CONTACT SITUATIONS FROM OBSERVED REACTION FORCES IN ASSEMBLY WITH UNCERTAINTY

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Abstract: A key problem in the automatic planning and execution of robotized assembly tasks is the estimation of the current contact situation using configuration and force information. This paper analyzes the force information processing necessary to achieve this goal in the case of planar movements with uncertainty. First, the off-line computation of the set of feasible reaction force directions in any contact situation of an assembly task is described. Then, the on-line classification of measured forces into one or more of the previous computed sets is presented. The method allows a fast classification of forces, compatible with the real time requirements of task execution.

Keywords: Robotics, Assembly, Force, Uncertainty, Measured values, Classification.

1. INTRODUCTION

The automatic planning and execution of assembly tasks with robots is still an open research field, particularly when the geometric uncertainty allows deviations of the same order as the part matting clearance. The uncertainty can be decreased by using reaction force information. Thus, sensing and control of reaction forces during task execution has become an important topic on which a lot of work has been done (Whitney, 1987).

In order to correctly perform an assembly task, some reaction force control strategies are needed. The off-line determination of these strategies can become quite difficult when geometric uncertainty is significant, and it is the main goal of the fine-motion planning (e.g. Dufay and Latombe, 1984; Lozano-Perez *et al.*, 1984; Erdmann, 1984; Buckley, 1987; Laugier, 1989; Xiao and Volz, 1989; Suárez and Basañez, 1991; Basañez and Suárez, 1992). In many cases, the execution of a fine-motion plan needs the identification of the current contact situation between the objects (e.g Basañez and Suárez, 1992; Xiao, 1993; Spreng, 1993), or the verification of some termination-conditions to finish a movement and to begin the execution of another command (e.g Lozano-Perez *et al.*, 1984; Erdmann, 1984; Buckley, 1987; Desai and Volz, 1989); in both cases the reaction forces that could be sensed in a given contact situation play an important role.

This paper deals with the use of the force information to achieve a fast on-line identification of the current contact situation of polygonal objects, considering planar movements, friction and different sources of uncertainty.

After this introduction, sections 2 and 3 briefly review the dual representation of forces and the uncertainty affecting the force measurements respectively. Section 4 presents an improved version of the method previously proposed by the authors (Suárez *et al.*, 1994) to compute the sets of possible reaction forces in the contact situations. This method is used in section 5 to tackle the next problem: determining if a measured force is compatible with a given contact situation. Finally, section 6 presents the conclusions of the work.

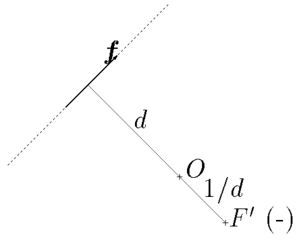

Fig. 1. Dual representation of a line of force.

2. DUAL REPRESENTATION OF FORCES

A force $\boldsymbol{f} = [f_x \;\; f_y]^T$ acting in the plane and producing a torque τ with respect to a reference origin O, is represented in a tridimensional force space $\mathcal{F}_3$ by a generalized force $\boldsymbol{g} = [f_x \;\; f_y \;\; \tau]^T$.

The direction[1] of the generalized force $\boldsymbol{g}$ in $\mathcal{F}_3$ can be represented by the coordinates of the intersecting point of the supporting line of $\boldsymbol{g}$ with the unitary torque plane jointly with the sign of τ, i.e. $P = [f_x/|\tau| \;\; f_y/|\tau|]^T$ and sign(τ). This representation of generalized force directions is easily obtained from the dual representation of pure forces acting in a plane (Brost and Mason, 1989).

The force $\boldsymbol{f}$ with force line $ax+by+c=0$ is represented by the dual point $F' = [a/c \;\; b/c]^T$. Geometrically, F' lies on the normal to the force line through the reference origin O and at a distance $1/d$ from O, d being the distance between the force line and O (figure 1). The sense of $\boldsymbol{f}$ is included by associating to F' the sign of the torque τ produced by $\boldsymbol{f}$ around O. P can be obtained by a $\pi/2$ clockwise rotation of F' around O. The dual representation does not preserve the force module though, in this case, it does not matter since the module is irrelevant for the estimation of the contact situation.

