A New Framework for Planning Three-Finger Grasps of 2D Irregular Objects

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***Abstract*— This paper presents a new approach to obtain three-finger robust grasps of 2D irregular objects. Given a discrete description of the object boundary, a partial representation of the force-closure space considering the position of two contact points is obtained. The computational cost of this representation is low and it still includes enough information to obtain suitable grasps as well as independent regions on the object boundary, such that a finger in each region ensures a force-closure grasp independently of the exact position of the contact points. The procedure has been implemented and several examples of the proposed methodology are included in the paper.**

I. Introduction

Grasping and manipulation of objects by means of mechanical hands has become an active research topic during the last decades. Several mechanical hands have been developed during this time, and an extensive theoretical background has been established [1].

One of the basic requirements of a grasp is the capability of ensuring the immobility of the object despite external disturbances, which is usually characterized by one of the following properties: *form-closure* when the position of the fingers ensures the object immobility, or *force-closure* when the forces applied by the fingers ensure its immobility [2]. In order to select a grasp among all the possible force-closure grasps, algorithms that optimize a quality criterion (for instance [3][4]) or algorithms based on heuristics criterions (for instance [5][6]) were developed. In both cases, these algorithms determine "precise" grasps, i.e. a set of contact points on the object boundary where the fingertips will be placed, requiring a good precision in the fingertip placements. In a real execution, the final grasp and the theoretical grasp may differ due to finger positioning errors. In order to provide grasp robustness in front of these errors, Nguyen [7] introduced the concept of independent regions, which are regions on the object boundary such that a finger in each region ensures a form or force-closure grasp independently of the exact contact point, and he developed geometrical approaches to determine the maximum independent regions on polygonal objects using four frictionless and two frictional contact points. Nguyen's approach was extended to grasp polygonal objects with three frictional contact points [8], and to grasp polyhedral objects with four frictional contact points [9]. A closely related problem is the determination of all the possible force-closure grasps of an object, called the force-closure space. In this line, algorithms to determine the force-closure space considering any number of fingers and polygonal objects can be found in [10][11], and the determination of independent regions using the force-closure space can be found in [12].

A common approach is the representation of the object boundary with a set of sampled points, and several iterative algorithms were proposed to search there a subset of contact points that produce a form/force-closure grasp [13] [14] [15].

The problem of finding independent regions on 2D irregular objects was tackled considering two frictional [16] and four frictionless [17] contact points, and a method to compute all the form-closure grasps considering four frictionless contacts can be found in [18]. For 3D objects, a general approach to determine independent regions based on initial examples was proposed in [19], although the selection of a good initial example remains as a critical step.

This paper presents a new methodology to obtain independent regions on 2D irregular objects considering three-finger grasps. Given a discrete description of the object boundary, a partial representation of the whole force-closure space is obtained using some characteristic points of the object. The computational cost of this representation is low, since it only considers the position of two of the three contact points, and it still includes enough information to obtain suitable grasps as well as independent regions on the object boundary. The proposed approach is also valid for polygonal objects, with the advantage with respect to previous works, that the independent regions are not constrained to lie on the same contact edge, but can have parts on several contiguous edges.

The paper is organized as follows. Section II presents some basic nomenclature and the force-closure conditions that the grasp has to satisfy. Section III describes the proposed methodology to construct a partial force-closure space and how to use it to determine the grasping points and the independent regions. Examples of the proposed methodology are included in Section IV. Finally, some concluding remarks and possible future lines to extend this work are pointed out in Section V.

II. Preliminary Concepts

A. Object and Force Representation

Let $\mathcal{B}(u)$ be a smooth and closed curve describing the actual object boundary parameterized on u, $\boldsymbol{p}_j = \mathcal{B}(u_j)$ be a point on the object boundary for a given u_j and θ_j be the inward normal direction at $\boldsymbol{p}_j$.

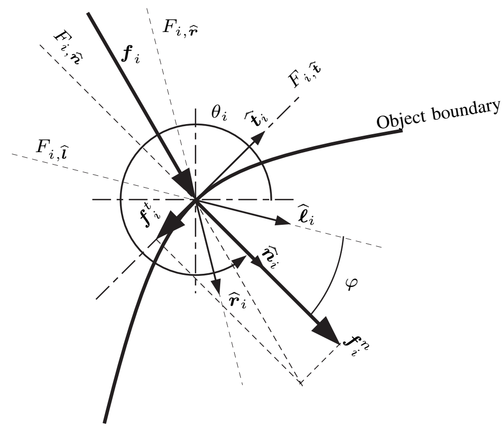


Fig. 1. Friction contact, where $\boldsymbol{f}_i$ is the applied force, $\boldsymbol{f}_i^n$ and $\boldsymbol{f}_i^t$ are its normal and tangent components with directions $\widehat{\boldsymbol{n}}_i$ and $\widehat{\boldsymbol{t}}_i$, respectively, $\widehat{\boldsymbol{r}}_i$ and $\widehat{\boldsymbol{\ell}}_i$ are the unitary primitive vectors, and $F_{i,\widehat{\boldsymbol{r}}}$, $F_{i,\widehat{\boldsymbol{l}}}$, $F_{i,\widehat{\boldsymbol{l}}}$ and $F_{i,\widehat{\boldsymbol{t}}}$ are the supporting lines of the these force components.

