathrm{T}} \neq R^s_i$

- If $c^s_m \in A^s_{12}$ then Π_p contains c_m.
- If $c^s_m \notin A^s_{12}$ and the centroid of A^s_{12} lies inside A^s_{12} then Π_p is fixed to contain the centroid of A^s_{12}.
- If $c^s_m \notin A^s_{12}$ and the centroid of A^s_{12} is not inside to A^s_{12} then trace a straight line, L_c, orthogonal to l_u passing through the centroid of A^s_{12}. Π_p is fixed to contain the middle point of $L_c \cap A^s_{12}$ (Figure 5c).

6. Quality of the Sets of Faces

In order to select a set of faces that allows a good FCG, the sets of faces are classified according to a quality measure defined considering:

- The triangle, △, defined by P_1, P_2, and P_3 should have the maximum possible area. It provides a larger dynamic stability [10].
- The centroid of △ should be as close as possible to the object center of mass. It gives a better response to gravitational forces and torques [1][5].
- The grasping forces f_i should have similar modules in absence of external perturbations (i.e. satisfy equation (1) for $F_{ex}=0$ and $M_{ex}=0$). It gives a greater range of variations of the applied forces to keep the FCG when there are external perturbations (i.e. satisfy equation (1) for $\boldsymbol{F}_{ex}\neq 0$ and $\boldsymbol{M}_{ex}\neq 0$) [7][9].

Let:

$\boldsymbol{n}^l_{bs}$ be the vector bisector of $C_i \cap C_j \cap (-C_k) \cap \Pi_l$, $\{i,j,k\}=\{1,2,3\}$ (the axis of C_k is the normal to the face that does not form the smallest angle between any two normals) for a set of three faces or be the vector parallel to the segment defined by $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ for a set of two faces.

I^s be the region $R^s \cap ConvexHull(A^s_1, A^s_2, A^s_3)$.

The proposed quality function uses three parameters d_1, d_2, and d_{ni} that are computed as follows considering the plane Π_p already determined (Section 5):

- Compute a region I^p as,
  - *For sets of three non-parallel faces*. I^p is the intersection of three regions limited each one by two parallel lines such that, for i=1,2,3:
    - The lines are parallel to $\boldsymbol{n}^l_i$ (Section 4).
    - The lines are tangent to a_i (Section 4).
  - *For sets of three parallel faces and* set of *two faces*. I^p is the intersection of the $ConvexHull(a_1,a_2,a_3)$ (note that $a_2=a_3$ for two faces with $R^s_{max\mathrm{T}} \neq R^s_i$) with a region limited by two parallel lines such that:
    - The lines are parallel to $\boldsymbol{n}^l_{bs}$.
    - The lines are tangent to a_1 (in the phase of selection by positions is assumed that the face A_1 has opposite direction to the faces A_2 and A_3).
- Find the centroid c^p_I of I^p.
- Then:
  d_1 is the distance between c^s_I and c^s_m.
  d_2 is the distance between c^p_I and c^p_m.
  d_{ni} is the distance between the extremes of each pair $\boldsymbol{n}^l_i$, i=1,2,3 for sets of three faces, and d_{ni} is the distance between the extremes of each pair $\boldsymbol{n}_i$, for set of two faces (in this case it is considered that $\boldsymbol{n}_2 = \boldsymbol{n}_3$).

Now, the proposed quality function that returns the quality of a set of faces as a value in the range [0,1] (being 1 the highest quality) is

$$Q = \prod_{i=1}^{5} q_i \qquad (5)$$

with

$$q_1 = \left\{ \left| \frac{d_{1\max} - d_1}{d_{1\max}} \right| \right. \qquad (6)$$

where $d_{1\max}$ is the maximum value of d_1 from all the valid sets of faces, q_1 indicate how close is c^s_I from c^s_m;

$$q_2 = \left\{ \left| \frac{d_{2\max} - d_2}{d_{2\max}} \right| \right. \qquad (7)$$

where $d_{2\max}$ is the maximum value of d_2 from all the valid sets of faces, q_2 indicate how close is c^p_I from c^p_m;

$$q_3 = \frac{1}{2}\left(\left| \frac{I^s}{I^s_{\max}} + \frac{I^p}{I^p_{\max}} \right| \right) \qquad (8)$$

