Synthesis of grasps with four contact points including at least three force-closure grasps of three contact points

Ricardo Prado and Raúl Suárez

Instituto de Organización y Control de Sistemas Industriales (IOC)
Universidad Politécnica de Cataluña (UPC), Barcelona, Spain
e-mails: sixto.ricardo.prado@upc.edu, raul.suarez@upc.edu

***Abstract*—The paper presents a method to build force-closure grasps on polyhedral objects with four frictional contact points such that three of the fingers can individually lose the contact with the object without making the three finger remaining grasp losing the force-closure property. This type of grasps is useful to allow the object manipulation via regrasp using finger gating. A necessary and sufficient condition for the determination of these grasps is proposed, as well as the algorithm to compute them.**

***Keywords*—robotics; force-closure grasps, regrasp, manipulation.**

I. Introduction

A force-closure grasp (FCG) can balance any external disturbance applied on the object by means of the contact forces applied by the fingers. Different approaches for the computation of force-closure grasps have been presented in the literature and in particular for four object-finger contacts on polyhedral objects [2] [3] [4] [5] [6] [7]. Nevertheless, in the grasps generated with these approaches the lost of a finger-object contact frequently implies the lost of the force-closure property, and the object may fall down (an example of this type of grasps is shown in Figure 1a). As a consequence, these grasps are unlikely to be considered as valids for a manipulatioin process with firger repositioning on the object (finger gating).

The methods to manipulate an object with four fingers and gating techniques start with an initial grasp that allows the individual lost of three contacts without losing the force-closure property [8]; nevertheless, these works do not describe how to obtain such initial grasp.

The approach describe in this paper computes, for a given FCG with three contact points, the regions on the object boundary such that a fourth contact anywhere on them allows the removement of two of the initial contacts without losing the force-closure property. First, a necessary and sufficient condition for the determination of a four-finger FCG that includes three three-fingers FCG is presented. Second, given a FCG with three contacts, the procedure to compute the contact regions on the object boundary for the fourth finger is given. Finally, a procedure to select a particular position for the fourth finger is given, trying to generate a good planar grasp or with the fourth finger close to the plane defined by the other three fingers (an example of this type of grasps is shown in Figure 1b); the intention here is to produce a "good enough grasp" [11], which best fits the potential reachability of an anthrophomorphic hand.

II. Proposed Approach

A. Necessary and sufficient condition

Three forces $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ and $\boldsymbol{f}_3$ applied on non colinear points on an object reach the equilibrium if and only if one of the following two conditions is satisfied [2] [12] (Figure 2):

1. $\boldsymbol{f}_1, \boldsymbol{f}_2$ and $\boldsymbol{f}_3$ are coplanar, span their supporting plane and their supporting lines intersect in a point.
2. $\boldsymbol{f}_1, \boldsymbol{f}_2$ and $\boldsymbol{f}_3$ are coplanar, parallel and that in the middle of the other two has different sense.

Let C_{fi} be the friction cone at contact *i*. It was shown in previous works [2] [12] [14] that if $\boldsymbol{f}_i \in C_{fi}$ ($\boldsymbol{f}_i$ lies strictly in the interior of C_{fi}) satisfy one of these conditions then the contact points P_i allow a FCG.

The method proposed in this paper is based on the following proposition. Let $\boldsymbol{n}_i$ be the unitary vector normal to the object boundary at contact point P_i.

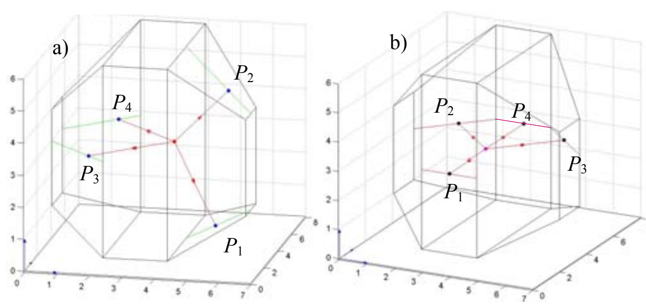

Fig. 1 TwoFCG: a) the loss of a finger-object contact implies the loss of the force-closure property, b) the loss of any of the finger-object contacts at P_2, P_3 y P_4 (one at a time) does not implies the loss of the force-closure property.

This work was partially supported by the CICYT projects DPI2005-00112 and DPI2007-63665.

