Determining Independent Grasp Regions on 2D Discrete Objects*

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Abstract— **This paper deals with the problem of determining independent grasps regions on the object boundary such that a four frictionless grasp with a contact point in each region assures a form-closure grasp independently of the exact position of the contact point. These regions are useful to provide some robustness to the grasp in front of finger positioning errors as well as in the design of fixtures. Given a discrete description of a 2D object, the methodology takes into account the uncertainty in the object description and it determines the independent regions without using hard iterative search procedures. The procedure has been implemented and an example of the proposed methodology is included in the paper.**

Index Terms— **Grasp synthesis, fixture design, form-closure independent regions, discrete objects.**

I. Introduction

Grasps capable of ensuring the immobility of the object despite external disturbances are characterized by one of the following properties: *form-closure* when the position of the fingers ensures the object immobility, or *force-closure* when the forces applied by the fingers ensure its immobility [1]. During the last two decades, several algorithms has been developed to determine form and force-closure grasps of objects. Some of these works consider that the model of the object is known and they use quality criterions (for instance [2][3][4]) or heuristics criterions (for instance [5][6]) to select the final grasp. Other works consider that the model of the object is not known, implying that the selection of the final grasps should consider the description of the object made by an artificial vision system [7] [8]. In both cases, these algorithms determine "precision" grasps, i.e. grasps formed by a set of contacts points on the object boundary where the fingertips will be placed, and they require a good precision in the fingertip placements. In a real execution, the final grasp and the theoretical grasp may differ due to finger positioning errors. A metric for measuring the sensitivity of a grasp with respect to positioning errors can be found in [9]. In order to provide robustness to the grasp in front of these errors, Nguyen [10] introduced the concept of independent regions, i.e. regions on the object boundary such that a finger in each region ensures a form or force-closure grasp independently of the exact contact point, and he developed a geometrical approach to determine the maximum independent regions on polygonal objects using four frictionless contacts and two friction contacts. The problem of determining independent regions considering polygonal or polyhedral objects has been treated in [11][12][13][14]. Nevertheless, the determination of independent regions considering non-polygonal objects has not aroused the same attention. In [15] a methodology to determine two friction contact grasps of 2D objects was developed and in [16] a general approach to determine independent regions on 3D objects based on initial examples was proposed, although the selection of a good initial example for a given object remains as a critical step. Closely related to the independent regions, some algorithms determine all the N-finger force-closure grasps of polygonal objects [17][18]. These algorithms have not been used to compute independent regions, although in [18] the most stable grasp considering finger positioning errors is determined.

This paper deals with the problem of determining independent regions on the boundary of irregular objects considering the minimum number of frictionless contacts (four for 2D objects [19]) such that a contact point in each region ensures a form-closure grasp (hereafter FC grasp). A sufficient condition is developed stating whether any two contact points are compatible with a FC grasp. The conditions that the other two contact points must satisfy to produce a FC grasp are also presented and, using this information, a procedure to obtain independent regions is developed. The procedure is valid for a discrete description of the object boundary with restricted uncertainty between two consecutive points, and it does not imply hard iterative search procedures. Besides, since the solution does not rely on friction, the proposed methodology is useful to design fixtures for planar objects [20].

Recently, an algorithm to determine form-closure grasps of 3D objects described by discrete points has been presented in [21]. This algorithm is based on an iterative search through the points. Even when the methodology presented here has been developed considering 2D objects, the main contributions respect to the application of [21] to 2D objects are: 1) Iterations are only needed to find some characteristic points of the object and they do not imply hard iterative search procedures with the risk of falling in local minima (local minima are the critical point in terms of computational cost in [21]); 2) The methodology can deals with some uncertainty between the discrete points in the object description.

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This paper is organized as follows. Section II summarizes some results of previous works ([3][14]) obtained for polygonal objects that are the starting point of the developments for irregular objects. Section III tackles the problem of finding FC grasps of irregular objects and presents a method to obtain independent regions on the object boundary. An example of the proposed methodology is included in Section IV. Finally, some concluding remarks and possible future lines to extend this work are pointed out in Section V.

II. Form-Closure Grasps of Polygonal Objects

Consider a polygonal object and let θ_i be the direction of the applied force on the edge i. In the absence of friction, θ_i is the inward direction normal to the contact edge, and an unitary applied force on this edge is given by $\boldsymbol{f}_i = [\cos\theta_i \;\; \sin\theta_i]$.

