On Computing Form-Closure Grasps/Fixtures for Non-Polygonal Objects [†]

Jordi Cornellà Raúl Suárez
Institut d'Organització i Control de Sistemes Industrials (IOC-UPC)
Av. Diagonal 647 Planta 11, 08028 Barcelona, SPAIN
Emails: jordi.cornella@upc.edu, raul.suarez@upc.edu

Abstract

Form-closure independent regions are parts of the object edges such that a grasp with a finger in each region ensures a form-closure grasp. These regions are useful to provide some robustness to the grasp in the presence of uncertainty as well as in the design of fixtures. The paper presents a new approach to compute independent regions for four frictionless contacts. A sufficient condition is stated and used to obtain combinations of two contact points that allow a form-closure grasp. Then, selecting one of these combinations, a set of four independent regions on the object boundary is determined. An example of the proposed methodology is included in the paper.

1 Introduction

Grasps capable of ensuring the immobility of the object despite external disturbances are characterized by one of the following properties: *form-closure* when the position of the fingers ensures the object immobility, or *force-closure* when the forces applied by the fingers ensure its immobility [1]. These two properties are closely related and many theoretical aspects are valid for both of them. A necessary and sufficient condition that form/force-closure grasps must satisfy and a quality criterion to select a grasp can be found in [2] and [3], respectively. In practice, the difference between form-closure and force-closure grasps relies in the field of application: while form-closure grasps are used to determine robust grasps that do not rely on friction, for instance, the fixture of objects to be manufactured, assembled or inspected in industrial processes, force-closure grasps rely on friction and they are used to manipulate objects with a low number of contact points, for instance, with grippers of two and three fingers.

During the last two decades, a lot of algorithms has been developed to determine form/force-closure grasps of polygonal and polyhedral objects [4][5][6], among others. These algorithms require the linealization of irregular objects, generating a large number of edges or introducing errors in the computation of the finger placements. For irregular objects, algorithms to compute antipodal points (i.e., points whose normal vectors are collinear and in opposite directions) can be found in [7] and [8], algorithms to compute three finger grasps of 2D and 3D objects can be found in [9] and a general algorithm to determine force-closure grasps of 3D objects was proposed in [10]. These algorithms determine contact points and the obtained grasps require a good precision in fingertips placements. In order to provide robustness to the grasp in front of finger positioning errors, the concept of independent regions was introduced in [11] as regions on the object boundary such that a finger in each region ensures a force-closure grasp independently of the exact contact point. The determination of independent regions was treated with different considerations: two fingers and polygonal [11] or non-polygonal [12] objects, four fingers and polygonal objects [11][13], three fingers and polygonal objects [14] and four fingers and polyhedral objects [15]. These works are specific for a given number of fingers. General algorithms to determine all the N-finger force-closure grasps on polygonal objects can be found in [16] and [17]. These algorithms have not been used to compute independent regions, although in [17] the most stable grasp considering finger positioning errors is determined. A general algorithm to obtain independent regions on polygonal objects can be found in [18], and an approach to determine independent regions on 3D objects based on initial examples was proposed in [19].

This paper deals with the problem of determining independent regions on the boundary of non-polygonal objects considering the minimum number of frictionless contacts (four for 2D objects [2]) such that a contact point in each region ensures a form-closure grasp (hereafter FC grasp). The approach developed here uses the knowledge of the antipodal points of the object to determine the combinations of two contact points that allow a FC grasp; then, selecting one of these combinations, a set of four independent regions on the object boundary is determined. Since the

[†]This work was partially supported by the CICYT projects DPI2004-03104 and DPI2002-03540

solution does not rely on friction, the proposed approach is useful to design fixtures for planar objects.

The basic assumptions considered in this work are: *1)* The grasped object is planar; *2)* The boundary of the object is known; *3)* Forces applied by the fingers act only against the object boundary; *5)* The fingertip is a point.