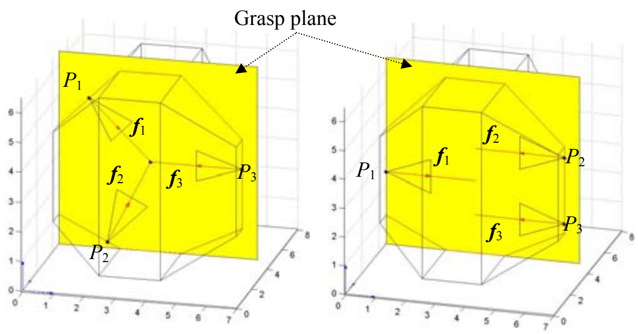

Fig. 2 The two cases of three forces that reach the equilibrium.

Proposition 1. A FCG with contact points P_i, i=1,2,3,4, includes at least three FCG with three contact points each one if and only if (Figure 3a):

C1. $\exists\ P_i, P_j$, with $i,j \in \{1,2,3,4\}$ and $i \neq j$ defining a straight line L_p such that:

1. The projections of $\boldsymbol{n}_i$ and $\boldsymbol{n}_j$ on L_p have different sense (Figure 3b).
2. $L_p \subset C_{fi} \cap C_{fj}$ and L_p lies strictly in the interior of C_{fi} and C_{fj}.

C2. The angle φ between $\boldsymbol{n}_k$ and the plane defined by P_i, P_j and P_k (Figure 3c), and the angle λ between $\boldsymbol{n}_l$ and the plane defined by P_i, P_j y P_l (Figure 3d), are both smaller than the friction cone half-angle α, and also $L_p \cap C_{fk} \cap C_{fl} \neq \varnothing$ with $\{i,j,k,l\}=\{1,2,3,4\}$.■

Proof

Sufficient Condition

Let $\boldsymbol{n}_i^L$ and $\boldsymbol{f}_i^L$ be the projections of $\boldsymbol{n}_i$ and $\boldsymbol{f}_i$ on L_p, and $\boldsymbol{n}_i^{L\perp}$ and $\boldsymbol{f}_i^{L\perp}$ the projections of $\boldsymbol{n}_i$ and $\boldsymbol{f}_i$ orthogonal to L_p.

From *C1* forces $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ can be appied such that $\boldsymbol{f}_i^L$ and $\boldsymbol{f}_j^L$ have different senses, while $\boldsymbol{f}_i^{L\perp}$ and $\boldsymbol{f}_j^{L\perp}$ may have any sense. From *C2* $L_p \cap C_{fk} \neq \varnothing$ and $L_p \cap C_{fl} \neq \varnothing$, this means that there exist forces $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$ whose supporting lines intersect with L_p (i.e. they are coplanar). From *C1* $L_p \subset C_{fi} \cap C_{fj}$ then $C_{fi} \cap C_{fj} \cap C_{fk} \neq \varnothing$ and $C_{fi} \cap C_{fj} \cap C_{fl} \neq \varnothing$ (since φ<α and λ<α then $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$ are not on the limits of C_{fk} and C_{fl}). Now, if $\boldsymbol{f}_k$=0 and $\boldsymbol{f}_l \neq 0$ then a positive lineal combination of $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ can balance $\boldsymbol{f}_l^L$ ($\boldsymbol{f}_i^L$ and $\boldsymbol{f}_j^L$ have different senses and $\boldsymbol{f}_i^{L\perp}$ and $\boldsymbol{f}_j^{L\perp}$ have any sense). This means that $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_l$ can reach the equilibrium, and therefore P_i, P_j and P_l allow a FCG. The same reasoning can be applied when $\boldsymbol{f}_k \neq 0$ and $\boldsymbol{f}_l$=0, obtaining that P_i, P_j and P_k allow a FCG (the forces $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ that balance $\boldsymbol{f}_k$ are not same that the forces $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ that balance $\boldsymbol{f}_l$).

From *C2,* $L_p \cap C_{fk} \cap C_{fl} \neq \varnothing$ implies that there exist coplanar sets $\boldsymbol{f}_i$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$, and $\boldsymbol{f}_j$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$, and since $\boldsymbol{f}_i^L$ and $\boldsymbol{f}_j^L$ have different senses then $\boldsymbol{f}_i$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$ (with $\boldsymbol{f}_j=\varnothing$) or $\boldsymbol{f}_j$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_l$ (with $\boldsymbol{f}_i=\varnothing$) reach the equilibrium (both cases are possible for an extremely large friction cone); therefore P_i, P_k and P_l and/or P_j, P_k and P_l allows a FCG. Then (P_i, P_k, P_l), (P_i, P_j, P_k) and (P_i, P_j, P_l) determine, each one, a FCG.