Let τ_i be the torque produced by $\boldsymbol{f}_i$ with respect to the object's center of mass. Since $\boldsymbol{f}_i$ is known given the contact edge i, there is an univocal relation between τ_i and the exact contact point $\boldsymbol{p}_i$ on the edge i. Based on this relation and considering $i=1,...,4$ the following concepts are defined [14].

Definition 1: The *Real Range* of τ_i, R_i, is the set of values of τ_i produced by the contact force $\boldsymbol{f}_i$ that are physically possible due to the length of the contact edge. ◇

Definition 2: The *Directional Range* of τ_i, R_{fc_i}, is the set of values of τ_i produced by the contact force $\boldsymbol{f}_i$ that allow a FC grasp given any other three wrenches and considering that the contact edge has infinite length (i.e. only the "direction" of the edge is considered). ◇

Let $\mathcal{P}_f$ be the polygon defined by the unitary forces $\boldsymbol{f}_i$ with $i = 1,...,4$ in the force space (i.e. the space defined by pure forces as in Fig. 1), and consider that $\mathbf{0} \in \mathcal{P}_f$ (otherwise the contact edges do not allow a FC grasp [22]). From the two definitions above, the existence of a FC grasp implies that $R_i \cap R_{fc_i} \neq \emptyset$. Since R_i is known, the set of valid torques that produces a FC grasp can be determined by finding R_{fc_i}.

The following remarks related with the Directional Ranges will be used here (they are proved in [14]):

1. There are two types of Directional Range R_{fc_i}: *Infinite* if R_{fc_i} has only one finite extreme and the other tends to $\pm\infty$, and *Limited* if R_{fc_i} has two finite extremes.
2. The number of finite extremes and, therefore, the type of Directional Range R_{fc_i}, can be determined knowing how many pairs $\beta_{i,jk}$ and $\beta_{i,kj}$ are non-positive, being:

$$\beta_{i,jk} = \frac{\sin(\theta_i - \theta_k)}{\sin(\theta_j - \theta_k)} \qquad (1)$$

$$\beta_{i,kj} = \frac{\sin(\theta_j - \theta_i)}{\sin(\theta_j - \theta_k)} \qquad (2)$$

   where θ_i, θ_j and θ_k are the directions of the forces $\boldsymbol{f}_i$, $\boldsymbol{f}_j$ and $\boldsymbol{f}_k$ with $\{i,j,k\} \in \{1,2,3,4\}$ and $i \neq j \neq k$. $\beta_{i,jk}$ and $\beta_{i,kj}$ cannot be simultaneously null.
3. There are always at least two Infinite Directional Ranges that correspond to the torques generated by two forces that define consecutive vertices of $\mathcal{P}_f$ and lie between the negated of the other two forces (Fig. 1 shows two different examples).
4. Let R_{fc_i} and R_{fc_j} be two Infinite Directional Ranges with $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ defining two consecutive vertices of $\mathcal{P}_f$. In a FC grasp R_{fc_i} tends to $\pm\infty$ and R_{fc_j} tends to $\mp\infty$.

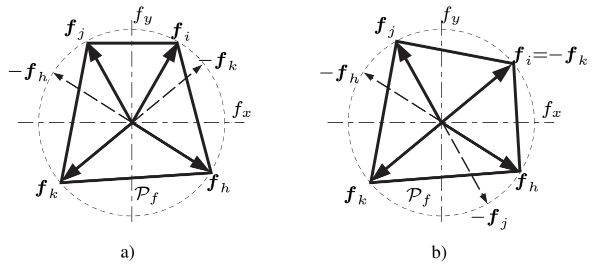

Fig. 1. *Examples of types of Directional Ranges from the applied forces: a) R_{fc_h} and R_{fc_k} are Limited and R_{fc_i} and R_{fc_j} are Infinite; b) R_{fc_k} is Limited and R_{fc_h}, R_{fc_i} and R_{fc_j} are Infinite.*

Based on these four remarks, the following necessary and sufficient condition for the existence of a FC grasp was enunciated and proved [14].