This paper is organized as follows. Section 2 summarizes some results of previous works ([18][20]) obtained for polygonal objects that are the starting point of the developments for non-polygonal objects. Section 3 tackles the problem of finding FC grasps of non-polygonal objects and presents a method to obtain independent regions on the object boundary. An example of the proposed methodology is included in Section 4, and finally, some concluding remarks and possible future lines to extend this work are pointed out in Section 5.

2 Form-Closure Grasps of Polygonal Objects

Consider a polygonal object described as a set of edges. Let $\boldsymbol{f}_i$ be the normalized applied force at a contact point $\boldsymbol{p}_i$ on an edge. In the absence of friction, $\boldsymbol{f}_i$ is normal to the edge and it produces a torque τ_i with respect to the object's center of mass. Since $\boldsymbol{f}_i$ is normalized and its direction is known for each contact edge, there is an univocal relation between the torque produced by the normal force and the exact contact point on the edge. Based on this relation the following terms are defined.

Definition 1: The *Real Range* of τ_i, R_i, is the set of values of τ_i produced by the contact force $\boldsymbol{f}_i$ that are physically possible due to the length of the contact edge. ◇

Definition 2: The *Directional Range* of τ_i, R_{fc_i}, is the set of values of τ_i produced by the contact force $\boldsymbol{f}_i$ that allow a FC grasp for any other given three torques and considering that the contact edge has infinite length (i.e. only the "direction" of the edge is considered). ◇

Let $\boldsymbol{f}_i$, $i=1,...,4$, be the applied forces on the object edges, let $\mathcal{P}_f$ be the polygon defined by these forces in the force space and consider that $\mathbf{0} \in \mathcal{P}_f$ (otherwise a FC grasp is not possible). From the two definitions above, the existence of a FC grasp implies that $\tau_i \in R_i \cap R_{fc_i}$. Since R_i is known, the set of valid torques that produces a FC grasp can be determined by finding R_{fc_i}.

The Directional Range was introduced in [18] and [20], where the following remarks were stated and proved:

1. There are two types of Directional Range: *Infinite* if R_{fc_i} has only one finite extreme and the other tends to $\pm\infty$, and *Limited* if R_{fc_i} has two finite extremes.

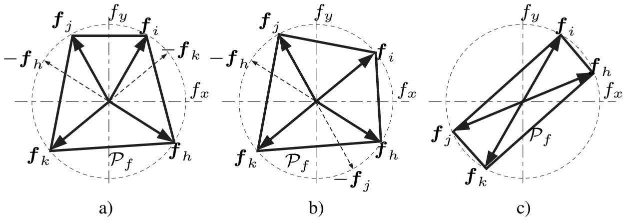

Figure 1: Examples of the determination of the types of Directional Ranges from the applied forces: a)R_{fc_h} and R_{fc_k} are Limited and R_{fc_i} and R_{fc_j} are Infinite; b) R_{fc_k} is Limited and R_{fc_h}, R_{fc_i} and R_{fc_j} are Infinite; c) The four Directional Ranges are Infinite.

2. The number of finite extremes and, therefore, the type of Directional Range R_{fc_i}, can be determined knowing how many pairs $\beta_{i,jk}$ and $\beta_{i,kj}$ are non-positive, being:

$$\beta_{i,jk} = \frac{\sin(\theta_i - \theta_k)}{\sin(\theta_j - \theta_k)} \quad (1)$$

$$\beta_{i,kj} = \frac{\sin(\theta_j - \theta_i)}{\sin(\theta_j - \theta_k)} \quad (2)$$

   where θ_i, θ_j and θ_k are the directions of the forces $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_k$ with $i, j, k \in \{1, 2, 3, 4\}$ and $i \neq j \neq k$.
3. There are always at least two Infinite Directional Ranges that correspond to the torques generated by two forces that define consecutive vertices of $\mathcal{P}_f$ and lie between the negated of the other two forces (Fig. 1 shows different examples).
4. Let R_{fc_i} and R_{fc_j} be two Infinite Directional Ranges with $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ defining two consecutive vertices of $\mathcal{P}_f$. In a FC grasp R_{fc_i} tends to $\pm\infty$ and R_{fc_j} tends to $\mp\infty$.