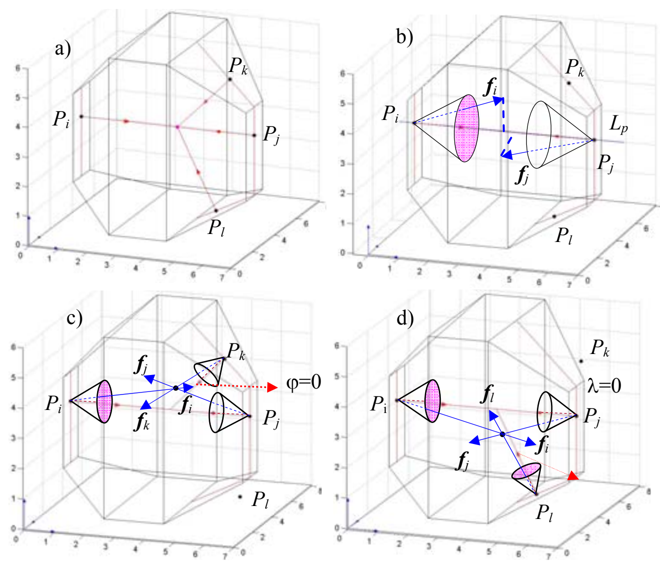

Fig. 3 a) FCG with coplanar P_i, i=1,2,3,4; b) P_i and P_j satisfy the condition *C1*; c) P_i, P_k and P_j allow a FCG (φ=0⇒φ<α); d) P_i, P_l and P_j allow a FCG (λ=0⇒λ<α).

Necessary Condition

If $\boldsymbol{n}_i^L$ and $\boldsymbol{n}_j^L$ have the same sense then $\boldsymbol{f}_i^L$ and $\boldsymbol{f}_j^L$ have also the same sense, which does not allow the application of forces $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_k$ (with $\boldsymbol{f}_l=\varnothing$) that are either parallel or coplanar spanning their supporting plane; then, the applied forces do not reach the equilibrium and therefore P_i, P_j and P_k do not allow a FCG. The same reasoning can be applied to $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_l$ (with $\boldsymbol{f}_k=\varnothing$), and therefore P_i, P_j and P_l neither allow a FCG. As a consequence, two out of the four possible sets of three contacts do not allow FCG, therefore the first parte of *C1* is necessary for the existence of a FCG that using the points P_i, i=1,2,3,4 allow three FCG with three contacts each one.

If P_i and P_j makes $L_p \not\subset C_{fi} \cap C_{fj}$ a grasp with a third point P_k can reach the equilibrium only if, on the grasp plane defined by P_i, P_j and P_k, the set $C_{fi} \cap C_{fj}$ and P_k lie on the same open half-plane defined by L_p, otherwise $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_k$ will not expand the grasp plane. Now, given four points such that $L_p \not\subset C_{fi} \cap C_{fj}$ for any two points P_i and P_j, since there will be always two combinations of points P_i and P_j that leaves $C_{fi} \cap C_{fj}$ and one of the remaining two points P_k and P_l on different sides of L_p, therefore at least two out of the four possible sets of three contacts do not allow a FCG. Thus, the second parte of *C1* is also necessary for the existence of the desired FCG.

A necessary condition for the existence of a FCG using P_i, i=1,2,3, is that the grasp plane defined by the contact points intersects with C_{fi}, i=1,2,3 [13], and this is possible only if the angle between $\boldsymbol{n}_i$ and the grasp plane is smaller than α. Therefore, if P_i, P_j and P_k allow a FCG then φ must be smaller than α; and if P_i, P_j and P_l allow a FCG then λ must be smaller than α. Thus, the condition *C2* is necessary