Necessary and sufficient condition: Four frictionless contacts allow a FC grasp iff for $\boldsymbol{f}_i$ and $\boldsymbol{f}_j$ defining two consecutive vertices of $\mathcal{P}_f$ and τ_i and τ_j having Infinite Directional Ranges

$$sign(\Gamma_i) \neq sign(\Gamma_j) \qquad (3)$$

where

$$\Gamma_\rho = \beta_{\rho,hk}\tau_h + \beta_{\rho,kh}\tau_k - \tau_\rho \quad \text{with } \rho \in \{i,j\} \qquad (4)$$

◇

Equation (4) has an useful geometrical property on the object space: the lines of action of the forces whose torques appear in equation (4) intersect at the same point when $\Gamma_\rho = 0$. In this case, the grasp is critical and it separates the FC grasps from the non-FC grasps [13]. Generally, the intersection involves the three lines of forces, although if either $\beta_{\rho,hk}$ or $\beta_{\rho,kh}$ are null (it happens when the angle between the directions of two forces is π), the intersection only involves the lines of action of two forces. An example of a FC grasp of a polygonal object and critical point positions is shown in Fig. 2.

III. Form-Closure Grasps of Irregular Objects

A. Object Description

Let $\mathcal{B}(u)$ and $\boldsymbol{p}_i = \mathcal{B}(u_i)$ be the actual object boundary parameterized on u and a point on it for a given u_i, respectively. $\mathcal{B}(u)$ is assumed to be a smooth and closed curve.

Let $\mathcal{B}_d(n) = \{\boldsymbol{q}_j, j = 1,...,n\}$, be a discrete description of $\mathcal{B}(u)$, with n being the number of points in the discretized boundary and $\boldsymbol{q}_j$ being one of its points. It is assumed that $\mathcal{B}_d$ and the normal direction θ_j on each of its points are known. Besides, it is also assumed that if $\boldsymbol{p}_i \in [\boldsymbol{q}_j, \boldsymbol{q}_{j+1}]$ then $\theta_i \in [\theta_j, \theta_{j+1}]$.

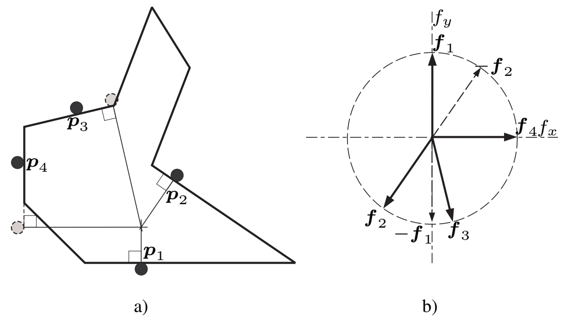

Fig. 2. *a) Example of a FC grasp of a polygonal object (black points) and of the determination of critical positions for p_3 and p_4 (grey points); b) Determination of the types of Directional Ranges from the applied forces: R_{fc_1} and R_{fc_2} are limited and R_{fc_3} and R_{fc_4} are Infinite.*

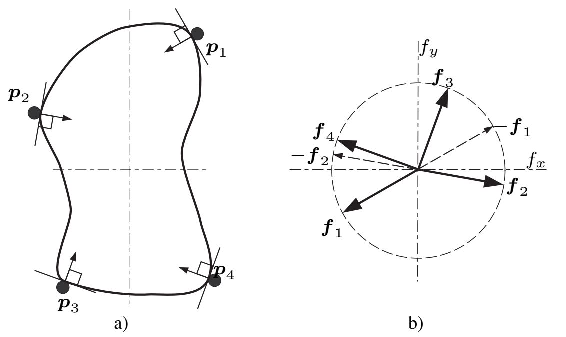

Fig. 3. *a) Example of a FC grasp of an irregular object; b) Determination of the types of Directional Ranges from the applied forces: R_{fc_1} and R_{fc_2} are limited and R_{fc_3} and R_{fc_4} are Infinite.*

B. Form-Closure Conditions

The necessary and sufficient condition enunciated for polygonal objects is based only on the directions of the applied forces. Given four contact points on the boundary of an irregular object the directions of the applied forces are also known, implying that the necessary and sufficient condition developed for polygonal objects can also be applied to check if these contact points allow a FC grasp (as in Fig. 3). Based on the necessary and sufficient condition for polygonal objects, the following lemma is enunciated.