Based on these four remarks, the following necessary and sufficient condition for the existence of a FC grasp was enunciated [18].

Necessary and sufficient condition: Given four contact edges, four frictionless contacts allow a FC grasp iff:

$$sign(\Gamma_i) \neq sign(\Gamma_j) \quad (3)$$

with

$$\Gamma_\rho = \beta_{\rho,hk}\tau_h + \beta_{\rho,kh}\tau_k - \tau_\rho \quad (4)$$

where $\rho \in \{i, j\}$, τ_i and τ_j have Infinite Directional Ranges and $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ define two consecutive vertices of $\mathcal{P}_f$. The coefficients $\beta_{\rho,hk}$ and $\beta_{\rho,kh}$ are determined from equations (1) and (2) and must be non-positive. ◇

Equation (4) has an useful geometrical property on the object space: the lines of action of the forces whose torques appear in equation (4) intersect at the same point when $\Gamma_\rho = 0$ (note that if the angle between θ_ρ and θ_k or θ_h is π, then $\beta_{\rho,hk} = 0$ or $\beta_{\rho,kh} = 0$, respectively, and only two

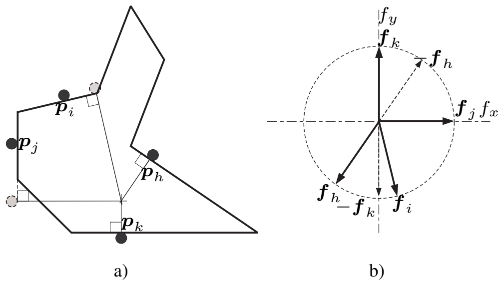

Figure 2: a) Example of a FC grasp of a polygonal object (black points) and the determination of critical points (shaded points); b) Determination of the types of Directional Ranges from the applied forces: R_{fc_h} and R_{fc_k} are limited and R_{fc_i} and R_{fc_j} are Infinite.

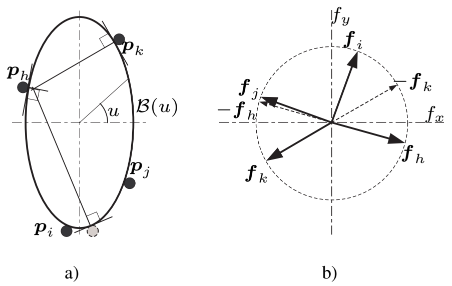

Figure 3: a) Example of a FC grasp of a non-polygonal object (black points) and the determination of a critical point (shaded point); b) Determination of the types of Directional Ranges from the applied forces: R_{fc_h} and R_{fc_k} are limited and R_{fc_i} and R_{fc_j} are Infinite.

torques appear in equation (4)). When $\Gamma_\rho = 0$ the grasp is critical and it separates the FC grasps from the non-FC grasps [13]. An example of a FC grasp of a polygonal object and critical point positions is shown in Fig. 2.

3 Form-Closure Grasps of Non-Polygonal Objects

3.1 Form-Closure Conditions

Let $\mathcal{B}(u)$ be the parametric description of an object boundary ($\mathcal{B}(u)$ is assumed to be a smooth and closed curve). At each point of this curve, $\boldsymbol{p}_i = \mathcal{B}(u_i)$, the tangent and the internal normal vectors are given by $\boldsymbol{t}_i$ and $\boldsymbol{n}_i$, respectively. Therefore, the normalized applied force $\boldsymbol{f}_i$ on the contact point $\boldsymbol{p}_i$ is given by $\boldsymbol{f}_i = \boldsymbol{n}_i / \|\boldsymbol{n}_i\|$.

The necessary and sufficient condition enunciated for polygonal objects is based only on the directions of the applied forces. Given four contact points on the object boundary, the directions of the applied forces are also known, implying that the necessary and sufficient condition developed for polygonal objects can also be applied to check if these contact points allow a FC grasp, as in Fig 3. Based on the necessary and sufficient condition for polygonal objects, the following lemma is enunciated.

