Assembly Cost Evaluation based on Necessary Adjustments due to Tolerances *

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Abstract

An assembly task, whether robotic or manual, may involve adjusting the positions of already assembled parts when a new part is been assembled, due to tolerance and position uncertainties of the parts. Such an adjusting operation can increase the cost of the product because of the adjusting time, fixturing costs, additional operations, etc. In this paper, a statistical approach to evaluate the cost of assembly is proposed. The cost is measured according to the number of objects that must be moved in order to successfully assemble the product. The approach uses a statistical analysis of the accumulated tolerances, the local clearances, and the possible adjustable zones due to clearances between the already assembled parts, to calculate the minimum number of objects that must be adjusted to complete the assembly. Special attention is given to the assembly of parallel chains. The complete procedure and simulation results are given.

1 Introduction

In real assembly, parts are not nominal but deviate slightly due to the design tolerances, thus introducing uncertainty in the position of the part features. Moreover, the uncertainty in the positions of the parts in an assembly increases due to the clearances (play) between them. Such uncertainty can make the product more difficult to be assembled by causing some nominal assembly strategies to fail.

The influence of the tolerances and clearances on the assemblability of a product has already been analyzed by Lee and Yi [1], where complete algorithms to calculate the assemblability of a product based on the tolerances and adjustable zones[1] are given. As a complement to that work, this paper focuses on the influence of the tolerances, clearances, and position uncertainty in the product assembly itself. The effect of these elements can be analyzed according to different criteria. In particular, in this work the minimum number of parts (and which are those parts) that have to be adjusted in order to successfully perform the assembly is considered.

This information is directly related to the cost of the assembly, since it is an indicator of whether or not a particular set of fixtures has to be used, or whether or not a set of extra movements has to be performed to re-fix the assembled parts in order to proceed with the assembly of the product. Therefore, this indicator can be used to select optimal assembly sequences that minimize the assembly cost or complexity.

The analysis is particularly interesting when the product contains serial chains of parts that must be simultaneously considered when the part that links them is added. Consider, for instance, the example of figure 1. Due to tolerances, clearances, and pose uncertainty, part M may not be assembled just by moving it to a nominal assembly pose, but it should be displaced to fit parts P_1 and P_2. Even if an adjustment in the pose of M is permitted, tolerances and clearances in chain 1 and 2 can produce deviations that may not allow the assembly unless the pose of P_1, P_2, or both is adjusted. In this case, it is necessary to use fixtures or to make additional movements of the robot prior to assembling part M.

Problem Statement: *What is the minimum number of parts (and which are they) that must be adjusted in order to perform a successful assembly, given a set of toleranced parts and a valid assembly sequence for the nominal dimensions of the parts?*

It must be noted that in some cases, adjusting the parts may not be enough to assemble them successfully because the deviations caused by tolerances may be too large to be compensated for by given clearances.

In this work the following assumptions are made: (a) Tolerances have a Gaussian probability distribution because specific manufacturing processes are not known at a design stage, many manufacturing pro-

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[1]An adjustable zone is a set of poses where a part is allowed to be placed with respect to its mating part in an assembly; it is defined by the clearance between two parts and the functionality of the product.