Lemma 1: Let θ_h, θ_i, θ_j and θ_k be the directions of the applied forces on any four points on $\mathcal{B}(u)$, and let τ_j and τ_k be the torques that have Infinite Directional Ranges (as in the necessary and sufficient condition developed for polygonal objects). In a FC grasp, these four directions must satisfy the following relations:

$$0 < \quad \theta_k - \theta_h \quad < \pi \tag{5}$$

$$\theta_h + \pi \leq \quad \theta_\rho \quad \leq \theta_k + \pi \quad \text{with } \rho \in \{i, j\} \tag{6}$$

◇

Proof: From equations (1) and (2) the coefficients $\beta_{\rho,hk}$ and $\beta_{\rho,kh}$ of equation (4) are:

$$\beta_{\rho,hk} = \frac{\sin(\theta_\rho - \theta_k)}{\sin(\theta_h - \theta_k)} \tag{7}$$

$$\beta_{\rho,kh} = \frac{\sin(\theta_h - \theta_\rho)}{\sin(\theta_h - \theta_k)} \tag{8}$$

and from the remark 2 of the previous section they must be non-positive. From equations (7) and (8) it can be checked that the coefficients are non-positive only if equations (5) and (6) are satisfied. ◇

Definition 3: An *opposite point*, $\boldsymbol{p}_i^o$ of a point $\boldsymbol{p}_i$, both on $\mathcal{B}(u)$, is a point such that $\boldsymbol{f}_i^o$ and $\boldsymbol{f}_i$ are in opposite directions. ◇

Definition 4: (From [23]) An *antipodal point*, $\boldsymbol{p}_i^a$ of a point $\boldsymbol{p}_i$, both on $\mathcal{B}(u)$, is a point on the object boundary such that $\boldsymbol{f}_i^a$ and $\boldsymbol{f}_i$ are in opposite directions and they are collinear (therefore $\boldsymbol{p}_i^a$ is also an opposite point of $\boldsymbol{p}_i$). ◇

From Lemma 1 the contact points $\boldsymbol{p}_\rho$, with $\rho \in \{i, j\}$, must be between opposite points of $\boldsymbol{p}_h$ and of $\boldsymbol{p}_k$. Considering a grasp formed by $\boldsymbol{p}_h$, $\boldsymbol{p}_k$, $\boldsymbol{p}_i = \boldsymbol{p}_h^o$ and $\boldsymbol{p}_j = \boldsymbol{p}_k^o$ (i.e. the extremes of the piece of boundary where $\boldsymbol{p}_\rho$ must lie) the necessary and sufficient condition developed for polygonal objects is equivalent to:

$$sign(\Gamma_h) \neq sign(\Gamma_k) \tag{9}$$

with

$$\Gamma_\nu = \sin\theta_\nu(x_\nu^o - x_\nu) - \cos\theta_\nu(y_\nu^o - y_\nu) \tag{10}$$

where $\boldsymbol{p}_\nu = (x_\nu, y_\nu)$, $\boldsymbol{p}_\nu^o = (x_\nu^o, y_\nu^o)$ and θ_ν is the direction of the applied force $\boldsymbol{f}_\nu$ for $\nu \in \{h, k\}$.

Given two contact points $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$, it is possible to assure that they allow a FC grasp with two other contacts points between $\boldsymbol{p}_h^o$ and $\boldsymbol{p}_k^o$ if they satisfy equation (9). Otherwise, it is not possible to assure anything even when they still may allow a FC grasp. Then, equation (9) can be used as a *sufficient condition* for a FC grasp.

Proposition 1: Considering frictionless contacts, a grasp including two antipodal points $\boldsymbol{p}_\nu$ and $\boldsymbol{p}_\nu^a$ is critical (i.e. equation (10) gives $\Gamma_\nu = 0$). ◇

Proof: The antipodal points are a subset of the opposite points. Then, equation (9), developed for two opposite points, also determines whether two antipodal points can produce a FC grasp. Geometrically, Γ_ν of equation (10) is the distance between the point $\boldsymbol{p}_\nu^o$ and the straight line containing $\boldsymbol{f}_\nu$ applied at $\boldsymbol{p}_\nu$. Since the forces applied at two antipodal points are collinear, $\boldsymbol{p}_\nu^a$ of $\boldsymbol{p}_\nu$ belongs to this straight line. As a result, $\Gamma_\nu = 0$ and a critical grasp is obtained. ◇