Let $\mathcal{B}_d = \{\boldsymbol{q}_i, i = 1, ..., N\}$ be a discrete description of $\mathcal{B}(u)$, with N sampled points $\boldsymbol{q}_i$. Given $\mathcal{B}_d$ and the inward normal direction θ_i at each point $\boldsymbol{q}_i$, it is assumed that if $\boldsymbol{p}_j \in [\boldsymbol{q}_i, \boldsymbol{q}_{i+1}]$ then $\theta_j \in [\theta_i, \theta_{i+1}]$ (the obtention of $\mathcal{B}_d$ and θ_i is outside of the scope of this paper, and it can be done as in [20]).

Let $\boldsymbol{f}_i$ be the force exerted by a finger on the object, either on a point $\boldsymbol{p}_i$ or $\boldsymbol{q}_i$, and let $\boldsymbol{f}_i^n$ and $\boldsymbol{f}_i^t$ be its components normal and tangent to the object boundary whose directions are given by the unitary vectors $\widehat{\boldsymbol{n}}_i = (\cos\theta_i \;\; \sin\theta_i)^T$ and $\widehat{\boldsymbol{t}}_i = (-\sin\theta_i \;\; \cos\theta_i)^T$, respectively (Fig. 1).

Based on the Coulomb model of friction, the finger slippage on the object boundary is avoided if:

$$\mu \|\boldsymbol{f}_i^n\| \geq \|\boldsymbol{f}_i^t\| \tag{1}$$

being μ the friction coefficient. Geometrically, equation (1) constraints the force applied by the finger to lie inside a friction cone centered on the direction normal to the object boundary and limited by the called primitive forces, $\boldsymbol{f}_i^r$ and $\boldsymbol{f}_i^\ell$, whose directions are given by the following unitary primitive vectors:

$$\widehat{\boldsymbol{r}}_i = [\cos(\theta_i - \varphi) \;\; \sin(\theta_i - \varphi)] \tag{2}$$

$$\widehat{\boldsymbol{\ell}}_i = [\cos(\theta_i + \varphi) \;\; \sin(\theta_i + \varphi)] \tag{3}$$

with $\varphi = \arctan\mu$. In the rest of the paper, $\boldsymbol{f}_i^c$ and $\widehat{\boldsymbol{c}}_i$ will be used to generically represent the primitive forces and the unitary primitive vectors, respectively (i.e. the supra-index c can mean either r or ℓ and $\widehat{\boldsymbol{c}}_i$ can mean either $\widehat{\boldsymbol{r}}_i$ or $\widehat{\boldsymbol{\ell}}_i$).

The straight lines that support each of the described forces at $\boldsymbol{p}_i$ will also be used. For a generic unitary vector $\widehat{\boldsymbol{v}}_i$, these straight lines are given by:

$$F_{i,\widehat{\boldsymbol{v}}} = \{(x, y) \;/\; (x, y) = \boldsymbol{p}_i + \alpha\widehat{\boldsymbol{v}}_i\} \tag{4}$$

where $-\infty < \alpha < \infty$ and (x, y) represents a point in the object space.

B. Generalized Opposite and Antipodal Points

The term *opposite points* has been often used in the literature (see [21], among others) to call a pair of points on the object boundary whose normal forces have opposite directions (i.e. $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are opposite points if $\widehat{\boldsymbol{n}}_i = -\widehat{\boldsymbol{n}}_j$). Besides, if the normal forces applied at two opposite points are collinear, these points are called *antipodal points* (i.e. $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are antipodal points if $F_{i,\widehat{\boldsymbol{n}}} = F_{j,\widehat{\boldsymbol{n}}}$).

In this paper, the concepts of opposite and antipodal points are generalized including also the primitive forces, giving the following types of opposite and antipodal points:

- *Primitive-Primitive Opposite (PPO) and Primitive-Primitive Antipodal (PPA) points.* Two points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are PPO if a primitive force applied at $\boldsymbol{p}_i$ and a primitive force applied at $\boldsymbol{p}_j$ have opposite directions. Besides, if these forces are collinear these points are also PPA.
- *Primitive-Normal Opposite (PNO) and Primitive-Normal Antipodal (PNA) points.* Two points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are PNO if a primitive force applied at $\boldsymbol{p}_i$ and the normal force applied at $\boldsymbol{p}_j$ have opposite directions. Besides, if these forces are collinear these points are also PNA.
- *Normal-Normal Opposite (NNO) and Normal-Normal Antipodal (NNA) points.* Two points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are NNO if the normal force applied at $\boldsymbol{p}_i$ and the normal force applied at $\boldsymbol{p}_j$ have opposite directions. Besides, if these forces are collinear these points are also NNA.