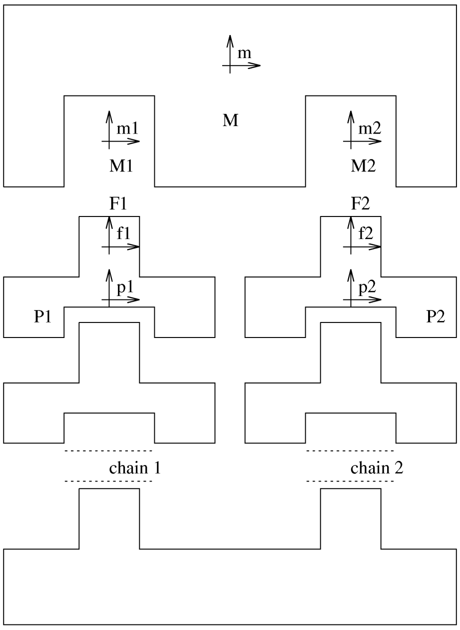

Figure 1: *A parallel chain and nomenclature.*

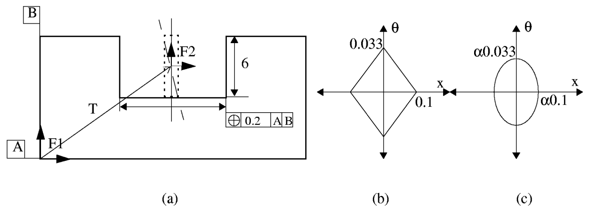

Figure 2: *Pose tolerance: a) position tolerance (ANSI) of a hole feature, b) real tolerance boundary in a deviation frame, c) approximated tolerance ellipse.*

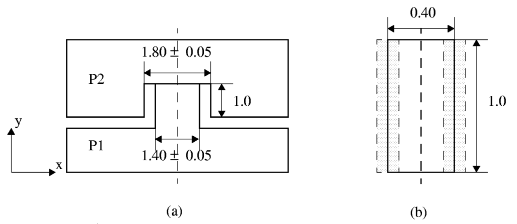

Figure 3: *a) Assembly of two parts P1 and P2, and b) the nominal adjustable zone and its minimum and maximum boundaries.*

cesses have a Gaussian probability distribution [2], and the central limit theorem [3] can be applied to the tolerance propagation; (b) Parts are rigid so they do not deform during and after the assembly; (c) The given assembly sequence is valid for nominal parts; (d) A serial chain is regarded as one object to be moved if the last part needs to be adjusted (adjusting the last part may involve adjusting more parts in the chain).

Related Works

Turner [4] showed that a tolerance specification can be expressed as an in-tolerance region (established by the tolerance limits) of a normed vector space. He developed the methods for tolerance analysis based on finding the relationship between in-tolerance regions and in-design regions, which is established by the design constraints. However, the analysis does not answer the problem of assemblability, where some dimensions (such as clearances) could be used to compensate for deviations caused by tolerances.

Bjorke [5] has proposed statistical approaches to a tolerance analysis based on functional dimensions[2] of simple tolerance chains (equivalent to parallel chains) and interrelated tolerance chains (equivalent to multichains). His goal was to derive a set of tolerance chain equations which can solve the functional dimensions. Then, these functional dimensions were checked against the given confidence limits. However, besides being limited to one dimension, similar to Turner'work, his objective (functionality analysis) is quite different from ours (assemblability analysis).

[2] A functional dimension is the one that affects the assembly more than other dimensions.

2 Representation: Tolerances and Adjustable Zones

It has been already studied [6] that most of the tolerance types described in ANSI [7] can be statistically approximated by an ellipsoid in a coordinate frame of kinematic parameters, called *deviation frame* in this paper. The origin of this frame indicates that the feature does not deviate from its nominal pose, and the ellipsoid approximates statistically the real tolerance bounds. A point in this ellipsoid represents a permitted deviation of a feature from its nominal pose.

For example, a feature (hole) of a part has a position tolerance of 0.2 (figure 2a). This means that the axis of the hole (shown with a solid vertical line) is permitted to deviate within the tolerance zone of 0.2 width (shown with a dotted rectangle). Then, the coordinate frame attached to the hole can deviate from its nominal pose as much as the axis is allowed to deviate from its nominal pose. The maximum boundary in a 2-dimensional deviation frame (θ and x) is shown in figure 2b, which shows that the coordinate frame attached to the nominal hole can rotate in θ at most ± 0.033 and can translate in the x-axis at most ± 0.1, but they are not independent. Figure 2c shows the approximation of the real tolerance boundary by an ellipse.

Figure 3a shows the adjustable zone between two parts, which is a rectangular zone of 0.40 by 1.0 for nominal parts. However, due to the size tolerances (± 0.05) of the features (peg and hole), the adjustable zone can vary within the maximum and minimum

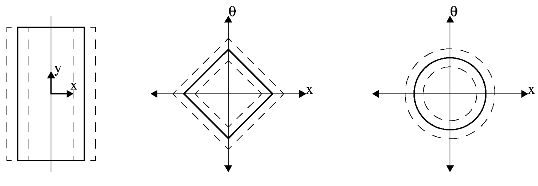

Figure 4: *a) Adjustable zone of P1 and P2, b) real boundary in a deviation frame, c) ellipse approximation.*

limits shown with dotted lines in figure 3b.