Proposition 2: Let $\boldsymbol{p}_h^a$ and $\boldsymbol{p}_k^a$ be two consecutive points with antipodal points on the object boundary. The result of equation (10) has the same sign for any point between $\boldsymbol{p}_h^a$ and $\boldsymbol{p}_k^a$. ◇

Proof: From Proposition 1, only antipodal points make equation (10) equal to zero. Therefore, since the sign of equation (10) can change only at the antipodal points and the object boundary is closed, all the points between two consecutive antipodal points make the solution of equation (10) to have the same sign. ◇

When the object boundary is described by $\mathcal{B}_d$ as a finite number of points, it is not possible to exactly compute the antipodal points. Nevertheless, it is possible to determine the regions where the antipodal points are located; it is done with the following algorithm.

Algorithm 1: For each pair of consecutive points $\boldsymbol{q}_n$ and $\boldsymbol{q}_{n+1}$ of $\mathcal{B}_d$, do:

1. Find two points $\boldsymbol{q}_v$ and $\boldsymbol{q}_w$ of $\mathcal{B}_d$ that satisfy:

$$\theta_v \leq \quad \theta_n^o \quad \leq \theta_w \tag{11}$$

$$\theta_v \leq \quad \theta_{n+1}^o \quad \leq \theta_w \tag{12}$$

   if more than two pairs of points satisfy equations (11) and (12) select those with minimum number between them.
2. Compute the straight lines containing the forces $\boldsymbol{f}_n$ $\boldsymbol{f}_{n+1}$ applied at $\boldsymbol{q}_n$ and $\boldsymbol{q}_{n+1}$, respectively.
3. If $\boldsymbol{q}_v$ and $\boldsymbol{q}_w$ are on different sides of the two straight lines determined in step 2, there is an antipodal point between $\boldsymbol{q}_n$ and $\boldsymbol{q}_{n+1}$. ◇

Let the $hk-$space be the 2D space defined by the parameters that fix the position of two points (the parameters are u_h and u_k for $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$, and h and k for $\boldsymbol{q}_h$ and $\boldsymbol{q}_k$). The values of these parameters that define antipodal points make a partition of the $hk-$space into rectangular cells (Fig. 4). If $\mathcal{B}$ was known this would be an exact partition. When the object boundary is described by $\mathcal{B}_d$, it is not possible to compute exactly the antipodal points, but it is possible to determine regions where they lie using Algorithm 1. Then, the partition of the $hk-$space is not exact and there are uncertainty regions between the cells. Fig.4a shows an example for an ellipse, where the exact antipodal points correspond to $u = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$. Fig. 4b shows the same ellipse described by 25 points and, in this case, there are some uncertainty in the localization of the antipodal points (the antipodal points are located in the portions of the boundary between crosses, which correspond to the shaded regions in the $hk-$space).

From Proposition 2, the result of equation (10) has the same sign for any contact point between two consecutive antipodal points. Then, all the combinations of any two contact points that belong to the same cell of the $hk-$space either satisfy equation (9), defining a FC cell, or do not satisfy equation (9), meaning that it is not possible to assure a FC grasp in that cell. Then, checking a combination of two contact points from each cell, it is possible to determine the FC cells (black cells in Fig.4) and the cells requiring a more exhaustive analysis (white cells), which are initially discarded.

C. Independent regions

In the previous subsection the FC cells has been determined such that for any two points from one of them it is possible to find other two contact points that allow a FC grasp. In this subsection, the selection of these other two contact points and a procedure to obtain independent regions that allow a FC grasp are presented.

Lemma 2: Let $\boldsymbol{p}_h$, $\boldsymbol{p}_i$, $\boldsymbol{p}_j$ and $\boldsymbol{p}_k$ be four contact points that satisfy Lemma 1. If these points produce a FC grasp, there are an odd number of critical points $\boldsymbol{p}_c$ between $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ such that $\boldsymbol{p}_c$, $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$ produce a critical grasp (i.e., the lines of action of $\boldsymbol{f}_c$, $\boldsymbol{f}_h$ and $\boldsymbol{f}_k$ intersect at the same point). ◇