Let $\widehat{\boldsymbol{v}}_i$ and $\widehat{\boldsymbol{v}}_j$ be two generic unitary vectors that define the directions of either a primitive force or the normal force applied at $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$. If these points are PPO, PNO or NNO, they will be denoted as:

$$\boldsymbol{p}_i^v = Op(\boldsymbol{p}_j^v) \Leftrightarrow \boldsymbol{p}_i^v = \{\mathcal{B}(u_i) \;/\; \widehat{\boldsymbol{v}}_i = -\widehat{\boldsymbol{v}}_j\} \tag{5}$$

and if $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are PPA, PNA or NNA, they will denoted as:

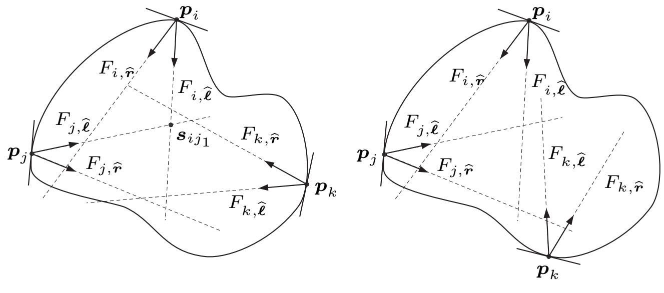
$$\boldsymbol{p}_i^v = Ap(\boldsymbol{p}_j^v) \Leftrightarrow \boldsymbol{p}_i^v = \{\mathcal{B}(u_i) \;/\; \widehat{\boldsymbol{v}}_i = -\widehat{\boldsymbol{v}}_j, F_{i,\widehat{\boldsymbol{v}}} = F_{j,\widehat{\boldsymbol{v}}}\} \tag{6}$$

where the supra-index v in equations (5) and (6) indicates the force component considered at each point (i.e. either r, ℓ or n) and, therefore, it determines the type of generalized opposite or antipodal points.

C. Force-closure Conditions

A set of contact points allows a force-closure grasp (hereafter FC grasp), if and only if the convex hull defined by the primitive wrenches contains the origin [22]. Even when this is a general necessary and sufficient condition and it can be applied considering 2D and 3D objects and any number of fingers, some authors have developed other necessary and sufficient conditions that avoid to compute the convex hull in some specifics cases. For 2D objects and three-finger grasps, the necessary and sufficient condition presented in the following proposition was stated in [23].

Proposition 1: (from [23]) Three contact points $\boldsymbol{p}_i$, $\boldsymbol{p}_j$ and $\boldsymbol{p}_k$ allow a FC grasp if and only if: (a) the unitary primitive vectors that bound the friction cones at these points positively



(a) (b)

Fig. 2. (a) FC grasp: at least the intersection point $\boldsymbol{s}_{ij_1}$ lies inside the friction cone defined by $F_{k,\widehat{\boldsymbol{r}}}$ and $F_{k,\widehat{\boldsymbol{\ell}}}$; b) Non-FC grasp: there is not an intersection between the supporting lines of two primitive forces inside the third friction cone.

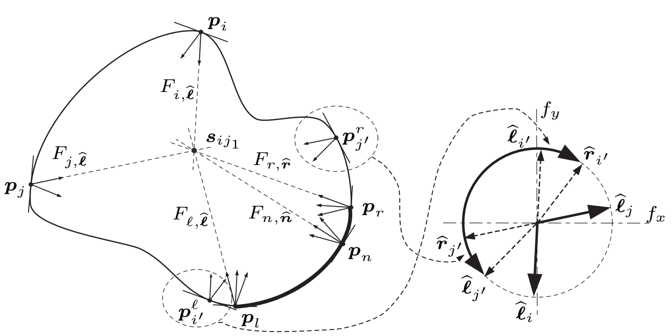


Fig. 3. Region where $\boldsymbol{p}_k$ has to be placed on the object boundary in order to obtain a FC grasp, considering the intersection between $F_{i,\widehat{\boldsymbol{\ell}}}$ and $F_{j,\widehat{\boldsymbol{\ell}}}$. The unitary vectors that bound the friction cone at any point between $\boldsymbol{p}_{i'}$ and $\boldsymbol{p}_{j'}$ spans the force space together with $\widehat{\boldsymbol{\ell}}_i$ and $\widehat{\boldsymbol{\ell}}_j$, and $\boldsymbol{s}_{ij_1}$ lies inside the friction cone at any point between $\boldsymbol{p}_l$ and $\boldsymbol{p}_r$; the intersection of these regions determines where $\boldsymbol{p}_k$ has to be placed.

span the force space, and (b) at least one intersection point between the supporting lines of two primitive forces lies inside the double-side friction cone at the other contact point. ◇

Fig. 2a shows an example of three contact points that satisfy the necessary and sufficient condition in Proposition 1, allowing a FC grasp, and Fig. 2b shows an example of three contact points that do not satisfy the condition.

From Proposition 1 the following two Lemmas can also be stated (illustrated in Fig. 3).

Lemma 1: Consider two contact points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, and let $\boldsymbol{s}_{ij_m}$ be the intersection point between the straight lines $F_{i,\widehat{\boldsymbol{c}}}$ and $F_{j,\widehat{\boldsymbol{c}}}$ (remember that $\widehat{\boldsymbol{c}}$ can be either $\widehat{\boldsymbol{r}}$ or $\widehat{\boldsymbol{\ell}}$). In order to obtain a FC grasp, the third point $\boldsymbol{p}_k$ must lie in the intersection of the following two regions on the object boundary:

- The region of points where the unitary vectors that bound the friction cone together with the unitary vectors of the primitive forces that determine $\boldsymbol{s}_{ij_m}$, positively span the force space. This region is bounded by the points $\boldsymbol{p}^{\ell}_{i'} = Op(\boldsymbol{p}^c_i)$ and $\boldsymbol{p}^r_{j'} = Op(\boldsymbol{p}^c_j)$ (note that this region is always continuous when the object is convex, while it can be discontinuous when the object is concave).
- The region of points whose friction cones contain $\boldsymbol{s}_{ij_m}$. This region is bounded by two points $\boldsymbol{p}_\ell$ and $\boldsymbol{p}_r$ defined as:

$$\boldsymbol{p}_\ell = \{\mathcal{B}(u_\ell) \;/\; \boldsymbol{s}_{ij_m} \in F_{\ell,\widehat{\boldsymbol{\ell}}}\} \quad (7)$$

$$\boldsymbol{p}_r = \{\mathcal{B}(u_r) \;/\; \boldsymbol{s}_{ij_m} \in F_{r,\widehat{\boldsymbol{r}}}\} \quad (8)$$

(analogously to the previous region, this one can be discontinuous when the object is concave) ◇

Proof: The proof is straightforward from Proposition 1. The first region includes the points of the object boundary that satisfy condition (a), and the second one includes the points that satisfy condition (b). Then, the intersection of these two regions contains the points where both conditions are satisfied at the same time. ◇

Lemma 2: Let $\boldsymbol{p}_\ell$ and $\boldsymbol{p}_r$ be two points on the object boundary defined by equations (7) and (8). It always exists a point $\boldsymbol{p}_n$ between them that satisfies:

$$\boldsymbol{p}_n = \{\mathcal{B}(u_n) \;/\; \boldsymbol{s}_{ij_m} \in F_{n,\widehat{\boldsymbol{n}}}\} \quad (9)$$

◇

Proof: The proof is straightforward since the object boundary is smooth and the direction of the force varies continuously. ◇

These two Lemmas will be used in the following section to obtain FC grasps.

III. Determination of Force-Closure Grasps

Definition 1: The *contact space* is the space defined by the three parameters that determine the positions of the three contact points on the object boundary. ◇

Definition 2: The *force-closure space, FC space,* is the subset of the contact space where FC grasps can be obtained. ◇

Efficient algorithms have been developed to obtain the FC space for any number of fingers and polygonal objects [10] [11] [12]. Instead, up to where we know, when the object is irregular and three fingers are considered there is no other proposed solution than checking by brute force all the possible combination of contact points. The computational cost of this approach is at least $O(N^3)$, where N is the number of sampled points, which must be large enough to get a good description of the object. Then, the obtention of the FC space is computationally expensive.

In this section, a procedure to obtain a partial representation of the FC space is presented. The computational cost of this approach is $O(N^2)$, since it only considers the position of two contact points and the existence of PNA points on the object boundary. This representation can be used to determine single FC grasps as well as independent regions.

A. Partial Force-Closure Space Representation

Consider an intersection point $\boldsymbol{s}_{ij_m}$ between the supporting lines, $F_{i,\widehat{\boldsymbol{c}}}$ and $F_{j,\widehat{\boldsymbol{c}}}$, of two primitive forces. From Lemma 1, the region where $\boldsymbol{p}_k$ must lie to allow the resulting unitary vectors positively span the force space is limited by PPO points of $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$. Instead of PPO points, it will be considered that this region is limited by PNO points of $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$. This reduces the actual range of possible directions of forces an amount equivalent to a friction cone semi-angle in each side (the neglected regions are extreme regions where a FC grasp is rarely possible or very close to a critical grasp). As a result, $\boldsymbol{p}_k$ must lie on the region of the object boundary limited by $\boldsymbol{p}^n_{i'} = Op(\boldsymbol{p}^c_i)$ and $\boldsymbol{p}^n_{j'} = Op(\boldsymbol{p}^c_j)$.

Let Γ^c_i and Γ^c_j be the torques produced by the unitary normal vectors at $\boldsymbol{p}^n_{i'}$ and $\boldsymbol{p}^n_{j'}$ with respect to the intersection point $\boldsymbol{s}_{ij_m}$. Since $\boldsymbol{p}^n_{i'} = Op(\boldsymbol{p}^c_i)$ the torque Γ^c_i does not depend on the position of $\boldsymbol{p}_j$ (the same reasoning can be applied considering Γ^c_j). Then, Γ^c_i and Γ^c_j can be determined as:

$$\Gamma^c_i = (\boldsymbol{p}^c_i - \boldsymbol{p}^n_{i'}) \times \widehat{\boldsymbol{n}}_{i'} \qquad (10)$$

$$\Gamma^c_j = (\boldsymbol{p}^c_j - \boldsymbol{p}^n_{j'}) \times \widehat{\boldsymbol{n}}_{j'} \qquad (11)$$

According to the signs of Γ^c_i and Γ^c_j, a combination of two contact points, $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, is classified as:

- *Odd Combination:*

$$sign(\Gamma^c_i) \neq sign(\Gamma^c_j) \qquad (12)$$

  In this case, there exists an odd number of points between $\boldsymbol{p}^n_{i'}$ and $\boldsymbol{p}^n_{j'}$, where the normal force produces a null torque with respect to $\boldsymbol{s}_{ij_m}$, and the sign of the torque produced by the normal force is different on each side of these points. In other words, there exists an odd number of points $\boldsymbol{p}_n$ (defined in Lemma 2) such that the supporting line of the normal force applied at $\boldsymbol{p}_n$ contains $\boldsymbol{s}_{ij_m}$. The existence of these points between $\boldsymbol{p}^n_{i'}$ and $\boldsymbol{p}^n_{j'}$ guaranties that the intersection between the regions defined in Lemma 1 is not null. Therefore, it is possible to assure that $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ together with a proper third contact point can produce a FC grasp, even when the exact position of the third point has not yet been determined.