An adjustable zone is approximated in a deviation frame by a nominal ellipsoid and its minimum and maximum limits (see figure 4). That is, the coordinate frame attached to the center of the axis of the hole of P2 is permitted to be adjusted as much as the adjustable zone allows with respect to the peg, as shown in figure 4(a). Figure 4(b) shows the adjustable zone in a deviation frame. The solid line is the nominal adjustable boundary, whereas the dotted lines indicate the minimum and maximum limits. Figure 4(c) shows an adjustable ellipse approximating the real adjustable boundary.

3 Parallel Chains

The interconnection of parts in an assembly may form loops or parallel chains. A parallel chain is closed by assembling the last part to two serial chains, e.g. part M, chain 1 and chain 2, in figure 1. The accumulated tolerances and adjustable zones at the ending feature of each serial chain (F_1 and F_2) must be computed in order to evaluate both the assemblability and difficulty of a parallel chain when part M is added. These parallel chains can make the assembly difficult or even impossible.

The tolerance accumulation in each serial chain is computed by adding the tolerances of all the features in the chain using a *sweep operation* on the two ellipsoids that represent the two tolerances. A sweep operation is similar to Minkowski addition [8], which is defined as an addition of all possible combinations of position vectors of two sets, except that the position vectors are generated as samples with some probability within the corresponding ellipsoids. Each sample models a random instance of a part deviation. Next, the result of the additions, a simulated solution, is optimally approximated by another ellipsoid using an analytic solution, a Gaussian function. Therefore, the tolerance ellipsoid of a serial chain can be computed by continuously applying the sweep operation to a tolerance ellipsoid of every feature in the chain.

The method to compute the adjustable zone accumulation of a serial chain is based on the addition of all the adjustable zones of the chain. The addition of two adjustable ellipsoids and their limits (minimum and maximum) can be optimally approximated by a new ellipsoid and limits. The numerical algorithm used is as follows: (1) Randomly generate an ellipsoid from each adjustable ellipsoid; (2) Compute the sweep volume of the two randomly generated ellipsoids; (3) Repeat steps (1) and (2) many times; (4) Optimally approximate the distribution of the accumulated sweep volumes with an ellipsoid and corresponding limits.

The approximation of step (4) can be done using a Gaussian-Sigmoid function[3] whose shape is very similar to the accumulation of the randomly generated ellipsoids (refer to [1] for details of the computation).

4 Statistical Evaluation of the Necessary Adjustments

Given an assembly sequence, the uncertainty introduced by each part due to tolerances of features and the adjustable zones allowed by local clearances is propagated as it was described in section 3.

The evaluation of the number of parts that has to be adjusted can be done for a serial chain[4] in order to decide if a fixture is needed to improve the average cost of the assembly operation, but this is a particular and simpler case of the general problem generated by parallel chains. This section presents a procedure to deal with two parallel chains of objects that can be systematically extended for the case of a manipulated object that links a larger number of parallel chains.

Let M be the manipulated object (the part that will link two serial chains), and P_1 and P_2 be the last two objects of the two serial chains to be assembled with M, (figure 1). The features (pegs) F_1 of P_1 and F_2 of P_2 have to be mated with the features (holes) M_1 and M_2 of M, respectively. The following nomenclature will be used:

$\mathbf{p}_i$: the pose[5] of part P_i (i=1,2).
$\mathbf{m}$: the pose of part M ($\mathbf{m}_o$: nominal value of $\mathbf{m}$).
$\mathbf{f}_i$: the pose of the feature F_i of part P_i (i=1,2). Note that $\mathbf{f}_i$ depends on $\mathbf{p}_i$ and the tolerances of part P_i.
$\mathbf{m}_i$: the pose of the feature M_i of part M (i=1,2)
L_i : the local clearance between P_i and M_i (i=1,2). $L_i(\mathbf{f}_i)$ will represent the valid assembly poses of M_i for the pose $\mathbf{f}_i$ of F_i, and $L_i(\mathbf{m}_i)$ will represent the valid assembly poses of F_i for the pose $\mathbf{m}_i$ of M_i.