Thus, the direction of a generalized force $\boldsymbol{g}$ in $\mathcal{F}_3$ associated with a force $\boldsymbol{f}$ in the working plane will be represented by F' and the corresponding sign.

3. FORCE UNCERTAINTY

Geometric uncertainty and friction affect the direction of the reaction forces that can really appear; the measurement of these forces is also affected by the uncertainty of the force/torque sensor.

Typical force/torque sensors give forces and torques as independent values with a specified resolution. Therefore, the acual value f_i of a measured component f_{im} satisfies $f_i \in [f_{im} - \epsilon_{f_i}, f_{im} + \epsilon_{f_i}]$, being ϵ_{f_i} the sensor resolution for f_i. Then, for planar movements the force uncertainty model in $\mathcal{F}_3$ is a rectangular parallelepiped U, of sides ϵ_{f_x}, ϵ_{f_y} and ϵ_τ centered in the measured generalized force $\boldsymbol{g}_m = [f_{xm} \;\; f_{ym} \;\; \tau_m]^T$. Figure 2 illustrates the uncertainty U of a measured force $\boldsymbol{g}_m$ and the representation on the plane $\tau = 1$ of the directions of the generalized forces with head in U. A $\pi/2$ rotation of the plane $\tau = 1$ counterclockwise around the τ-axis gives rise to the dual representation $\mathcal{U}'$of the forces in U.

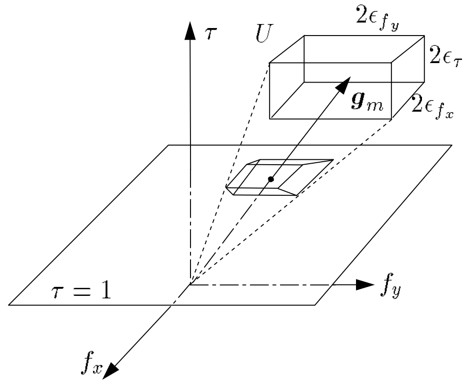

Fig. 2. Generalized force uncertainty U and its representation on the plane $\tau = 1$.

4. DOMAINS OF POSSIBLE REACTION FORCES

Due to the uncertainty, the contact point(s) and the direction of the reaction force cannot be exactly determined. Nevertheless, it is possible to compute the domain $\mathcal{G}$ of all the possible reaction forces for any contact situation of an assembly task by using the nominal models of the objects and the models of geometric uncertainty and friction. Geometric uncertainty includes manufacturing tolerances, inaccuracy in the observation of the object position, errors in the robot position and orientation, and slipping of the object in the gripper (Basañez and Suárez, 1991).

Due to the paper space constraints, only basic contacts between a mobile-object vertex and a static-object edge will be illustrated in this paper. Basic contacts between a mobile-object edge and a static-object vertex are solved in an analogous way. Let be:

- $\mathcal{V}$: region where the actual contact-vertex position should lie due to uncertainty (Basañez and Suárez, 1991).
- $\mathcal{V}'$: region of the dual points of force lines crossing $\mathcal{V}$.
- μ: friction coefficient.
- ϕ_l: orientation of the lower bound of the friction cone decreased with uncertainty ϵ_ψ of the edge normal orientation ψ, i.e. $\phi_l = \psi - \arctan(\mu) - \epsilon_\psi$.

[1] From now on, unless it is explicitly indicated, *direction* means *direction* and *sense*.

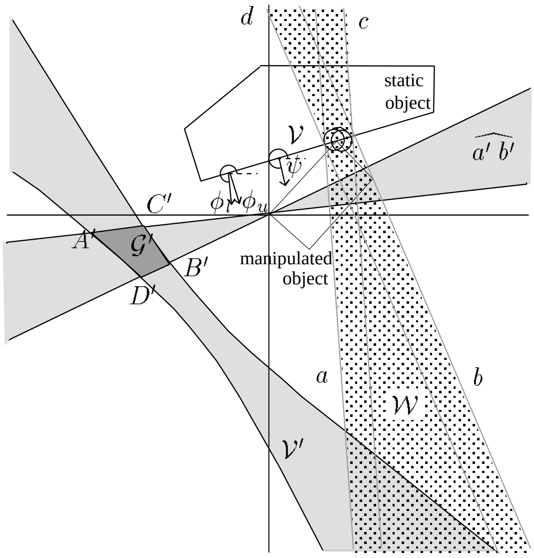

Fig. 3. Domain $\mathcal{G}'$ for one basic contact.