Lemma 1: Let θ_h, θ_i, θ_j and θ_k be the directions of the consecutive applied forces on any four points on $\mathcal{B}(u)$, and consider that τ_j and τ_k have Infinite Directional Ranges (as in the necessary and sufficient condition developed for polygonal objects). In a FC grasp, these four directions must satisfy the following relations:

$$0 < \quad \theta_k - \theta_h \quad < \pi \tag{5}$$

$$\theta_h + \pi \leq \quad \theta_\rho \quad \leq \theta_k + \pi \quad \text{with } \rho \in \{i, j\} \tag{6}$$

◇

Proof: From equations (1) and (2) the coefficients $\beta_{\rho,hk}$ and $\beta_{\rho,kh}$ of equation (4) are:

$$\beta_{\rho,hk} = \frac{\sin(\theta_\rho - \theta_k)}{\sin(\theta_h - \theta_k)} \tag{7}$$

$$\beta_{\rho,kh} = \frac{\sin(\theta_h - \theta_\rho)}{\sin(\theta_h - \theta_k)} \tag{8}$$

and from the remark 2 of the previous section they must be non-positive. From equations (7) and (8) it can be checked that the coefficients are non-positive only if equations (5) and (6) are satisfied. ◇

Definition 3: An *opposite point*, $\boldsymbol{p}_i^o$ of $\boldsymbol{p}_i$, is a contact point on the object boundary such that $\boldsymbol{f}_i^o$ and $\boldsymbol{f}_i$ are in opposite directions. ◇

Definition 4: (From [7]) An *antipodal point*, $\boldsymbol{p}_i^a$ of $\boldsymbol{p}_i$, is a contact point on the object boundary such that $\boldsymbol{f}_i^a$ and $\boldsymbol{f}_i$ are in opposite directions and they are collinear (therefore $\boldsymbol{p}_i^a$ is also an opposite point of $\boldsymbol{p}_i$). ◇

Consider a FC grasp formed by $\boldsymbol{p}_\nu = (x_\nu, y_\nu)$, $\nu \in \{h, k\}$ and their opposite points $\boldsymbol{p}_\nu^o = (x_\nu^o, y_\nu^o)$ and let $\boldsymbol{f}_\nu = (f_{x_\nu}, f_{y_\nu})$ be the applied force on $\boldsymbol{p}_\nu$. In this case, the necessary and sufficient condition developed for polygonal objects is equivalent to

$$sign(\Gamma_h) \neq sign(\Gamma_k) \tag{9}$$

with

$$\Gamma_\nu = f_{y_\nu}(x_\nu^o - x_\nu) - f_{x_\nu}(y_\nu^o - y_\nu) \qquad \nu \in \{h, k\} \tag{10}$$

Given two contact points $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$, it is possible to assure that they allow a FC grasp if both of them satisfy equation (9). Otherwise, it is not possible to assure anything. Then, equation (9) is a *sufficient condition* for a FC grasp.

Proposition 1: Considering frictionless contacts, equation (10) gives $\Gamma_\nu = 0$ (and as a consequence the grasp is critical) if and only if the grasp includes two antipodal points $\boldsymbol{p}_\nu^a$. $\diamond$

Proof: The antipodal points are a subset of the opposite points. Then, equation (9) developed for two opposite points, also determines whether two antipodal points can produce a FC grasp. Geometrically, Γ_ν of equation (10) is the distance between the point $\boldsymbol{p}_\nu^o$ and the straight line defined by the vector $\boldsymbol{f}_\nu$ and the point $\boldsymbol{p}_\nu$. Since the directions of two antipodal points are collinear, $\boldsymbol{p}_\nu^a$ of $\boldsymbol{p}_\nu$ belongs to the straight line defined by $\boldsymbol{f}_\nu$ and $\boldsymbol{p}_\nu$. As a result, $\Gamma_\nu = 0$ implying a critical grasp. $\diamond$