[3] A Gaussian-Sigmoid function is $2[\frac{1}{1+e^{G(X)/T}} - \frac{1}{2}]$, where $G(X)$ is a Gaussian function. This probability distribution has a flat-top bell shape determined by the parameters X and T.

[4] In a serial chain, the number of parts $N = \{0,1,2\}$ that has to be adjusted to be able to perform the assembly allows four possible solutions: $\emptyset$, $\{P\}$, $\{M\}$, and $\{P, M\}$, where P is the end part in the chain and M is the manipulated part.

[5] Poses are expressed in an absolute reference frame unless it is explicitly specified.

C_i : a set of possible poses of F_i (i=1,2), due to tolerances and adjustable zones of the parts.

$S(\mathbf{m})$: a set of possible poses of M for a given commanded pose $\mathbf{m}$ with probability distribution $d(\mathbf{m})$ due to the pose uncertainty of M.

$S_i(\mathbf{m})$: a set of possible poses of M_i for a given commanded pose $\mathbf{m}$ with probability distribution $d(\mathbf{m}_i)$ due to tolerances and the pose uncertainty of M (i=1,2).

The number of parts N={0,1,2,3} that has to be adjusted to be able to perform the assembly as well as which are these parts (P_1, P_2, and/or M) allow eight different solutions: $\emptyset$, $\{P_1\}$, $\{P_2\}$, $\{M\}$, $\{M,P_1\}$, $\{M,P_2\}$, $\{P_1,P_2\}$ and $\{M,P_1,P_2\}$. The statistical occurrence of each one is computed as follows:

1. Propagate the tolerances and clearances of each serial chain (using the methodology described in section 3). For $i = 1,2$ do:
   a. Statistically propagate the tolerances of the objects in the chain. The result will be a set T_i of possible accumulated deviations in the position of F_i with certain distribution of probability.
   b. Statistically propagate the clearances between the objects in the chain. The result will be a set D_i of possible sets of adjustable displacements for the part P_i with certain distribution of probability.
2. Compute $S(\mathbf{m})$ and $d(\mathbf{m})$ by considering the pose uncertainty of M.
3. Compute $S_1(\mathbf{m})$ and $S_2(\mathbf{m})$ with probability distributions $d(\mathbf{m}_1)$ and $d(\mathbf{m}_2)$ by considering the tolerances of M.
4. Set num$_\alpha$=0 for $\alpha = \emptyset$, $\{P_1\}$, $\{P_2\}$, $\{M\}$, $\{M,P_1\}$, $\{M,P_2\}$, $\{P_1,P_2\}$, and $\{M,P_1,P_2\}$.
5. Repeat n times (n large enough):
   a. Determine C_i by statistically choosing a deviation from T_i and a set of adjustable displacements from D_i.
   b. Randomly select particular instances of L_1 and L_2 from the tolerances of M, P_1 and P_2.
   c. Randomly select a pose $\mathbf{p}_1$ from C_1.
   d. Randomly select a pose $\mathbf{p}_2$ from C_2.
   e. Randomly select a pose $\mathbf{m}$ from $S(\mathbf{m}_o)$ according to $d(\mathbf{m})$.
   f. Randomly select a pose $\mathbf{m}_1$ from $S_1(\mathbf{m})$ according to $d(\mathbf{m}_1)$.
   g. Randomly select a pose $\mathbf{m}_2$ from $S_2(\mathbf{m})$ according to $d(\mathbf{m}_2)$.
   h. CALL *parts-to-be-adjusted* (flow chart in figure 5).
   i. Increment num$_\alpha$ according to the result obtained in the previous step.
6. Return $\frac{\text{num}_\alpha}{n}$ for each α.