- *Even Combination:*

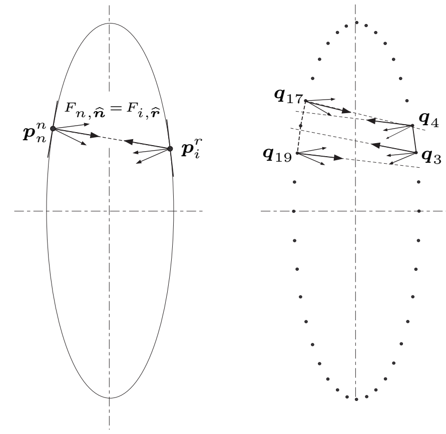
$$sign(\Gamma^c_i) = sign(\Gamma^c_j) \qquad (13)$$

  In this case, there exists none or an even number of points between $\boldsymbol{p}^n_{i'}$ and $\boldsymbol{p}^n_{j'}$ where the normal force produces a torque with respect to $\boldsymbol{s}_{ij_m}$ equal to zero, and the sign of the torque produced by the normal force is different on each side of these points. As a result, it is not possible to assure the existence of points $\boldsymbol{p}_n$ between them and the obtention of a FC grasp with $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ is uncertain.

The values of Γ^c_i and Γ^c_j are determined by equations (10) and (11) considering two PNO points, which implies that Γ^c_i and Γ^c_j can be zero only when these points are also PNA, i.e. $\Gamma^c_i = 0$ when $F_{i,\widehat{\boldsymbol{c}}} = F_{i',\widehat{\boldsymbol{n}}}$ and $\Gamma^c_j = 0$ when $F_{j,\widehat{\boldsymbol{c}}} = F_{j',\widehat{\boldsymbol{n}}}$. Therefore, the signs of Γ^c_i and Γ^c_j can change only at PNA points, and all the points between two of them produce values of Γ^c_i or Γ^c_j with the same sign.

When the object boundary is described by $\mathcal{B}_d$ as a finite number of points, it is not possible to exactly compute the PNA points, but it is possible to determine the regions where the antipodal points are located, using the following algorithm.

Algorithm 1: For each pair of consecutive points $\boldsymbol{q}_i$ and $\boldsymbol{q}_{i+1}$ of $\mathcal{B}_d$, do:



(a) (b)

Fig. 4. Determination of a PNA point on the boundary of an ellipse considering the primitive force $\boldsymbol{f}^r_i$ (a) Exact determination (b) Approximated determination for a discrete description of the object.

1. Find two points $\boldsymbol{q}_u$ and $\boldsymbol{q}_v$ of $\mathcal{B}_d$ with minimum number of points between them that satisfy:

$$\theta_u \leq \quad \theta_i + \pi \pm \varphi \quad \leq \theta_v \qquad (14)$$

$$\theta_u \leq \quad \theta_{i+1} + \pi \pm \varphi \quad \leq \theta_v \qquad (15)$$

   where φ is added or subtracted depending on the primitive forces that is being considered (see equations (2) and (3)). If the object is concave it may be more than one pair of points that satisfy equations (14) and (15).

2. Compute the equations of the supporting lines $F_{i,\widehat{\boldsymbol{c}}}$ and $F_{i+1,\widehat{\boldsymbol{c}}}$ and evaluate them for $\boldsymbol{q}_u$ and $\boldsymbol{q}_v$:

$$S_1 = \{F_{i,\widehat{\boldsymbol{c}}}(\boldsymbol{q}_u), F_{i+1,\widehat{\boldsymbol{c}}}(\boldsymbol{q}_u), F_{i,\widehat{\boldsymbol{c}}}(\boldsymbol{q}_v), F_{i+1,\widehat{\boldsymbol{c}}}(\boldsymbol{q}_v)\} \qquad (16)$$

3. Compute the equations of the supporting lines $F_{u,\widehat{\boldsymbol{n}}}$ and $F_{v,\widehat{\boldsymbol{n}}}$ and evaluate them for $\boldsymbol{q}_i$ and $\boldsymbol{q}_{i+1}$, obtaining:

$$S_1 = \{F_{u,\widehat{\boldsymbol{n}}}(\boldsymbol{q}_i), F_{v,\widehat{\boldsymbol{n}}}(\boldsymbol{q}_i), F_{u,\widehat{\boldsymbol{n}}}(\boldsymbol{q}_{i+1}), F_{v,\widehat{\boldsymbol{n}}}(\boldsymbol{q}_{i+1})\} \qquad (17)$$

3. If there are elements with different signs both in S_1 and S_2, then there is a PNA point between $\boldsymbol{q}_i$ and $\boldsymbol{q}_{i+1}$. ⋄

As an example, Fig. 4a shows the exact determination of a pair of PNA points on the boundary of an ellipse considering the primitive force $\boldsymbol{f}^r_i$, and Fig. 4b shows the bounds of the region where these PNA points are located when the boundary of the ellipse is described by a discrete set of points.