Proposition 2: Let $\boldsymbol{p}_h^a$ and $\boldsymbol{p}_k^a$ be two consecutive points with antipodal points on the object boundary. The result of equation (10) has the same sign for any point between $\boldsymbol{p}_h^a$ and $\boldsymbol{p}_k^a$. $\diamond$

Proof: From Proposition 1, only antipodal points make equation (10) equal to zero. Therefore, since the sign of equation (10) can change only at the antipodal points and the object boundary is closed, all the points between two consecutive antipodal points make the solution of equation (10) to have the same sign. $\diamond$

Given the parametric description of the object boundary $\mathcal{B}(u)$, it is possible to determine whether two contact points $\boldsymbol{p}_\nu = \mathcal{B}(u_\nu)$ with $\nu \in \{h, k\}$ satisfy equation (9) and, therefore, allow a FC grasp avoiding exhaustive search procedures given the antipodal points on $\mathcal{B}(u)$ (algorithms to compute antipodal points can be found in [7] and [8]). Let the $hk-$space be the 2D space defined by the parameters that fix the position of two contact points. The values of these parameters that define antipodal points make a partition of the $hk-$space into rectangular cells (Fig. 4). From Proposition 2, the result of equation (10) has the same sign for any contact point between two consecutive antipodal points. Then, all the combinations of any two contact points that belong to the same cell of the $hk-$space satisfy equation (9), defining a FC cell, or do not satisfy equation (9), meaning that it is a cell where it is not possible to assure a FC grasp. The FC cells are determined checking a combination of two contact points from each cell of the antipodal grid (black cells in Fig 4.b). The other cells require a more exhaustive analysis and they are discarded, implying a conservative and linear approximation (in Fig 4.b the discarded regions that actually allow a FC grasp are shadowed).

3.2 Independent regions

In the previous subsection, the cells where it is possible to obtain a FC grasp considering only two contact points were determined. In this subsection, an approach to select the other two contact points and a procedure to obtain independent regions on the object boundary that allow a FC grasp are presented.

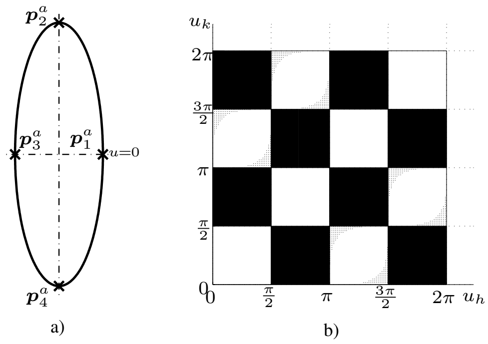

Figure 4: a) Antipodal points on an ellipse; b) Partition of the hk-space: Regions where a FC grasp is possible (black cells), regions where a FC grasp is not possible (white cells) and regions that allow a FC grasp but are discarded (shaded regions).

Lemma 2: Let $\boldsymbol{p}_h$, $\boldsymbol{p}_i$, $\boldsymbol{p}_j$ and $\boldsymbol{p}_k$ be four contact points that satisfy Lemma 1 and produce a FC grasp. Then, there are an odd number of critical points $\boldsymbol{p}_{c_n}$ between $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ such that $\boldsymbol{p}_{c_n}$, $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$ produce a critical grasp (i.e., the lines of action of $\boldsymbol{f}_{c_n}$, $\boldsymbol{f}_h$ and $\boldsymbol{f}_k$ intersect at the same point), with $\boldsymbol{p}_{c_n} = (x_{c_n}, y_{c_n})$ being the solution of:

$$f_{y_{c_n}}(x_{c_n} - x_{hk}) - f_{x_{c_n}}(y_{c_n} - y_{hk}) = 0 \qquad (11)$$

where (x_{hk}, y_{hk}) is the intersection point of the lines of action of $\boldsymbol{f}_h$ and $\boldsymbol{f}_k$. $\diamond$