The conditions in the flow-chart of figure 5 have been implemented for a two-dimensional case (i.e. ellipsoids are reduced to ellipses) but they are valid for any number of degrees of freedom (i.e. ellipsoids are actually hyper-ellipsoids of the corresponding dimension). The physical meaning and geometrical solutions of conditions c_i for i=1,...,8 are:

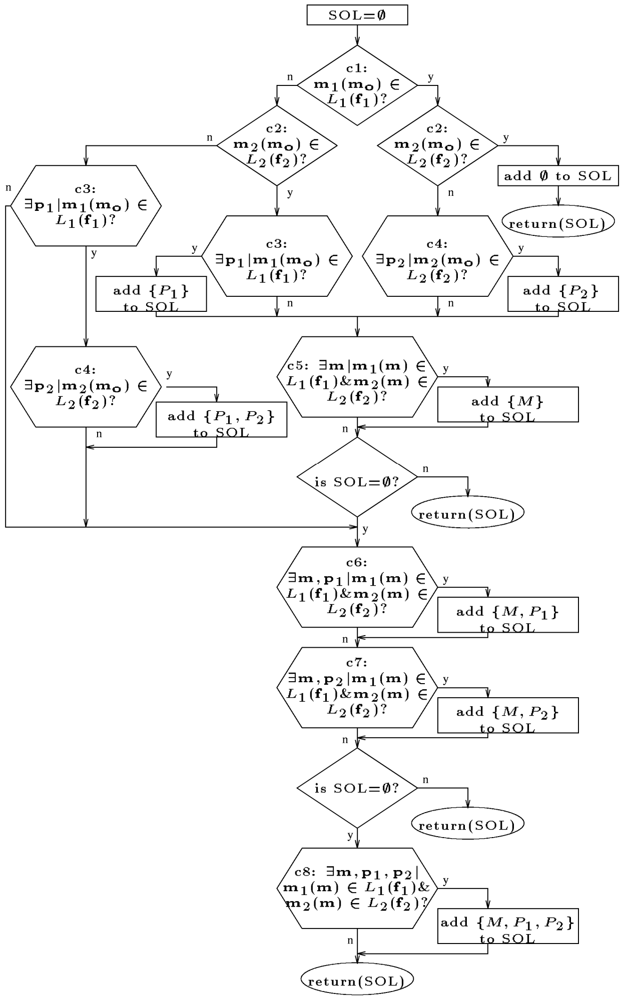

Figure 5: *Flow chart of the procedure* **parts-to-be-adjusted** *used to determine which parts have to be adjusted in order to be able to perform the assembly.*

c1. *Does the local clearance L_1 allow to mate features F_1 and M_1 for the nominal position $\mathbf{m}_o$ of M and despite tolerances and previous clearances?* Solved by testing if a point lies inside an ellipsoid: $\mathbf{m}_1(\mathbf{m}_o) \in L_1(\mathbf{p}_1)$.

c2. Idem to **c1** replacing subindex 1 with 2.

c3. *Is it possible to adjust the pose $\mathbf{p}_1$ of part P_1 by using previous clearances such that for the nominal pose $\mathbf{m}_o$ of M, the local clearance L_1 allow features F_1 and M_1 to mate despite tolerances?* Solved by testing if two ellipsoids intersect each other: $C_1 \cap L_1(\mathbf{p}_1)$.

c4. Idem to **c3** replacing subindex 1 with 2.

c5. *Is it possible to adjust the pose* $\mathbf{m}$ *of part* M *such that the local clearances* L_1 *and* L_2 *simultaneously allow feature* F_1 *to mate* M_1 *and* F_2 *to mate* M_2 *despite tolerances and previous clearances?* Solved by:

a) propagating the tolerances between the two features of M; considering the local clearance this means the translation of an ellipsoid: $L_1(\mathbf{p}_1)$ to $L_1(\mathbf{p}_1')$ (or $L_2(\mathbf{p}_2)$ to $L_2(\mathbf{p}_2')$). The transformation between $\mathbf{p}_1$ and $\mathbf{p}_1'$ is the transformation between $\mathbf{f}_1$ and $\mathbf{f}_2$ (or the opposite if L_2 is translated).

b) testing if two ellipsoids intersect each other: $L_1(\mathbf{p}_1') \cap L_2(\mathbf{p}_2)$ (or $L_2(\mathbf{p}_2') \cap L_1(\mathbf{p}_1)$).