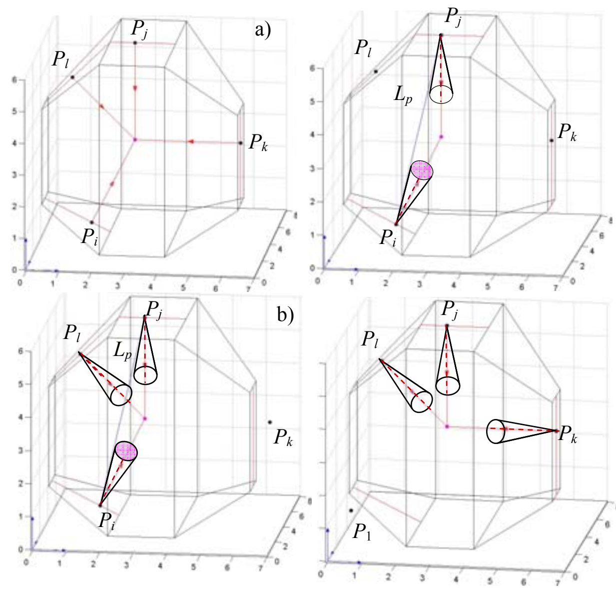

Fig. 4 a) There are not two points P_i and P_j satisfying the condition *C1*; b) the sets (P_i, P_l, P_j) and (P_j, P_k, P_l) do not allow a FCG.

for the existence of the desired FCG. ■

Note that from this proof it can also be concluded that a sufficient condition for P_i, i=1,2,3, to allow a FCG is that two contact points satisfy the condition *C1* and the friction cone of the third contact point intersect with the line L_p; this means that $L_p \cap C_{fi} \cap C_{fj} \cap C_{fk} \neq \varnothing$, with $\{i,j,k\}=\{1,2,3\}$.

Conditions *C1* and *C2* in Proposition 1 are used in the next subsection to obtain the desired four-contact FCG.

B. Determining the contact regions R_i

Given three contact points (P_i, P_j, P_k), $\{i,j,k\}=\{1,2,3\}$, that allow a FCG, the contact region R_i such that a fourth point $P_4 \in R_i$ makes the sets (P_i, P_j, P_4) and (P_i, P_k, P_4) also allow a FCG are computed in two steps:

1. Compute the region R'_i / $\forall P_4 \in R'_i$, P_4 and P_i satisfy the condition *C1*.
2. Compute $R_i \subseteq R'_i$ / $\forall P_4 \in R_i$, the sets (P_i, P_j, P_4) and (P_i, P_k, P_4) satisfy the condition *C2*.

If $\forall i$ $R_i=\varnothing$ then the desired grasp cannot be obtained. A conceptual example of the regions R_i is shown in Figure 5. Note that R_i may be a discontinuous region (e.g. R_2 in Figure 5), having parts on differents object faces. The proposed procedure takes into account this situation checking all the faces that may contain a portion of R_i.

Let A_l, l=1,…,m, be the object faces. For each A_l, the following procedure determines whether the position and orientation of A_l allows the existence of a region $R'_i \neq \varnothing$ (steps 1 and 2), and, if $R'_i \neq \varnothing$ computes the region $R_i \subseteq R'_i$ (step 3). Given three contact points P_i, P_j, and P_k that allow a FCG, the procedure to determine R_i is:

1. Compute the angle γ between $\boldsymbol{n}_l$ and $-\boldsymbol{n}_i$. If $\gamma \geq 2\alpha$ then

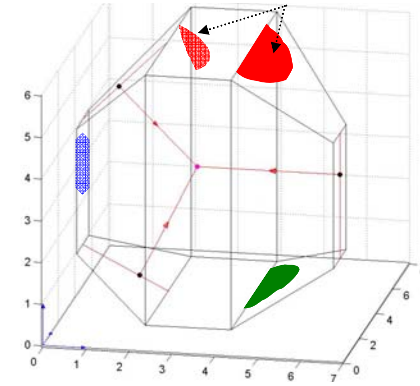

Fig. 5 Conceptual representation of the regions $R_i \neq \varnothing$ for the inicial FCG given by P_1, P_2 and P_3.

Return(***invalid face***).

If $\gamma \geq 2\alpha$ then at least one of the two parts of condition *C1* is not satisfied.

2. Compute $R'_i = A_l \cap C_{fo} \cap C_{fi}$, where C_{fo} is the cone with half-angle α, axis with the direction of $\boldsymbol{n}_l$ and the origin at P_i (Figure 6) (note that since $\gamma<2\alpha$ then $C_{fo} \cap C_{fi} \neq \varnothing$). If $R'_i=\varnothing$ then Return(***invalid face***).

If $R'_i=\varnothing$ then P_i and any P_4 on A_l do not satisfy the second part of the condition *C1*, even when $A_l \cap C_{fi} \neq \varnothing$. If $R'_i \neq \varnothing$ (note that $R'_i \subset C_{fi}$) then P_i and any $P_4 \in R'_i$ always satisfy the condition *C1*.