Proof: A FC grasp satisfies equation (3), implying that Γ_i and Γ_j have different signs. The values of Γ_i and Γ_j are obtained from equation (4) considering the contact points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, respectively. Since the object boundary is smooth, $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ define a continuous piece of the object boundary, and since in a FC grasp the signs of Γ_i and Γ_j must be different, there must be an odd number of critical points $\boldsymbol{p}_{c_n}$ between $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ that make equations (4) equal to zero and allow the result of equation (4) to change its sign. As a result, the lines of action of the applied forces $\boldsymbol{f}_h$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_{c_n}$ intersect at the same point, and given $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$, the critical points $\boldsymbol{p}_{c_n}$ can be determined from equation (11). $\diamond$

Let $\boldsymbol{p}_h = \mathcal{B}(u_h)$ and $\boldsymbol{p}_k = \mathcal{B}(u_k)$ be two contact points with u_h and u_k belonging to the same FC cell of the hk-space. In a FC grasp, the directions of the applied forces $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ must satisfy equation (6), determining a lower and an upper limit points for $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, $\boldsymbol{p}_l = \mathcal{B}(u_l)$ and $\boldsymbol{p}_u = \mathcal{B}(u_u)$, where u_l and u_u are functions of u_h and u_k. As a result of Proposition 2, there must be an odd number

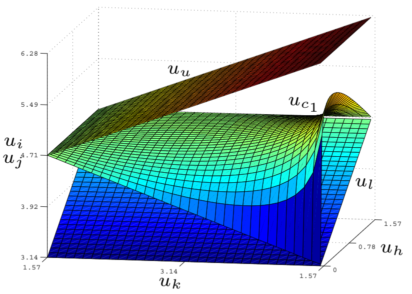

Figure 5: Limit and Critical surfaces, u_l, u_u and u_{c_1}, for the FC cell $u_h=[0,\pi/2]$ and $u_k=[\pi/2,\pi]$ of the ellipse in Fig. 4.

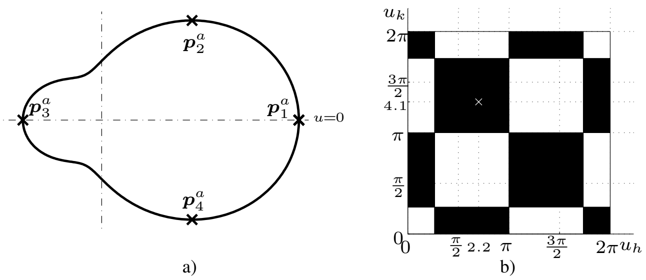

Figure 6: a) Object and antipodal points on its boundary; b) hk-space, FC cells (black cells) and positions u_{h_0} and u_{k_0} of the initial points (white cross).

of critical points $\boldsymbol{p}_{c_n}=\mathcal{B}(u_{c_n})$ between $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, where u_{c_n} is also a function of u_h and u_k. Considering the ranges of u_h and u_k that define a FC cell, u_l and u_u describe two limit surfaces and each u_{c_n} (there is an odd number) describes a critical surface, as in Fig. 5. Therefore, a set of independent regions is obtained determining two parallelepipeds between the limit surfaces with the same projection on the $hk-$space such that there are an odd number of critical surfaces between them. The edges of these parallelepipeds define on the object boundary independent regions where a FC grasp is possible.

Based on this methodology a procedure to obtain a set of independent regions is proposed as follows:

1. Select two contact points $\boldsymbol{p}_{h_0}=\mathcal{B}(u_{h_0})$ and $\boldsymbol{p}_{k_0}=\mathcal{B}(u_{k_0})$ with u_{h_0} and u_{k_0} belonging to the same FC cell.
2. Determine u_l, u_u and all the u_{c_n}.
3. Compute the middle point between each two consecutive points obtained in step 2, and select two of these middle points with an odd number of critical points between them (note that if there is only one critical point, there will be only two middle points).
4. Select two of the middle points obtained in step 3 as u_{i_0} and u_{j_0} generating a FC grasp.
5. Determine two parallelepipeds with the range of u_i and u_j containing u_{i_0} and u_{j_0} with the same ranges of u_h and u_k and without intersecting the limit surfaces and the critical surface.
6. The projections of these two parallelepipeds on each axis determine the independent regions on the object boundary.