c6. *Is it possible to simultaneously adjust the poses* $\mathbf{m}$ *and* $\mathbf{p}_1$ *of parts* M *and* P_1 *such that the local clearances* L_1 *and* L_2 *simultaneously allow feature* F_1 *to mate* M_1 *and* F_2 *to mate* M_2 *despite tolerances and previous clearances?* Solved by:

a) propagating the tolerances between the two features of M; considering the local clearance this means the translation of an ellipsoid: $L_1(\mathbf{p}_1)$ to $L_1(\mathbf{p}_1')$. Again, The transformation between $\mathbf{p}_1$ and $\mathbf{p}_1'$ is the transformation between $\mathbf{f}_1$ and $\mathbf{f}_2$.

b) performing a Minkowsky sum: $C_2' = C_2 \oplus L_2(\mathbf{p}_2)$.

c) testing if two ellipsoids intersect each other: $L_1(\mathbf{p}_1') \cap C_2'$.

c7. Idem to **c6** replacing subindices 1 with 2.

c8. *Is it possible to simultaneously adjust the poses* $\mathbf{m}$*,* $\mathbf{p}_1$ *and* $\mathbf{p}_2$ *of parts* M*,* P_1 *and* P_2 *such that the local clearances* L_1 *and* L_2 *simultaneously allow feature* F_1 *to mate* M_1 *and* F_2 *to mate* M_2 *despite tolerances?* Solved by:

a) performing the Minkowsky sums: $C_1' = C_1 \oplus L_1(\mathbf{p}_1)$ and $C_2' = C_2 \oplus L_2(\mathbf{p}_2)$

b) considering that one of the features of M has been assembled, propagate the tolerance from this feature to the other; this means the translation of an ellipsoid: C_1' to C_1'' (or C_2' to C_2'').

c) testing if two ellipsoids intersect each other: $C_1'' \cap C_2'$ (or $C_2'' \cap C_1'$).

Note that if not all the conditions are satisfied, the product cannot be assembled. The percentage of assemblable products (assemblability) can be computed in this way as an alternative procedure to that introduced in [1].

5 Example

The evaluation of assembly cost is illustrated in this section using the four-part example assembly shown in figure 6. The bottom part (P1) has two holes

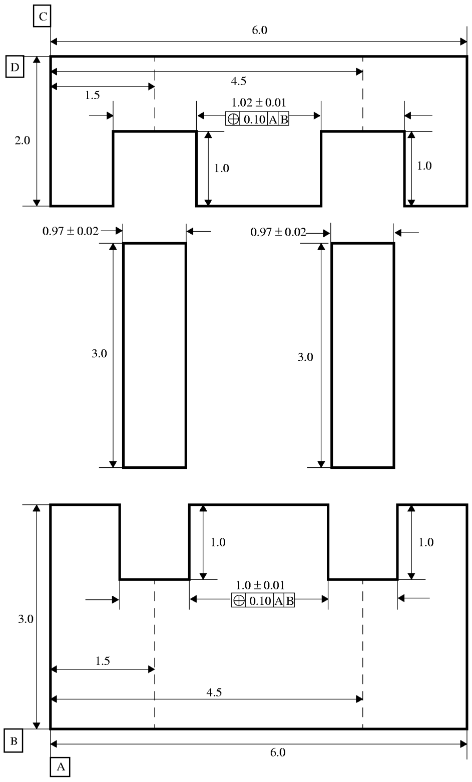

Figure 6: *ANSI dimension and tolerance specification for the four part 2D example assembly: 1 Bottom (P1), 2 Shafts (P2 & P3), 1 Cover (P4).*

where the left shaft (P2) and the right shaft (P3) are inserted into. The top part (P4) with two holes is assembled with the two shafts. The holes of part P1 have a diameter of 1.0 with size tolerance of ± 0.01 and position tolerance of 0.10. The holes of part P4 have a diameter of 1.02 with size tolerance of ± 0.01 and position tolerance of 0.10. The shafts, P2 and P3, have a diameter of 0.97 with size tolerance of ± 0.02.

The simulation counts the number of objects that must be moved in order to successfully assemble the parts. Therefore, it assumes that the cost is the same for moving the same number of objects (i.e. moving two objects {P1,P2} or {P1,M} costs the same), although they may be different. It must be noted that the algorithm described in section 4 accounts for all possible results including the case of different objects.