3. Compute R_i according to one of the following four cases:
Let: L_{ij} be the straight line defined by P_i and P_j.
L_{ik} be the straight line defined by P_i and P_k.
Remind that L_p was defined in generic way as a line through points P_i and P_j that satisfy certain conditions, it does not necessarily coincide with L_{ij} as defined here.

3.1 If $L_{ij} \subset C_{fj}$ and $L_{ik} \subset C_{fk}$ then $R_i=R'_i$ (Figure 7).
In this case, $P_i \in L_{ij}$ and $L_{ij} \subset C_{fj} \Rightarrow P_i \subset C_{fj}$, therefore $C_{fi} \cap C_{fj} \neq \varnothing$. Now, from Step 1 is satisfied that P_i and

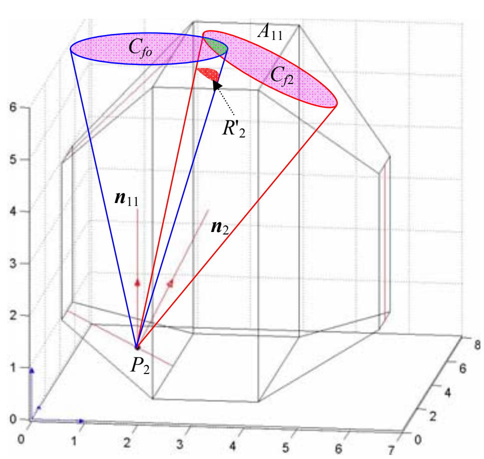

Fig. 6 Computation of R'_2 on the face A_{11}(i.e. i=2 and l=11).

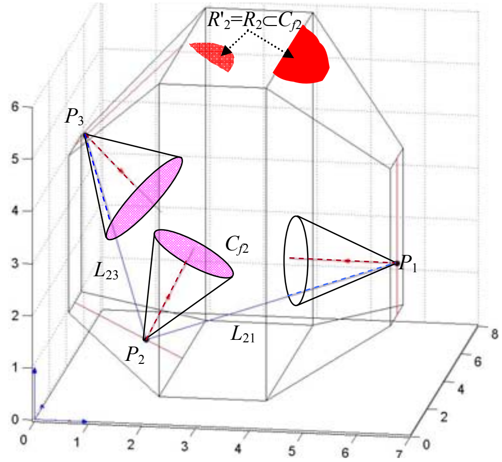

Fig. 7 $L_{ij}\subset C_{fi}$ y $L_{ik}\subset C_{f3}\Rightarrow R'_2=R_2$.

any $P_4\in R'_i$ satisfy the condition *C1* (remind that $R'_i\subset C_{fi}$), therefore $L_p\subset C_{fi}\cap C_{f4}$, and as $C_{fi}\cap C_{fj}\neq\varnothing$ then $L_p\cap C_{fi}\cap C_{fj}\cap C_{f4}\neq\varnothing$. Then P_i, P_j and P_4 satisfy the sufficient condition to determine a FCG.

With an analogous reasoning, since $P_i\in L_{ik}$ and $L_{ik}\subset C_{fk}$ $\Rightarrow P_i\subset C_{fk}$, and therefore P_i, P_k and P_4 allow a FCG.

As a conclusion, (P_i, P_j, P_k), (P_i, P_j, P_4) and (P_i, P_k, P_4) allow a FCG (remind that P_i, P_j and P_k define the initial FCG).

3.2 If $L_{ij}\subset C_{fj}$ and $L_{ik}\not\subset C_{fk}$ then:

- Find the sub-space, S_{ik}, limited by two planes tangent to C_{fk} and containing L_{ik}, such that $C_{fk}\subset S_{ik}$ (Figure 8).

  Any plane that intersects with C_{fk} and that contains P_i and P_k always belongs to S_{ik}. Since in a FCG with three contact points the grasp plane (defined by the three contact points) always intersects with the three friction cones, then the grasp plane of P_i, P_k and any other contact point that allow a FCG is always contained in S_{ik}.
- Compute $R_i=R'_i\cap S_{ik}$.

  If $P_4\in R'_i\cap S_{ik}$ then is satisfied that $L_p\subset C_{fi}\cap C_{f4}$ (Step 1) and $L_p\subset S_{ik}$, and as by construction $C_{fk}\subset S_{ik}$ (C_{fk} is tangent to S_{ik}) then $L_p\cap C_{fk}\neq\varnothing$. This implies that $L_p\cap C_{fi}\cap C_{fk}\cap C_{f4}\neq\varnothing$, therefore P_i, P_k and P_4 satisfy the sufficient condition to determine a FCG (note that the grasp plane defined by P_i, P_k and P_4 lies in S_{ik}).