4 Example

An example of the proposed methodology is presented in this section. The object used in the example has been taken from [12], where independent regions for two friction contacts were determined. The boundary of the object is described by:

$$\mathcal{B}(u) = 0.6 + 0.6\cos(u) + 0.8\cos^2(u) \qquad (12)$$

for $0\leq u\leq 2\pi$. Fig. 6 shows the object, the antipodal points on its boundary and the antipodal grid with the FC cells. The procedure described in Subsection 3.2 is applied for the cell $u_h\in(0.834,\pi)$ and $u_k\in(\pi,5.449)$, obtaining the following result:

1. Selection of $u_{h_0}=2.2$ and $u_{k_0}=4.1$ (white cross in Fig. 6b).
2. Determination of u_l=5.540, u_u=7.025 and u_{c_1}=6.296.
3. Computation of the middle points: $u_{i_0}=(u_{c_1}+u_l)/2=5.918$, $u_{j_0}=(u_u+u_{c_1})/2=6.661$.
4. Since there are only two middle points, $u_{i_0}=5.918$ and $u_{j_0}=6.661$ are the selected middle points.
5. Determination of the parallelepipeds as $u_{\nu_0}\pm 0.2$ for $\nu\in\{h,i,j,k\}$. The resulting parallelepipeds do not intersect the limit surfaces nor the critical surface.
6. The independent regions are: $u_h\in[2,2.4]$, $u_k\in[3.9,4.3]$, $u_i\in[5.718,6.118]$ and $u_j\in[6.461,6.861]$.

Fig.7 shows the limit and the critical surfaces and one of the parallelepipeds that defines the independent regions (the second parallelepiped lies behind the critical surface) and Fig.8 shows on the physical object the independent regions and the points $\boldsymbol{p}_{h_0}$, $\boldsymbol{p}_{i_0}$, $\boldsymbol{p}_{j_0}$ and $\boldsymbol{p}_{k_0}$ obtained with the proposed procedure.

5 Conclusions and future works

This paper presents a new approach to determine independent regions on the object boundary considering four frictionless contacts. Since the placement of a finger in each one of these regions ensures a FC grasp despite the exact position of the contact point, the determination of these

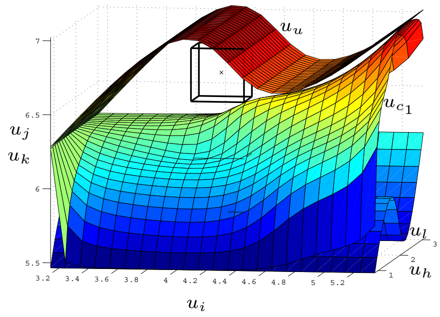

Figure 7: Limit surfaces, u_l and u_u, Critical surface u_{c_1} and independent regions (the second parallelepiped is behind the critical surface) for the FC cell $u_h = [0.834, \pi]$ and $u_k = [\pi, 5.449]$.

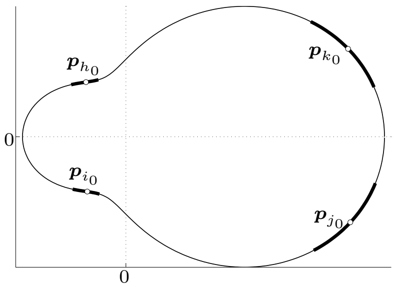

Figure 8: Independent regions (black regions) and initial points (white points) on the object boundary.

regions is useful to provide some robustness in front of finger positioning errors as well as in the design of fixtures.

The major part of the algorithms previously proposed to compute independent regions have been developed considering polygonal or polyhedral objects and they can not be applied when non-polygonal objects are considered. Only a few algorithm has been done considering non-polygonal object. The approach developed here can be applied for any object given a parametric description of its boundary, although the amount of computation increases with the complexity of the object boundary.

Future works includes the obtention of the independent regions following some quality criterion and the adaptation of the proposed methodology to the case where more than two friction contacts are considered.