Consider the 2D space defined by the parameters that fix the positions of $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$. The bounds of the regions containing the PNA points obtained with the primitive forces $\boldsymbol{f}^r_i$ and $\boldsymbol{f}^r_j$ make a partition of this space into rectangular cells. Based on the property that the signs of Γ^r_i and Γ^r_j are equal for all the points between two of these PNA points, each cell of the partition is classified as one of the following three types:

Odd cells: Contain the points whose coordinates represent two contact points that satisfy equation (12). Therefore, it is always possible to obtain a FC grasp for any combination of two contact points that define a point into an odd cell.

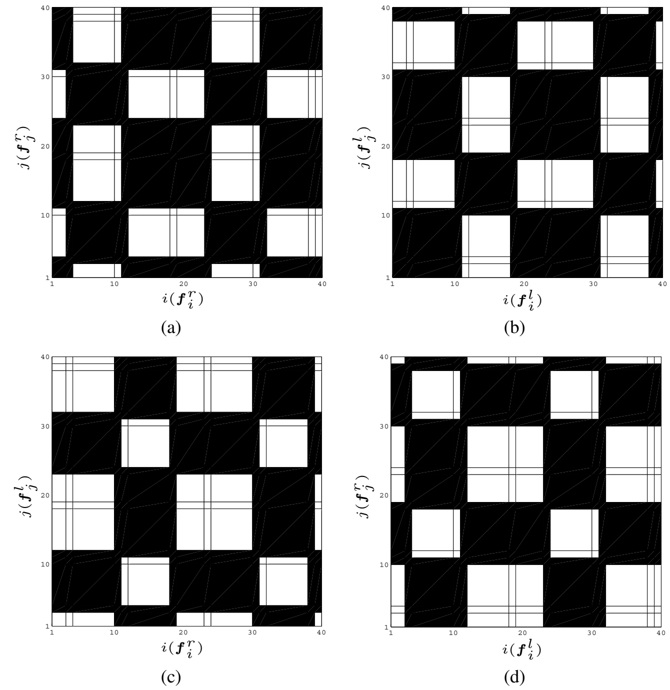


Fig. 5. Partial Primitive-Normal ($\mathcal{P}_{pn_m}$) partitions associated with an ellipse (a) $\mathcal{P}_{pn_1}$ partition considering $\boldsymbol{f}_i^r$ and $\boldsymbol{f}_j^r$; (b) $\mathcal{P}_{pn_2}$ partition considering $\boldsymbol{f}_i^\ell$ and $\boldsymbol{f}_j^\ell$; (c) $\mathcal{P}_{pn_3}$ partition considering $\boldsymbol{f}_i^r$ and $\boldsymbol{f}_j^\ell$; (d) $\mathcal{P}_{pn_4}$ partition considering $\boldsymbol{f}_i^\ell$ and $\boldsymbol{f}_j^r$;

Even cells: Contain the points whose coordinates represent two contact points that satisfy equation (13). Therefore, the obtention of a FC grasp with any combination of two contact points that define a point into an even cell is uncertain.

Uncertain cells: These cells are due to the discrete description of the object boundary, and contain the PNA points.

Fig. 5a shows an example of this partition considering the ellipse of Fig. 4b, where the odd cells are white and the even and the uncertain cells are black.

The same partition can be done in this 2D space taking into account the possible combinations between $\boldsymbol{f}_i^r$, $\boldsymbol{f}_i^\ell$, $\boldsymbol{f}_j^r$ and $\boldsymbol{f}_j^\ell$, and the resulting PNA points. Then, four different partitions are obtained, which will be called *Partial Primitive-Normal Partitions ($\mathcal{P}_{pn_m}$ partition)*, with $m = 1, ..., 4$. Fig. 5 shows these four $\mathcal{P}_{pn_m}$ partitions for the ellipse of Fig. 4b.

The union of the four $\mathcal{P}_{pn_m}$ partitions gives the *Total Primitive-Normal Partition ($\mathcal{T}_{pn}$ partition)*, where the 2D space is divided into cells composed by the parameters that fix the positions of two contact points that are classified as an odd combination either in all the $\mathcal{P}_{pn_m}$ partitions, in some of them or none of them. The index $\mathcal{C} = 0, ..., 4$ indicates the number of times that each combination of two contact points is classified as an odd combination in the original partitions, i.e. $\mathcal{C} = 0$ indicates that the combination is not classified as odd in any of the four $\mathcal{T}_{pn}$ partitions, while $\mathcal{C} = 4$ indicates that the combination is classified as odd in all of them. Following with the example of the ellipse, Fig. 6 shows its $\mathcal{T}_{pn}$ partition

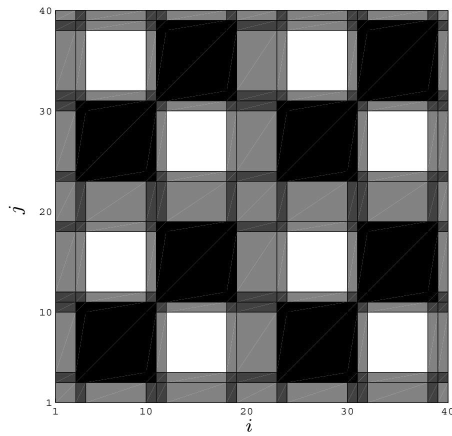


Fig. 6. Total Primitive-Normal ($\mathcal{T}_{pn}$) partition of an ellipse and where the index $\mathcal{C}$ is represented as a gray-scale (a black cell means $\mathcal{C} = 0$ while a white cell means $\mathcal{C} = 4$).

where the index $\mathcal{C}$ is represented as a gray-scale such that a black cell means $\mathcal{C} = 0$ and a white cell means $\mathcal{C} = 4$.