where $I^s_{\max}$ and $I^p_{\max}$ are the maximum values of I^s and I^p from all the valid sets of faces, q_3 indicate the area ratio between I^s and I^p with $I^s_{\max}$ and $I^p_{\max}$, respectively;

$$q_4 = \left| 1 - \frac{\varphi}{\alpha} \right| \qquad (9)$$

q_4 indicates how close are the extremes of $\boldsymbol{n}_i$ to Π_p;

$$q_5 = \begin{cases} \begin{cases} \dfrac{d_{min}}{d_{max}} & \text{if } 0 \in convexhull(n^l_i) \\ 0 & \text{otherwise} \end{cases} & non\text{-}parallel \\ \begin{cases} 1\text{-}\dfrac{d_{min}}{d_{max}} & \text{if } 0 \in convexhull(n^l_i) \\ 0 & \text{otherwise} \end{cases} & parallel \\ 1\text{-}\dfrac{d_{min}}{d_{max}} & \text{two faces} \end{cases} \qquad (10)$$

where d_{max} and d_{min} are the maximum and minimum values of d_{ni} i=1,2,3, respectively. q_5 indicates

- For a set of three non-parallel faces, whether the triangle defined by $\boldsymbol{n}^l_1$ $\boldsymbol{n}^l_2$ and $\boldsymbol{n}^l_3$ contains the origin and, if this is the case, which is the relation between the maximum and minimum edges of the triangle.
- For a set of three parallel faces, how close are $\boldsymbol{n}^l_2$ and $\boldsymbol{n}^l_3$ from the supporting line of $\boldsymbol{n}^l_1$.
- For a set of two faces**,** how close is $\boldsymbol{n}_2$ from the supporting line of $\boldsymbol{n}_1$.

The set of faces with largest Q is selected for the grasp.

7. Determination of the Contact Points

The position of the contact point P_i on the face A_i is determined on the segment a_i, i=1,2,3 (note that for a set of two faces the points P_2 and P_3 are on the same face A_2) being $a_i=\Pi_p \cap A_i$. Let:

c_a be the centroid of $ConvexHull(a_1,a_2,a_3)$,

L_i be the straight line on Π_p passing through c^p_I (centroid of I^p) with the direction of $\boldsymbol{n}^l_i$, i=1,2,3.

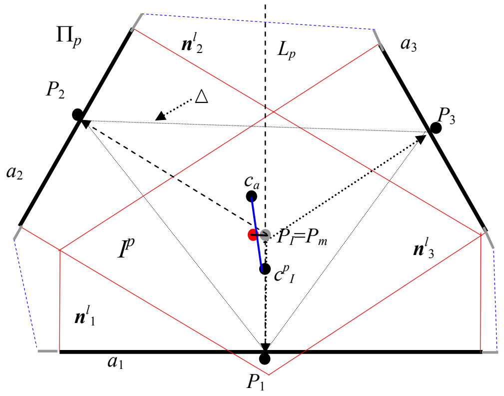

Figure 6. Example of the determination of P_i for a set of non-parallel faces.

The process used to determine each contact point P_i, for each type of set of faces, assures that c_a belongs to the triangle defined by the three points P_i and that is close to the triangle centroid.

In order to avoid contact points close to the boundary of the faces below a security limit, the segments a_i can be shortened a given security distance from their actual vertices. The following subsections describe the procedure for the contact point determination for each type of sets of faces.

7.1 Contact points for three non-parallel faces

In this case the following procedure is used (Figure 6):