ϕ_u: orientation of the upper bound of the friction cone enlarged with uncertainty ϵ_ψ of the edge normal orientation ψ, i.e. $\phi_u = \psi + \arctan(\mu) + \epsilon_\psi$.
a, c: tangents to region $\mathcal{V}$ with orientation ϕ_l.
b, d: tangents to region $\mathcal{V}$ with orientation ϕ_u.
$\mathcal{W}$: union of cones[2] $\widehat{a\,b}$ and $\widehat{c\,d}$.
$\widehat{a'\,b'}$: cone containing the dual points of force lines with directions in the range $[\phi_l, \phi_u]$ (note that a' and b' are lines through the reference origin and perpendicular to a and b respectively).
$\mathcal{H}(P_1, ..., P_n)$: convex hull defined by points $P_1, ..., P_n$.

The following subsections describe an improved method to compute the dual representation $\mathcal{G}'$ of the possible reaction forces in a given contact situation. The method is based on a previous proposal by the authors (Suárez *et al.*, 1994).

4.1. *Case of One Basic Contact*

The domain $\mathcal{G}$ of a contact situation with only one basic contact is composed of the forces satisfying the following two conditions:

1) Contact-point condition: the line of the reaction force must intersect region $\mathcal{V}$.

2) Direction condition: the reaction force direction ϕ must belong to the range $[\phi_l, \phi_u]$.

[2] A cone on the plane, represented by $\widehat{a\,b}$, a and b being two straight lines, refers to the sector swept by line a when it is rotated counterclockwise around the intersection point of a with b until b is reached.

Then, $\mathcal{G}'$ is computed from two sets of dual points as:

$$\mathcal{G}' = \mathcal{V}' \cap \widehat{a'b'} \qquad (1)$$

$\mathcal{V}'$ and $\widehat{a'\,b'}$ being the regions representing forces satisfying, respectively, the contact-point condition and the direction condition (figure 3). It must be noted that the vertices of $\mathcal{G}'$ are the dual points A', B', C' and D' of the lines a, b, c and d, respectively.

4.2. *Case of More than One Basic Contact*

The domain $\mathcal{G}$ for several basic contacts is the set of the forces resulting from the composition of all possible compatible reaction forces, one at each basic contact. Therefore, the dual representation $\mathcal{G}'$ is the set of all non-negative linear combinations of possible compatible dual reaction forces, one at each basic contact.

From now on, force domains will be particularized by a subscript indicating the set of related basic contacts. Let be:
S: a set of n compatible basic contacts.
s: any sub-set of S with $\inf(n-1, 3)$ basic contacts.

Proposition 1:

$$\mathcal{G}_S \supset \bigcup_{\forall s \subset S} \mathcal{G}_s \qquad (2)$$

Proof: If a generalized reaction force $\boldsymbol{g}_m$ satisfies $\boldsymbol{g}_m \in \mathcal{G}_s$ then it is the resultant of one force at each of the basic contacts of s and zero force at the other(s). Therefore $\boldsymbol{g}_m \in \mathcal{G}_S$ $\diamond$

From proposition 1, $\mathcal{G}'_S$ can be expressed as:

$$\mathcal{G}'_S = [\bigcup \mathcal{G}'_s] \cup [\mathcal{E}'_S] \qquad (3)$$

$\mathcal{E}'_S$ being a dual region associated to the basic contacts of S.

The following propositions indicate how the sets $\mathcal{G}'$ are computed according to the number of involved basic contacts. The proofs can be found in (Basañez *et al*, 1995).

Proposition 2: For a contact situation with only one basic contact i, $\mathcal{E}'_{\{i\}} = \mathcal{G}'_{\{i\}}$.