The index $\mathcal{C}$ can be easily determined just checking a combination of any two contacts from each cell of the $\mathcal{T}_{pn}$ partition. Moreover, since the $\mathcal{T}_{pn}$ partition is always symmetric with respect the straight line $i = j$, it is only necessary to check a half of the space.

The $\mathcal{T}_{pn}$ partition can be used for grasp planning since any combination of two contact points whose $\mathcal{C} \neq 0$ allows a FC grasp. Let $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ be two contact points, $\mathcal{C}$ can be geometrically interpreted as the number of intersection points, $\boldsymbol{s}_{ij_m}$, between the supporting lines $F_{i,\widehat{\mathbf{c}}}$, $F_{j,\widehat{\mathbf{c}}}$ that lies inside the friction cone of the third point $\boldsymbol{p}_k$ (Fig. 2). Therefore, $\mathcal{C}$ can be used as a measure of the robustness of the FC grasp, obtaining the most robust FC grasps when $\mathcal{C} = 4$ (this is the maximum possible value). In the next subsection the $\mathcal{T}_{pn}$ partition (especially the cells with maximum $\mathcal{C}$) is used to obtain independent regions on the object boundary.

B. *Independent regions*

The independent regions were defined as regions on the object boundary such that a finger in each region ensures a FC grasp with independence of the exact contact point [7]. These regions are useful to provide robustness to the grasp in front of finger positioning errors, as well as for grasp planning.

Geometrically, the independent regions define a parallelepiped fully contained in the FC space (definition 2). In order to obtain a good (or eventually an optimal) set of regions, the length of the shortest edge of the parallelepiped, which represents the shortest region on the object boundary, is commonly used as quality measure and the criterion is to maximize it [8]. Since the object is represented by a set of sampled points, the length of the edge is determined by the number of points enclosed in the shortest region (this measure is valid when the sampled points are well distributed on the actual object boundary). With this criterion, and taking into account the cells of the $\mathcal{T}_{pn}$ partition where $\mathcal{C}$ is maximum, the determination of the independent regions is done with the following algorithm.

Algorithm 2: Select a cell of the $\mathcal{T}_{pn}$ partition where $\mathcal{C}$ is maximum, and do:

1. Initialize the sets I and J with the coordinates of the midpoint of the selected cell, and determine the set K applying Lemma 1 considering these initial points.
2. Compute the resultant region of applying Lemma 1 considering the neighbor points of I and J.
3. Add to I and J the neighbor points such that I and J define a rectangle in the $\mathcal{T}_{pn}$ partition and the length of the minimum region is increased.
4. Repeat step 2 and 3 until a decreasing in the minimum region is detected (in this way, the other two regions are enlarged).
5. Return I, J and K. $\diamond$

The $\mathcal{T}_{pn}$ partition is obtained considering the position of two contact points, and the unique information that provide about the third point is whether it allows a FC grasp or not. This implies that, in some cases, the final independent region for the third finger obtained from Algorithm 2 could be very short (although it always exists), even when the index $\mathcal{C}$ is maximum. To overcome this problem, Algorithm 2 can be applied for several cells of $\mathcal{T}_{pn}$, starting with the cells of largest edges and taking as the final grasp the one with largest independent regions, instead of trying to obtain more information about the third point, which could increase more the computational cost of the proposed procedure.

IV. Examples

Two examples of the proposed methodology are presented in this section. In the first one, the object to be grasped is the ellipse used through the paper to illustrate the methodology, and in the second one an irregular object is used. The friction coefficient used in both examples is $\mu = 0.3$.

Example 1. Consider the ellipse described by 40 points shown in Fig. 4b and its associated $\mathcal{T}_{pn}$ partition shown in Fig. 6. There are four different white cells where $\mathcal{C} = 4$ (remember that the $\mathcal{T}_{pn}$ partition is symmetric with respect $i = j$). Moreover, due to the own symmetry of the object, only two of these four cells give different results.

Consider the Cell 1 defined by $i = [12, ..., 18]$ and $j = [4, ..., 10]$. The following independent regions are obtained using Algorithm 2:

$$I = [12, 18],\ J = [4, 10] \text{ and } K = [31]$$

Fig. 7a shows these independent regions on the object boundary. Note that, although the four intersection points between the supporting lines of the primitive forces applied at any point of I and K lie inside the friction cone at K, this independent region contains only one point, since the $\mathcal{T}_{pn}$ partition only considers the position of $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$.