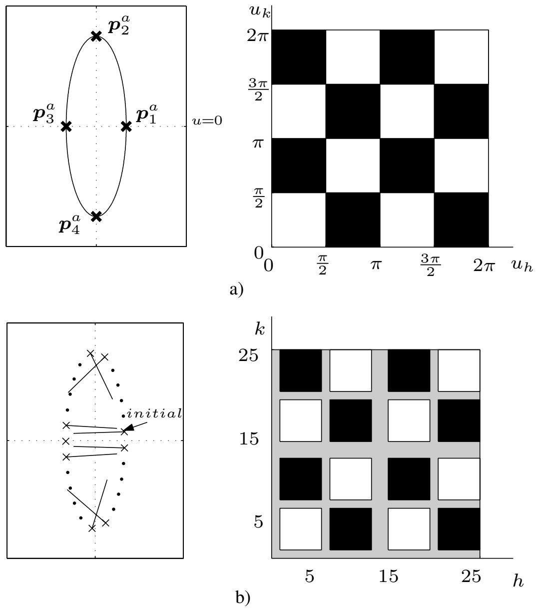

Fig. 4. *a) Exact antipodal points on an ellipse and partition of the $hk-$space (the FC cells are the black ones); b) Discrete description of an ellipse with the regions where the antipodal points are located (portions of the boundary between crosses) and partition of the $hk-$space with uncertainty regions (shaded regions).*

Proof: In order to obtain a FC grasp, equation (3) must be satisfied implying that Γ_i and Γ_j have different signs. The values of Γ_i and Γ_j are obtained from equation (4) considering the contact points $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, respectively. Since the object boundary is smooth, $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ define a continuous piece of the object boundary. As a result, there must be an odd number of critical points $\boldsymbol{p}_c$ between them that make equation (4) equal to zero (i.e. $\boldsymbol{f}_h$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_c$ produce a critical grasp) and allow the result of equation (4) to change its sign. ◇

Based on Lemmas 1 and 2 the following algorithm was developed to obtain the points $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$ that allow a FC grasp.

Algorithm 2: Let ($\boldsymbol{q}_h$, $\boldsymbol{q}_{h+1}$) and ($\boldsymbol{q}_k$, $\boldsymbol{q}_{k+1}$) be two pairs of consecutive positions of $\mathcal{B}_d$ with h, $h+1$, k and $k+1$ belonging to the same FC cell. For these points, do:

1. Compute the straight lines containing the forces $\boldsymbol{f}_h$, $\boldsymbol{f}_{h+1}$, $\boldsymbol{f}_k$ and $\boldsymbol{f}_{k+1}$ applied at their respective points.
2. Intersect the line containing $\boldsymbol{f}_h$ and the line containing $\boldsymbol{f}_{h+1}$ with the lines containing $\boldsymbol{f}_k$ and $\boldsymbol{f}_{k+1}$. Four intersection points are obtained.
3. Determine $\boldsymbol{q}_{l^-}$ and $\boldsymbol{q}_{l^+}$ that satisfy $\theta_{l^-} \leq \theta_h^o \leq \theta_{l^+}$ and $\theta_{l^-} \leq \theta_{h+1}^o \leq \theta_{l^+}$ with the minimum number of points between them.
4. Determine $\boldsymbol{q}_{s^-}$ and $\boldsymbol{q}_{s^+}$ that satisfy $\theta_{s^-} \leq \theta_k^o \leq \theta_{s^+}$ and $\theta_{s^-} \leq \theta_{k+1}^o \leq \theta_{s^+}$ with the minimum number of points between them.
5. Determine $\boldsymbol{q}_\rho$ that satisfy $\theta_{l^+} \leq \theta_\rho \leq \theta_{s^-}^o$ for $\rho \in \{i, j\}$

(it is equivalent to satisfy Lemma 1).

6. Compute the straight lines containing $\boldsymbol{f}_\rho$ and $\boldsymbol{f}_{\rho+1}$ applied at their respective points.
7. Evaluate the positions of the points obtained in step 2 with respect to the straights lines obtained in step 6. If these points are on different sides of the two straight lines, there are critical points between $\boldsymbol{q}_\rho$ and $\boldsymbol{q}_{\rho+1}$.
8. Determine $\boldsymbol{q}_{c^-}$ and $\boldsymbol{q}_{c^+}$ as the minimum and maximum values of $\boldsymbol{q}_\rho$ that satisfy step 7. $\boldsymbol{q}_{c^-}$ and $\boldsymbol{q}_{c^+}$ define a critical region where the critical points lie.
9. Chose $\boldsymbol{q}_i \in [\boldsymbol{q}_{l^+}, \boldsymbol{q}_{c^-}]$ and $\boldsymbol{q}_j \in [\boldsymbol{q}_{c^+}, \boldsymbol{q}_{s^-}]$. Then, $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$ jointly with $\boldsymbol{q}_h$ and $\boldsymbol{q}_k$ satisfy Lemma 2, and they form a FC grasp. ⋄