Proposition 3: For a contact situation with two basic contacts i and j, $\mathcal{G}'_{\{i,j\}} = \mathcal{G}'_{\{i\}} \cup \mathcal{G}'_{\{j\}} \cup \mathcal{E}'_{\{i,j\}}$, with $\mathcal{E}'_{\{i,j\}} = \mathcal{H}(A'_i, B'_i, A'_j, B'_j) \cup \mathcal{H}(A'_i, B'_i, C'_j, D'_j) \cup \mathcal{H}(C'_i, D'_i, A'_j, B'_j) \cup \mathcal{H}(C'_i, D'_i, C'_j, D'_j)$.

Figure 4 shows the four convex hulls that compose the region $\mathcal{E}'_{\{i,j\}}$ and figure 5 shows the domain $\mathcal{G}'_{\{i,j\}} = \mathcal{G}'_{\{i\}} \cup \mathcal{G}'_{\{j\}} \cup \mathcal{E}'_{\{i,j\}}$.

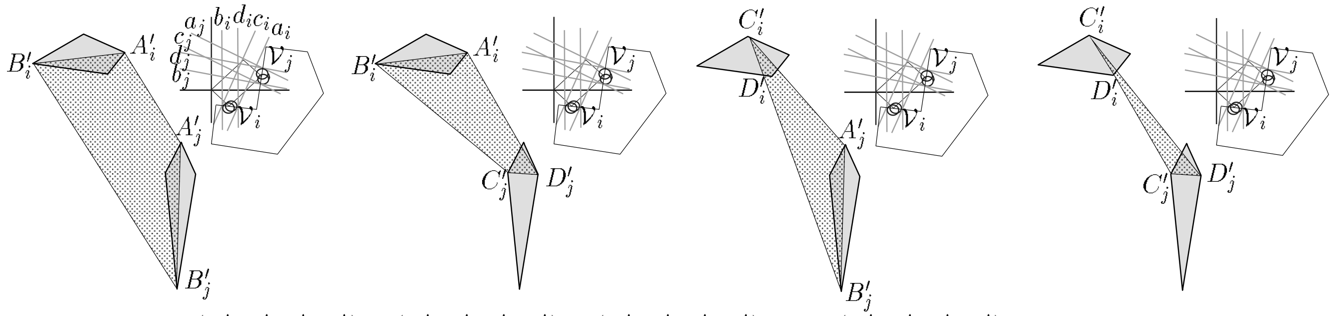

Fig. 4. Regions $\mathcal{H}(A'_i, B'_i, A'_j, B'_j)$, $\mathcal{H}(A'_i, B'_i, C'_j, D'_j)$, $\mathcal{H}(C'_i, D'_i, A'_j, B'_j)$ and $\mathcal{H}(C'_i, D'_i, C'_j, D'_j)$.

Proposition 4: For a contact situation with three basic contacts i, j and k, $\mathcal{G}'_{\{i,j,k\}} = \mathcal{G}'_{\{i,j\}} \cup \mathcal{G}'_{\{i,k\}} \cup \mathcal{G}'_{\{j,k\}} \cup \mathcal{E}'_{\{i,j,k\}}$, with $\mathcal{E}'_{\{i,j,k\}} = \mathcal{H}(F'_i, F'_j, F'_k)$, for any $F'_i \in \mathcal{G}'_{\{i\}}$, $F'_j \in \mathcal{G}'_{\{j\}}$ and $F'_k \in \mathcal{G}'_{\{k\}}$.

Proposition 5: For a contact situation with $n > 3$ basic contacts $\mathcal{E}'_S = \emptyset$.

5. CLASSIFICATION OF OBSERVED REACTION FORCES

5.1. *General procedure*

A generalized reaction force $\boldsymbol{g}_m$ measured with uncertainty U is compatible with the contact situation determined by a set S of n basic contacts if and only if

$$U \cap \mathcal{G}_S \neq \emptyset \tag{4}$$

It must be noted that, due to uncertainty, $\boldsymbol{g}_m$ could be compatible with more than one contact situation. Condition (4) is expressed in the dual plane as

$$\mathcal{U}' \cap \mathcal{G}'_S \neq \emptyset \tag{5}$$

and, from equation (3), this condition is satisfied if and only if at least one of the following two conditions is satisfied:

$$\mathcal{U}' \cap [\bigcup \mathcal{G}'_s] \neq \emptyset \tag{6}$$

$$\mathcal{U}' \cap \mathcal{E}'_S \neq \emptyset \tag{7}$$

Given a sensed reaction force $\boldsymbol{g}_m$ and the domain $\mathcal{G}'_S$ of a contact situation, the following algorithm evaluates the above conditions. The algorithm returns "compatible" when $\boldsymbol{g}_m$ is compatible with $\mathcal{G}'_S$ and "incompatible" otherwise; this result is stored in a global variable C_S that is used to speed up further evaluations of condition (6) for other contact situations. The function test($\mathcal{E}'_S$, $\boldsymbol{g}_m$) directly evaluates condition (7) for a given measured reaction force $\boldsymbol{g}_m$; it returns "true" when the condition is satisfied and "false" otherwise.