Two assembly sequences were tested and compared in simulation: (1) {P1,P2,P3} $\cup$ {P4} and (2) {P4,P2,P3} $\cup$ {P1}, where {.} denotes a subassembly and $\cup$ denotes the assembly operation. It must be noted that the subassemblies {P1,P2,P3} and {P4,P2,P3} are serial chains whose assembly cost has been ignored in this simulation. We only illustrate here the cost of assembling the parallel chain. Although the deviation frame requires three axes, the y-component

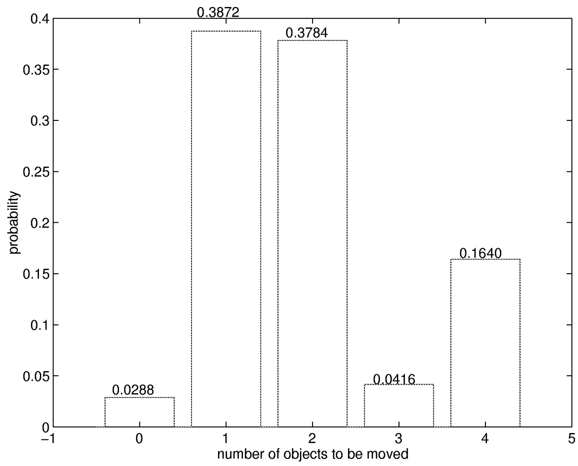

Figure 7: *Probability bar graph for the assembly sequence 1: {P1, P2, P3} ∪ {P4}*

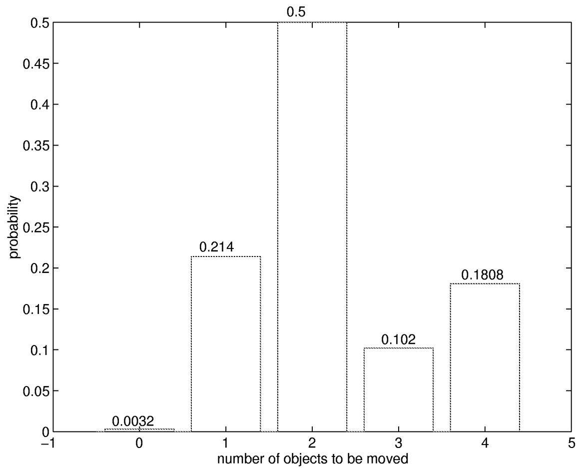

Figure 8: *Probability bar graph for the assembly sequence 2: {P4, P2, P3} ∪ {P1}.*

has been ignored since it hardly affects the results and these can be better illustrated.

In simulation (1), tolerance and clearance accumulation ellipses are computed at the upper end of the shafts, P2 and P3, propagated from P1. In simulation (2), tolerance and clearance accumulation ellipses are computed at the lower end of the shafts, P2 and P3, propagated from P4. Figures 7 and 8 show the probabilities of having to move $N \in \{0, 1, 2, 3, 4\}$ number of objects (where 4 denotes the infeasible assembly) for the simulations (1) and (2) respectively.

Assembly sequence (1) can be executed without moving any object about 2.8% of the time, while in sequence (2) it is only 0.32%. Also, for the case of only one adjustment the percentage in sequence (2) is smaller. The opposite situation occurs for the case of two and three adjustments. Assuming that the cost is directly related to the number of objects to be moved, then the assembly sequence (1) is better than the assembly sequence (2). The difference of infeasible assembly probabilities between the assembly sequences (1) and (2) is the result of approximations and numerical computation errors.

6 Conclusion

This work complements the work previously done by Lee and Yi [1]. Using those results and algorithms, this work proposes a novel approach to evaluating the assembly by statistically counting the number of objects that must be moved for a successful assembly. The approach can be used to select a particular assembly sequence from a pool of possible assembly sequences. A complete algorithm and simulation results were presented.

The contributions of this paper are:

- A new cost metric is proposed based on the concept of adjustability of the assembly.
- The proposed approach can compute both the assemblability and the cost of assembly.
- Assembly sequences can be evaluated and selected using this metric, thus bringing the assembly sequence evaluation closer to more realistic problems.

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