The largest independent regions are obtained when the cell 2 defined by $i = [24, ..., 30]$ and $j = [12, ..., 18]$ is considered. In this case, the following independent regions are obtained using Algorithm 2:

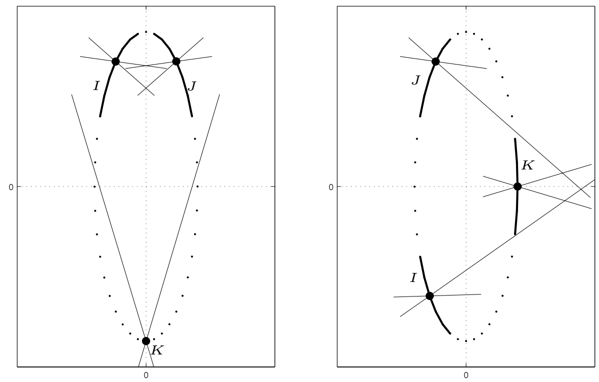


Fig. 7. Independent regions on the ellipse and friction cones at three possible contact points considering two different cells of the $\mathcal{T}_{pn}$ partition: (a) Cell 1 (b) Cell 2.

$$I = [24, 29],\ J = [13, 18] \text{ and } K = [39, 3]$$

Fig. 7b shows these independent regions on the object boundary. Since these independent regions are larger, this grasp is more robust in front of finger positioning errors than the previous one.

Example 2. Consider the object described by 223 points shown in Fig. 8a. The initial data are the coordinates of each point and its normal inward direction (Fig. 8b). These initial data has been obtained using the approach presented in [20].

The first step is to determine the PNA points using Algorithm 1, and to construct the $\mathcal{T}_{pn}$ partition, shown in Fig. 9. Two different good independent regions are obtained considering the Cells 1 and 2.

The Cell 1 is defined by $i = [110, ..., 130]$ and $j = [39, ..., 49]$. The following independent regions are obtained using Algorithm 2:

$$I = [115, 130],\ J = [39, 53] \text{ and } K = [154, 168]$$

The Cell 2 is defined by $i = [110, ..., 130]$ and $j = [162, ..., 175]$. The following independent regions are obtained using Algorithm 2:

$$I = [110, 126],\ J = [162, 175] \text{ and } K = [94, 108]$$

Fig. 10 shows the independent regions on the object boundary considering both cells. Both solutions are adequate since the difference between the shortest regions is just one sample point.

V. Conclusions and Future Works

In this paper a new framework for planning and determining tree-finger FC grasps on 2D irregular objects has been presented. The proposed approach obtains a partial representation of the FC space, called *Total Primitive-Normal ($\mathcal{T}_{pn}$) Partition*, which contains enough information to determine FC grasps as well as independent regions on the object boundary. Besides, it is not necessary to obtain the whole FC space, which reduces the computational cost of the proposed approach.

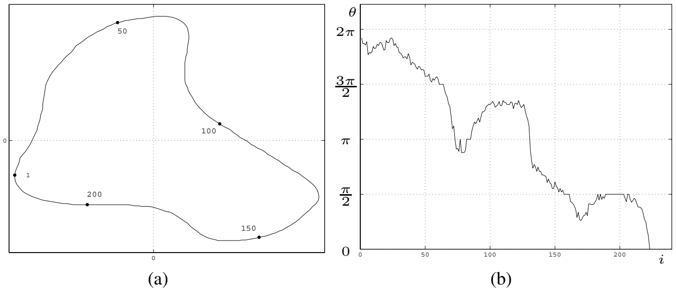


Fig. 8. Irregular object: (a) Object boundary (223 points); (b) Normal direction at each point.

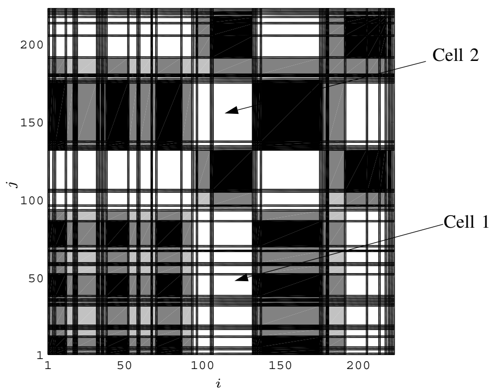


Fig. 9. Total Primitive-Normal ($\mathcal{T}_{pn}$) partition of to the irregular object and cells used to determine the two sets of independent regions.

Future works include the extension of the proposed approach considering more constraints on the contact points in the construction of the $\mathcal{T}_{pn}$ partition, like the kinematics of a mechanical hand or task requirements. Another related field where the proposed approach can be applied is the planning of regrasp sequences in order to reach a final grasp from an initial one, doing in this way dexterous manipulation of irregular objects.

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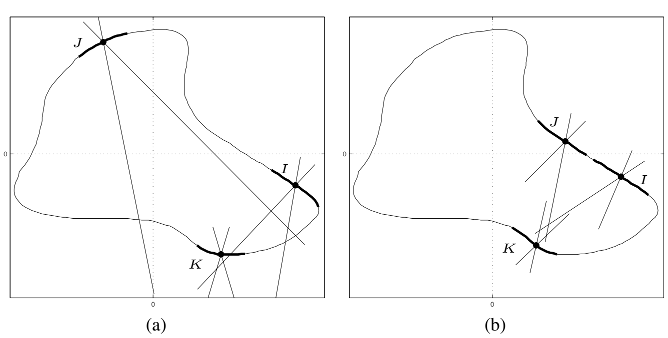

Fig. 10. Independent regions on the irregular object boundary and friction cones at three possible contact points considering two different cells of the $\mathcal{T}_{pn}$ partition: (a) Cell 1 (b) Cell 2.

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