If Algorithm 2 is applied considering the entire ranges of a FC cell (i.e., the values of h and k that define the FC cell) the values of l^+ and s^- (step 5 of Algorithm 2) describe two limit surfaces, and c^- and c^+ (step 8) describe two critical surfaces (Fig. 5 shows an example considering a FC cell for an ellipse). Therefore, a set of independent regions is obtained by determining two parallelepipeds between the limit surfaces with the same projection on the $hk-$space such that the critical surfaces c^- and c^+ are between them. The ranges of the edges of these parallelepipeds define, on the object boundary, the independent regions where a FC grasp is possible. The maximization of the shortest independent region is used as optimization criterion (as in [11]). With this criterion the determination of the independent regions is done with the following algorithm.

Algorithm 3: For each FC cell, do:

1. Determine an initial set of independent regions in the FC cell. It is done as:
   1.1. Select the intervals $H=[h,h+1]$, $K=[k,k+1]$ such that they define a square in the middle of the FC cell.
   1.2. Determine the maximum values of l^+ and c^+ and the minimum values of c^- and s^- for the vertices of the square obtained in step 1.1. $H=[h,h+1]$, $I=[l^+,c^-]$, $J=[c^+,s^-]$ and $K=[k,k+1]$ are independent regions.
2. Maximize the length of the minimum independent regions, as:
   2.1. Increase H and K with its neighbor positions and determine I and J in analogous way as in step 1.2.
   2.2. If the length of the minimum region increase, then update H, K, I and J and go to step 2.1. Else return H, K, I and J. ⋄

Actually, step 2.2. can be applied until a decreasing in the minimum region is detected, producing in this way the same minimum region but enlarging some or all of the other three regions.

IV. Example

An example of the proposed methodology is presented in this section. The object used in the example has been taken from [23]. The exact object boundary $\mathcal{B}(u)$ is described by the following parametric function:

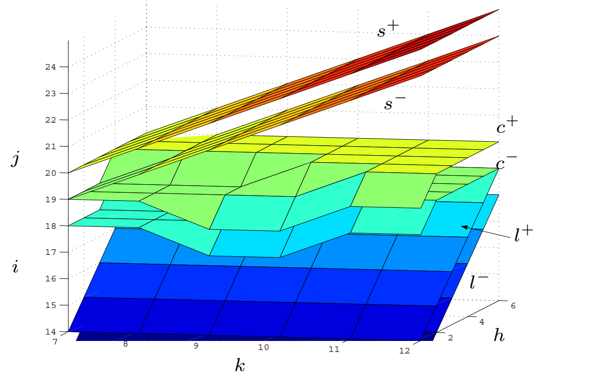

Fig. 5. *Limit surfaces l^+ and s^- and critical region between c^- and c^+, for the FC cell $h=[1,6]$ and $k=[7,12]$ of the ellipse in Fig. 4.*

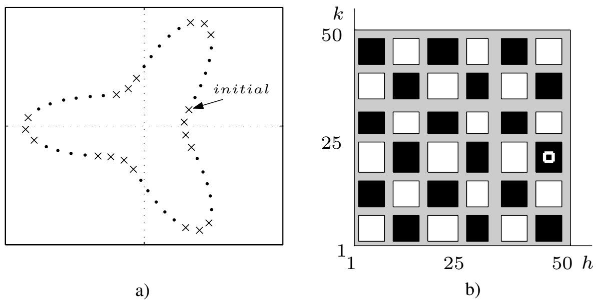

Fig. 6. *a) Object and regions where the antipodal points lies (portions of the boundary between crosses); b) $hk-$space and FC cell used in the example (black cell with a white square).*

$$\mathcal{B}(u) = \left(\frac{3\cos(u)}{1+0.5\cos(3u)}, \frac{3\sin(u)}{1+0.5\cos(3u)}\right) \qquad (13)$$

for $0 \le u \le 2\pi$. $\mathcal{B}(u)$ is used to check the results of the discrete method proposed in this paper using $\mathcal{B}_d$. Fig. 6a shows the discrete description $\mathcal{B}_d$ of the object boundary using 50 points and the regions where the antipodal points lie (portions of the boundary between crosses) determined by Algorithm 1. Fig. 6b shows the $hk-$space with the FC cells and the uncertainty regions.