1. Compute the straight line L_i, i=1,2,3, closest to c_a. Let L_p be the L_i closest to c_a.
2. Compute the projection, P_m, of the middle point of the segment $\overline{c_a c_I^p}$ on L_p.
3. Determine a point P_I such that:
   - If $P_m \in I^p$ then $P_I = P_m$.
   - If $P_m \notin I^p$ then P_I is the extreme of $L_p \cap I^p$ closest to a_2.
4. Trace three straight lines through P_I with the directions of $\boldsymbol{n}^l_1$, $\boldsymbol{n}^l_2$ y $\boldsymbol{n}^l_3$. Let $'P_1$, $'P_2$ and $'P_3$ be the intersection points of these lines with a_1, a_2, and a_3 respectively.
5. Let $'\triangle$ be the triangle determined by $'P_1$, $'P_2$ and $'P_3$. Then:
   - If $P_I \in {'\triangle}$ then $P_1={'P_1}$, $P_2={'P_2}$ and $P_3={'P_3}$.
   - If $P_I \notin {'\triangle}$ then trace three straight lines through c^p_I with the directions of $\boldsymbol{n}^l_1$, $\boldsymbol{n}^l_2$ y $\boldsymbol{n}^l_3$. The intersection points of these lines with a_1, a_2, and a_3 respectively determine P_1, P_2 and P_3.

7.2 Contact points for three parallel faces and of two faces with $R^s_{max\mathrm{T}} = R^s_i$

In this case the following procedure is used (Figure 7):

1. Compute the intersection, ${}^*P_1$, of the line containing a_1 with the line passing through c_a with direction $\boldsymbol{n}^l_{bs}$.
   - If ${}^*P_1 \in a_1$ then $P_1 = {}^*P_1$.

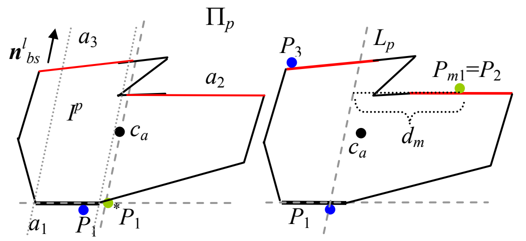

Figure 7. Example of the determination of P_i for a set of parallel faces.

   - If ${}^*P_1 \notin a_1$ then P_1 is the extreme of a_1 closest to ${}^*P_1$.
2. Determine the middle points, P_{m1} and P_{m2}, of the portions of a_2 and a_3 on each side of the straight line, L_p, that passes through P_1 with the direction of $\boldsymbol{n}^l_{bs}$.
3. Select P_2 as the point P_{m1} or P_{m2} that is more far away from L_p.
4. Determine the distance, d_m, from P_1 to L_p.
5. Select a point ${}^*P_3$ on the supporting line of a_3 at a distance d_m from L_p, now:
   - If ${}^*P_3 \in a_3$ then $P_3 = {}^*P_3$.
   - If ${}^*P_3 \notin a_3$ then P_3 is the extreme of a_3 closest to ${}^*P_3$.

7.3 Contact points for two faces with $R^s_{max\mathrm{T}} \neq R^s_i$

In this case the following procedure is used:

1. Compute the segments a_1 and a_2. The largest will contain two contact points (without loss of generality let us consider here that $a_2 \geq a_1$).
2. Compute the intersection, ${}^*P_1$, of the line containing a_1 with the line passing through c_a with direction of $\boldsymbol{n}^l_{bs}$.
   - If ${}^*P_1 \in a_1$ then $P_1 = {}^*P_1$.
   - If ${}^*P_1 \notin a_1$ then P_1 is the extreme of a_1 closest to ${}^*P_1$.
3. Trace the straight line, L_p, passing through P_1 with direction of $\boldsymbol{n}^l_{bs}$.
4. Compute $P_s = L_p \cap a_2$. Let s_{21} and s_{22} be the two parts of a_2 delimited by P_s.
5. Select P_2 as the middle point of s_{21} or s_{22} that is more far away from L_p.
6. Determine the distance, d_m, from P_2 to L_p.
7. Select a point ${}^*P_3$ on the supporting line of a_2 at a distance d_m from L_p in the direction opposite to P_2:
   - If ${}^*P_3 \in a_2$ then $P_3 = {}^*P_3$.
   - If ${}^*P_3 \notin a_2$ then P_3 is the extreme of a_2 closest to ${}^*P_3$.