  On the other hand, since $L_{ij}\subset C_{fj}\Rightarrow P_i\subset C_{fj}$ (this condition is similar those of the Case 3.1) and $P_4\in R'_i\cap S_{ik}$ then $L_p\cap C_{fi}\cap C_{fj}\cap C_{f4}\neq\varnothing$, this implies that P_i, P_j and P_4 determine a FCG.

  Therefore (P_i, P_j, P_k), (P_i, P_j, P_4) and (P_i, P_k, P_4) determine each a FCG.

3.3 If $L_{ij}\not\subset C_{fj}$ and $L_{ik}\subset C_{fk}$ then:

This case is analogous to the previous one, therefore with the same reasoning and swapping C_{fk} by C_{fj} and L_{ij} by L_{ik} results $R_i=R'_i\cap S_{ik}$ (S_{ik} is the subspace equivalent to S_{ij}).

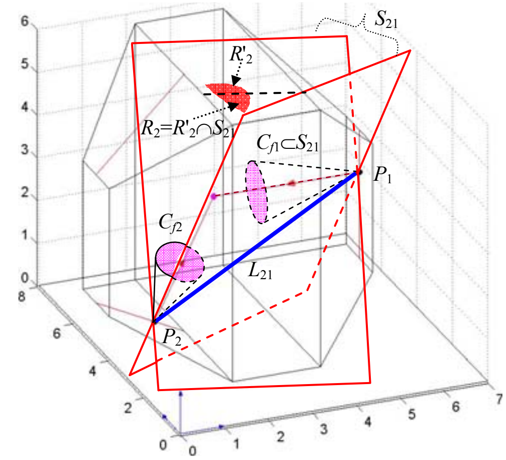

Fig. 8 The sub-space S_{21} contains C_{f1} (C_{f1} is tangent to the limits of S_{21}) and $R_2=R'_2\cap S_y$.

3.4 If $L_{ij}\not\subset C_{fj}$ and $L_{ik}\not\subset C_{fk}$ then:

- Find the sub-space, S_{ik}, limited by two planes tangent to C_{fk} and containing L_{ik} such that $C_{fk}\subset S_{ik}$ (Figure 9), and the sub-space, S_{ij}, limited by two planes tangents to C_{fj} and containing L_{ij} such that $C_{fj}\subset S_{ij}$.

  As it was stated before, any plane that intersects with C_{fk} and that contains P_i and P_k always belongs to S_{ik}, and analogously, any plane that intersects with C_{fj} and that contains P_i and P_j always belongs to S_{ij}.
- Compute $R_i=R'_i\cap S_{ik}\cap S_{ij}$.

  If $P_4\in R'_i\cap S_{ik}\cap S_{ij}$ then is satisfied that $L_p\subset C_{fi}\cap C_{f4}$ (step 1), $L_p\subset S_{ik}$ and $L_p\subset S_{ij}$, and as by construction $C_{fk}\subset S_{ik}$ and $C_{fj}\subset S_{ij}$ (C_{fk} and C_{fj} are tangent to S_{ik} and to S_{ij}, respectively) then $L_p\cap C_{fk}\neq\varnothing$ and $L_p\cap C_{fj}\neq\varnothing$. This implies that $L_p\cap C_{fi}\cap C_{fk}\cap C_{f4}\neq\varnothing$ and $L_p\cap C_{fi}\cap C_{fj}\cap C_{f4}\neq\varnothing$. Therefore ($P_i$, P_k, P_4) and (P_i, P_j, P_4) satisfy the sufficient condition to determine a FCG.

  As a conclusion, (P_i, P_j, P_k), (P_i, P_j, P_4) and (P_i, P_k, P_4) allow a FCG.

Note that if $P_4\in R_i\cap R_j\neq\varnothing$, $i\neq j$, then any of the three fingers located contacting on the original contact points P_i, P_j, P_k can be removed without losing the FCG.

Note that there was no assumption about the position of the contact points on the object faces, thus the approach is valid for contact points on two, three or four faces.