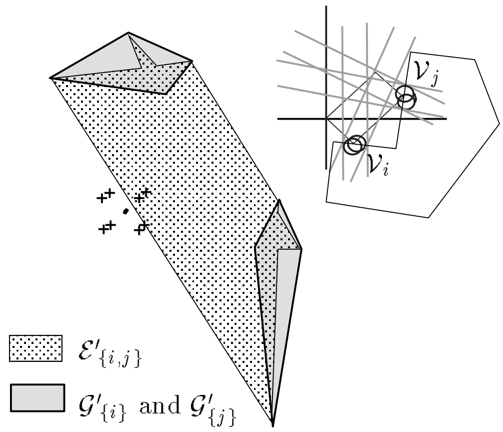

Fig. 5. Domain $\mathcal{G}'_{\{i,j\}} = \mathcal{G}'_{\{i\}} \cup \mathcal{G}'_{\{j\}} \cup \mathcal{E}'_{\{i,j\}}$.

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classify(G'_S, g_m)
    FOR EACH s ∈ S                                      }
      IF G'_s has not been yet classified               }
      THEN C_s = classify(G'_s, g_m)                    }
      END IF                                            }  cond
      IF C_s = "compatible"                             }  (6)
      THEN C_S = "compatible"                           }
            RETURN(C_S)                                 }
      END IF                                            }
    END FOR                                             }
    IF test(E'_S, g_m) = "true"                         }
    THEN C_S = "compatible"                             }  cond
    ELSE C_S = "incompatible"                           }  (7)
    END IF                                              }
    RETURN(C_S)                                         }
end
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5.2. *The function* test($\mathcal{E}'_S$, g_m)

For a given sensed force $\boldsymbol{g}_m$, let be:

- V'_q: the dual point representing the force on the vertex q of U, with $q \in \{1, ..., 8\}$.
- e'_q: the straight segment containing the dual points representing forces on the edge q of U, with $q \in \{1, ..., 12\}$.

$\mathcal{P}'_q$: the polygon containing the dual points representing forces on the face q of U, with $q \in \{1, ..., 6\}$.

E_q: the point in the physical space whose dual line contains e'_q.

The function test evaluates condition (7) by sequentially testing the following three conditions and returning "true", as soon as one of them is satisfied, or "false" otherwise:

$$V'_q \in \mathcal{E}'_S, \quad \text{for any} \quad q \in \{1, ..., 8\} \tag{8}$$
$$e'_q \cap \mathcal{E}'_S \neq \emptyset, \quad \text{for any} \quad q \in \{1, ..., 12\} \tag{9}$$
$$\mathcal{P}'_q \supset \mathcal{E}'_S, \quad \text{for any} \quad q \in \{1, ..., 6\} \tag{10}$$

The evaluation of any of the conditions (8) to (10) assumes that the previous ones are not satisfied and it is performed in a different way depending on the number of the basic contacts involved, as it is specified below.

One Basic Contact i: From proposition 2, $\mathcal{E}'_{\{i\}} = \mathcal{G}'_{\{i\}}$, and from equation (1), $\mathcal{G}'_{\{i\}} = \mathcal{V}' \cap \widehat{a'b'}$, then:

• *Condition (8).* $V'_q \in \mathcal{E}'_{\{i\}}$ if it belongs to both $\mathcal{V}'$ and $\widehat{a'b'}$, which is tested as:

$$[V'_q \in \mathcal{V}'] \cap [V'_q \in \widehat{a'b'}] \tag{11}$$

Figure 6a shows a case in which the dual representations of two vertices of U are inside $\mathcal{G}'_{\{i\}}$.