References

[1] A. Bicchi, "On the Closure Properties of Robotics Grasping", *Int. J. Robot. Res.*, 14(4) pp.319-334, 1995.

[2] B. Mishra, J.T. Schwartz and M. Sharir, "On the Existence and Synthesis of Multifinger Positive Grips", *Algorithmica, Special Issue:Robotics,* 2(4)pp.541-558,1987.

[3] C. Ferrari and J. Canny, "Planning Optimal Grasps," *In Proc. IEEE ICRA*, pp.2290-2295, 1992.

[4] Y.C. Park and G.P. Starr, "Grasp Synthesis of Polygonal Objects" *In Proc. IEEE ICRA,* pp.1574-1580, 1990

[5] R.C. Brost and K.Y. Goldberg, "A Complete Algorithm for Designing Planar Fixtures Using Modular Components," *IEEE Trans. Robot. Automat.*, 12(1) pp.31-46, 1996.

[6] D. Ding, Y-H. Liu and S. Wang, "Computation of 3-D Form-Closure Grasps," *IEEE Trans. Robot. Automat.*, 17(4) pp.515-522, 2001.

[7] I.M. Chen and J.W. Burdick, "Finding Antipodal Point Grasps on Irregularly Shaped Objects," *IEEE Trans. Robot. Automat.*, 9(4) pp.507-512, 1993.

[8] Y-B. Jia, "Computation on Parametric Curves with an Application in Grasping", *Int. J. Robot. Res.*, 9(7-8), pp.825-855, 2004.

[9] B. Mirtich and J. Canny, "Easily Computable Optimum Grasps in 2D and 3D", *In Proc. IEEE ICRA,* pp.739-747, 1994.

[10] X. Zhu, H. Ding and J. Wang, "Grasp Analysis and Synthesis Based on a New Quantitative Measure," *IEEE Trans. Robot. Automat.*, 19(6) pp.942-953, 2003.

[11] V.D. Nguyen, "Constructing Force-Closure Grasps", *Int. J. Robot. Res.*, 7(3) pp.3-16, 1988.

[12] B. Faverjon and J. Ponce, "On Computing Two-Finger Force-Closure Grasps Of Curved Objects", *In Proc. IEEE ICRA*, pp. 424-429, 1991.

[13] A.F. Stappen, C. Wentink and M.H. Overmars, "Computing immobilizing grasps of polygonal parts", *Int. J. Robot. Res.*, 19(5) pp.467-479, 2000.

[14] J. Ponce and B. Faverjon, "On Computing Three Finger Force-Closure Grasp of Polygonal Objects", *IEEE Trans. Robot. and Aut.*, 11(6) pp.868-881, 1995.

[15] J. Ponce, S. Sullivan, A. Sudsang, J.D. Boissonnat, and J.P. Merlet, "On Computing Four-Finger Equilibrium and Force-Closure Grasps of Polyhedral Objects", *Int. J. Robot. Res.*, 16(1), pp. 11-35, 1997.

[16] Y-H. Liu, "Computing n-Finger Force-Closure Grasps on Polygonal Objects", *In Proc. IEEE ICRA*, pp.2734-2739, 1998.

[17] Y. Li, Y. Yu and S. Tsujio, "An Analytical Grasp Planning on Given Object with Multifingered Hand", *In Proc. IEEE ICRA*, pp. 3749-3754, 2002.

[18] J. Cornellà and R. Suárez. "Fast and Flexible Determination of Force-Closure Independent Regions to Grasp Polygonal Objects", *to appear in Proc. IEEE ICRA,* 2005.

[19] N.S. Pollard, "Closure and Quality Equivalence for Efficient Synthesis of Grasps from Examples", *Int. J. Robot. Res.*, 23(6) pp.595-613, 2004.

[20] J.Cornellà and R.Suárez, "On 2D 4-Finger Frictionless Optimal Grasps", *In Proc. IEEE/RSJ IROS,* pp.3680-3685, 2003.