C. Determination of the fourth contact point on R_i

In previous works [10] [11] [15] it was shown that considering (for a precision grasp) the geometry and kinematics of an anthropomorphic hand, the contact points of the best grasp are coplanar or, at least, the contact point of the middle finger is close to the plane defined by the other three contact points (the referenced works use four finger

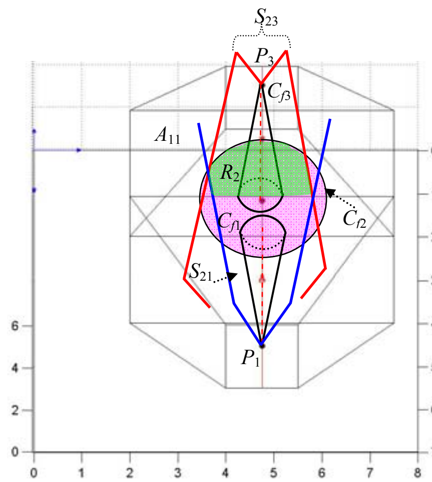

Fig. 9 Computation of $R_2 \subset A_{11}$ according the case $L_{ij} \not\subset C_{fj}$ and $L_{ik} \not\subset C_{fk}$.

anthropomorphic mechanical hands). It is also interesting to remark that, in general, in the examples presented in these works, the line defined by the contact points of the middle finger and the thumb is equidistant from the other two contact points. Then, even when the work presented here cannot assure the reachability of the resulting grasp by a given hand, considering these results, P_4 is selected on R_i looking for:

- A coplanar grasp (i.e. P_i, i=1,2,3,4 are coplanar) (Figure 10), or with P_4 as close as possible to the grasp plane defined by the initial grasp.
- Similar distances between L_p and the two contact points that do not define L_p (i.e. the points that satisfy the condition *C2*).
- A minimum distance between the projections on L_p of the two contact points that do not define L_p.

III. Examples

The application of the proposed methodology is illustrated using two different objects. It is considered a constant friction coefficient μ=0,36. The procedure was implemented in Matlab and run on a server INTEL Biprocessor Pentium III 1,4 GHz. Figures 11 and 12 show several examples of the obtained grasps on the two objects using different initial FCG (the initial grasp is always given by P_1, P_2 and P_3). The computation time needed to determine a grasp with four contact points that satisfy the two conditions of Proposition 1 depends on the case (remind that there are four cases in the procedure described in Section II-B), and varies from 80 ms up to 150 ms.

IV. Conclusion

The proposed procedure determines, given a FCG with three contact points, the regions on the object boundary where the positioning of a fourth finger produces a grasp

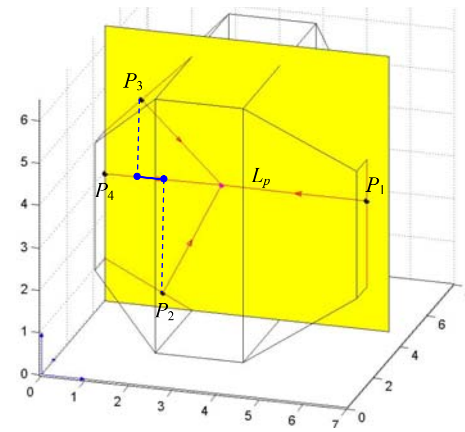

Fig. 10 Computation of P_4.

that allows the loss of two of the three original contacts without losing the force closure property.

This type of grasp can take place on sets of two, three or four object faces, either parallel or non parallel, and is useful to allow the object manipulation by changing the finger contacts on the object (*finger gating*). The proposed approach is based on geometric operations in the 3D physical space. First, a necessary and sufficient condition for the existence of this type of grasp was stated; second, an algorithm for the computation of the contact region for the fourth finger was presented (the region may not exist if the desired type of grasp is not possible); and, finally, how to determine a convenient contact point within this region was described (trying to obtain a planar grasp reachable with an anthropomorphic hand, but this is not formally assured).

As future work it is considered the development of manipulation strategies based on a sequence of grasps determined with the approach presented in this work.