The application of Algorithm 3 for the FC-cell defined by $h=42,...,48$ and $k=17,...,24$, gives the following result:

1. Determination of the initial independent regions: $H_1=[45,46]$, $I_1=[26,30]$, $J_1=[36,40]$, $K_1=[20,21]$. Minimum length = 1.
2. Maximization of the minimum region length: *1rst* iteration: $H_2=[44,47]$, $I_2=[26,30]$, $J_2=[36,40]$, $K_2=[19,22]$. Minimum length = 3. *2nd* iteration: $H_3=[43,47]$, $I_3=[26,30]$, $J_3=[36,40]$, $K_3=[18,22]$. Minimum length = 4 (last iteration).

Fig.7 shows the limit surfaces, the critical surfaces and the parallelepipeds that define the independent regions. Fig.8a shows the initial independent regions H_1 and K_1 on the $hk-$space and the iterations to obtain the final regions H_3 and K_3. Fig.8b shows the final result on the object.

Considering the real object described by equation (13), the final independent regions correspond to

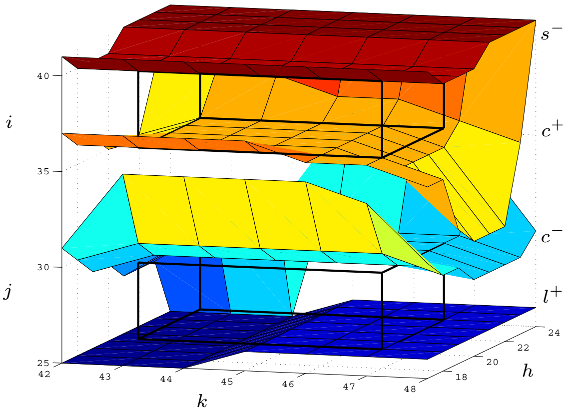

Fig. 7. *Limit surfaces, l^+ and s^-, critical surfaces c^- and c^+ and independent regions defined by the two polyhedrons.*

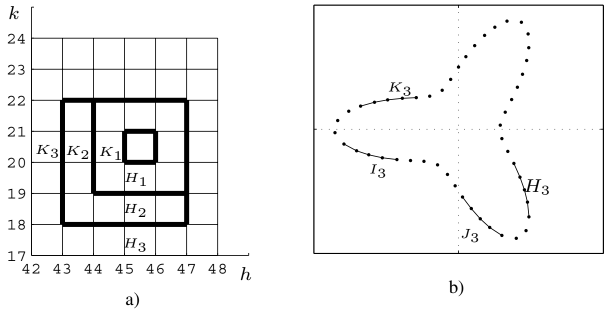

Fig. 8. *a) Steps to obtain the final Independent regions in the $hk-$space: H_1 and K_1 are the initial independent regions, H_2 and K_2 are the regions obtained in the first iteration and H_3 and K_3 are the final independent regions; b) Final independent regions (black regions) on the object boundary.*

$\triangle u_h = [5.39, 5.73]$, $\triangle u_i = [3.27, 3.6]$, $\triangle u_j = [4.77, 5.1]$ and $\triangle u_k = [2.5, 2.91]$. It can be checked that any combination of points on the actual object boundary that belong to these regions produce a FC grasp.

V. Conclusions and Future Works

Most of the algorithms to compute FC grasps and fixtures for 2D determine a set of contact points on the object boundary where the fingertips will be placed, requiring a good precision in the fingertips placement. In the real execution of the grasps, the final grasp and the theoretical grasp may differ due to finger positioning errors. The determination of independent regions was presented as a possible solution to this problem. Existing solution approaches has being mainly developed for polygonal or polyhedral objects, and only a few works tackle the problem considering non-polygonal objects. In this paper a new approach to compute independent regions for irregular 2D objects and four frictionless contacts has been presented. The procedure is valid for a discrete description of the object boundary with restricted uncertainty between two consecutive points, and it does not imply hard iterative search procedures.

Future works include the consideration of friction contacts in the proposed methodology (following the approach in [14]), as well as the extension of the methodology to 3D objects.

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