8. Examples

Six examples are given in order to illustrate the proposed approach. In all the cases it is assume a constant friction coefficient μ=0,3. The implementation was done using Matlab and executed on a server INTEL Biprocessor Pentium III 1,4 GHz. The six objects with numbered faces are shown on the left column of Figure 8 and the objects with the resulting grasping points are shown on the right column of the same figure, even when the implementation is not particularly oriented to time optimization, the required processing time is given in

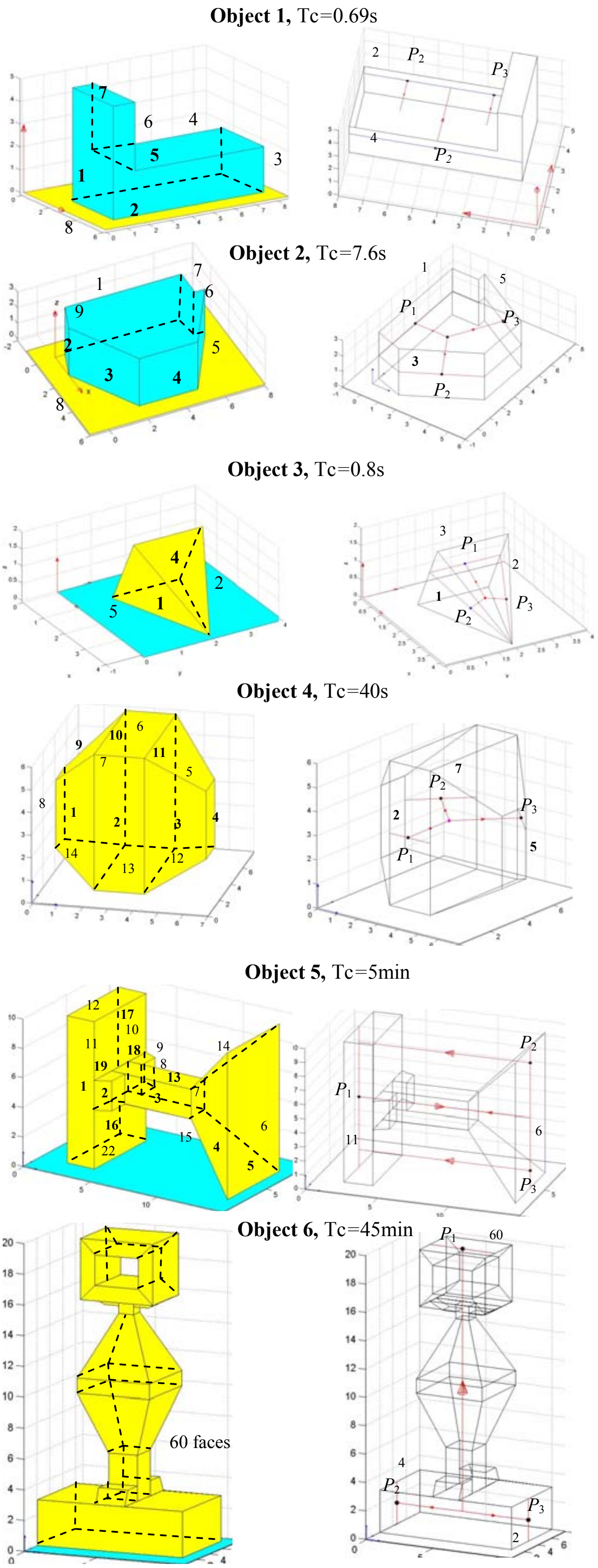

Figure 8. Six examples of FCG obtained with the proposed approach.

each case showing that it strongly increases with the number of faces. Note that the obtained FCG make physical sense. Nevertheless, even when the FCG condition can be assured, no formal measure of the grasp quality was yet implemented (for instance the criteria of the maximum minimum wrench presented in [9]).

9. Summary

A heuristic approach to the determination of force-closure grasps for polyhedral objects using three contact points with friction was presented in the paper. The approach can obtain a force closure grasp on two or three object faces. First, the best set of faces was selected from those whose relative orientations and positions allow force closure grasps, and then, the grasp contact points are determined on the selected set of faces using geometric reasoning and heuristics to avoid iterative procedures. Since all the possible sets of faces are initially considered, the time used in the selection of the best one clearly increases with the number of faces, on the other hand, once the contact faces were selected, the determination of the contact points in not time consuming. The results for different objects show that the obtained grasps make sense and are robust. Future work includes the comparison of the obtained grasps with the optimum one according to different optimizations criteria, which requires the implementation of a procedure to find the optimum grasp.

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