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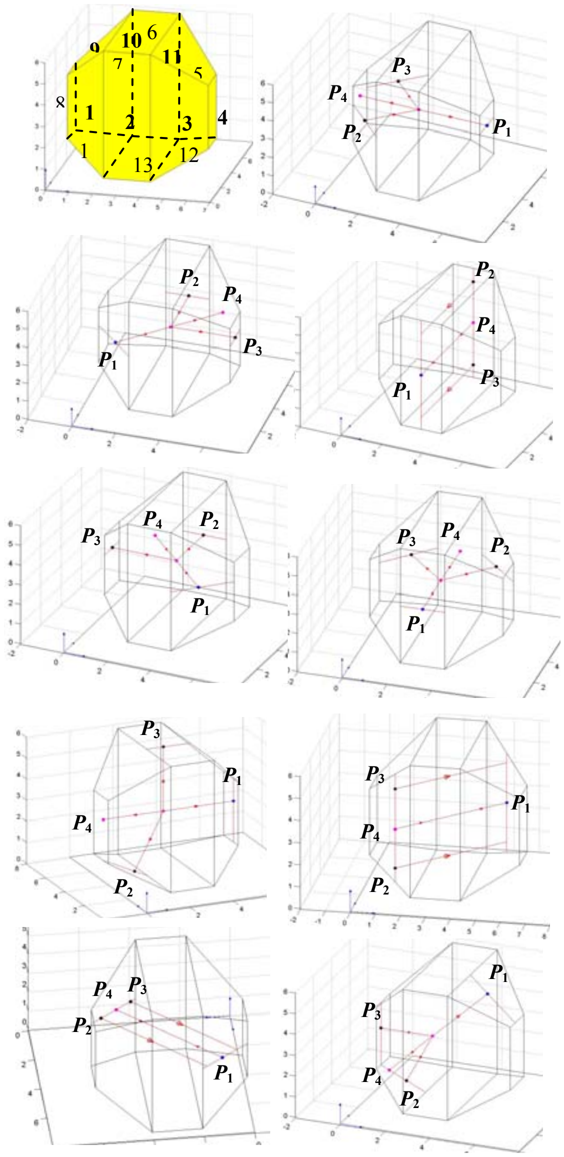

Fig. 11. Different FCG determined by the approach proposed on an object of 14 faces.

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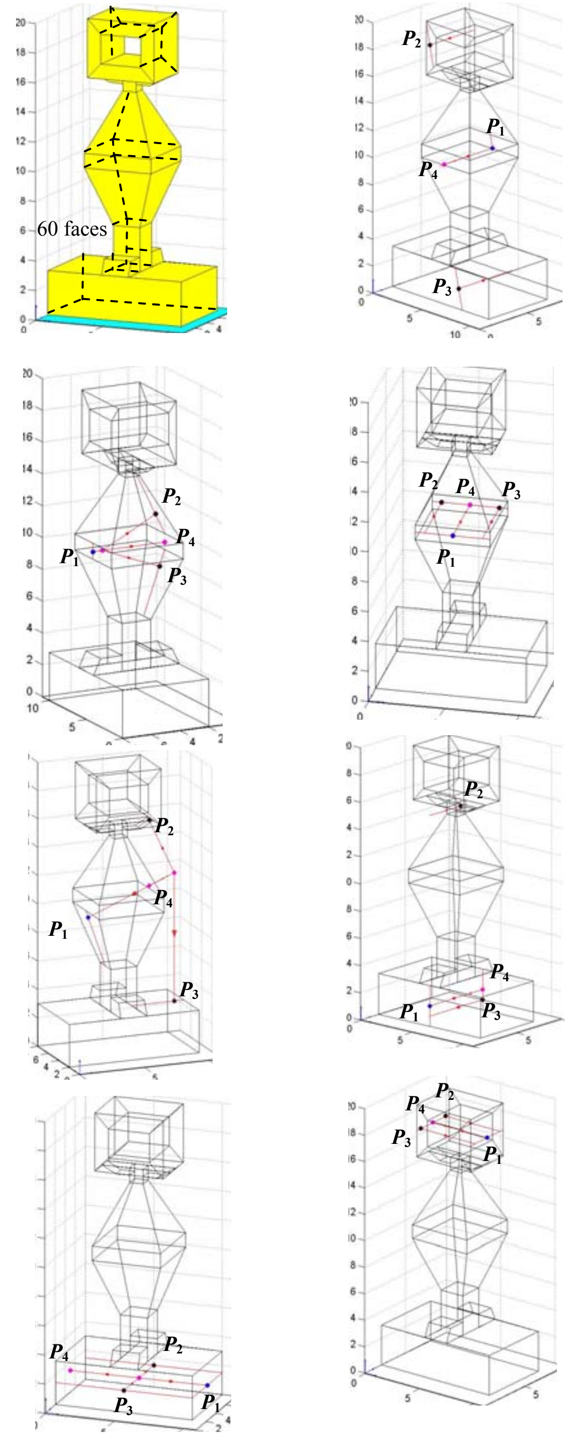

Fig. 12. Different FCG determined by the approach proposed on an object of 60 faces.

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