• *Condition (9).* A segment e'_q crosses $\mathcal{E}'_{\{i\}}$ if it crosses either $\mathcal{V}'$ being inside $\widehat{a'b'}$, or $\widehat{a'b'}$ being inside $\mathcal{V}'$, or both regions being its supporting line the dual line of a point of $\mathcal{W}$. All this is tested as:

$$\begin{aligned}&[(e'_q \subset \mathcal{V}') \cap (e'_q \cap \widehat{a'b'} \neq \emptyset)] \cup \\ &[(e'_q \cap \mathcal{V}' \neq \emptyset) \cap (e'_q \subset \widehat{a'b'})] \cup \\ &[(e'_q \cap \mathcal{V}' \neq \emptyset) \cap (e'_q \cap \widehat{a'b'} \neq \emptyset) \cap (E_q \in \mathcal{W})]\end{aligned} \tag{12}$$

Figures 6b and 6c show two situations in which an edge e'_q intersects $\mathcal{G}'_{\{i\}}$.

• *Condition (10).* A polygon $\mathcal{P}'$ whose edges do not cross $\mathcal{E}'_{\{i\}}$ contains $\mathcal{E}'_{\{i\}}$ if any arbitrary point (e.g. vertex A') of $\mathcal{G}'_{\{i\}}$ belongs to $\mathcal{P}'$. So, the condition is tested as:

$$A' \in \mathcal{P}'_q \tag{13}$$

Figure 6d shows a case in which a face $\mathcal{P}'_q$ (dashed) contains $\mathcal{G}'$.

Two Basic Contacts i and j: From proposition 3, $\mathcal{E}'_{\{i,j\}} = \mathcal{H}(A'_i, B'_i, A'_j, B'_j) \cup \mathcal{H}(A'_i, B'_i, C'_j, D'_j) \cup \mathcal{H}(C'_i, D'_i, A'_j, B'_j) \cup \mathcal{H}(C'_i, D'_i, C'_j, D'_j)$, then:

• *Condition (8).* $V'_q \in \mathcal{E}'_{\{i,j\}}$ if V'_q belongs to any of the four polygons that compose $\mathcal{E}'_{\{i,j\}}$, which is tested as:

$$\begin{aligned}&[V'_q \in \mathcal{H}(A'_i, B'_i, A'_j, B'_j)] \cup \\ &[V'_q \in \mathcal{H}(A'_i, B'_i, C'_j, D'_j)] \cup \\ &[V'_q \in \mathcal{H}(C'_i, D'_i, A'_j, B'_j)] \cup \\ &[V'_q \in \mathcal{H}(C'_i, D'_i, C'_j, D'_j)]\end{aligned} \tag{14}$$

Figure 5 shows a case in which a sensed reaction force is compatible with a domain $\mathcal{G}'_{\{i,j\}}$ because the dual representations of three vertices of U lies inside $\mathcal{E}'_{\{i,j\}}$.

• *Condition (9).* Since it is already known that the vertices of e'_q are not inside $\mathcal{E}'_{\{i,j\}}$, then $e'_q \cap \mathcal{E}'_S \neq \emptyset$ if e'_q intersects any arbitrary polygon of those composing $\mathcal{E}'_{\{i,j\}}$. Selecting one of these polygons the condition is tested as:

$$e'_q \cap \mathcal{H}(A'_i, B'_i, A'_j, B'_j) \neq \emptyset \tag{15}$$

• *Condition (10).* It is never satisfied since $\mathcal{P}_q \not\supset \mathcal{G}'_i$, $\mathcal{P}_q \not\supset \mathcal{G}'_j$ $\forall q \in \{1, ..., 6\}$ and $e'_q \cap \mathcal{E}'_{\{i,j\}} = 0$ $\forall q \in \{1, ..., 12\}$.

Three Basic Contacts i, j and k: From proposition 4, $\mathcal{E}'_{\{i,j,k\}} = \mathcal{H}(F'_i, F'_j, F'_k)$, and by construction, the border of $\mathcal{E}'_{\{i,j,k\}}$ satisfies $\partial\mathcal{E}'_{\{i,j,k\}} \subset (\mathcal{G}'_{\{i,j\}} \cup \mathcal{G}'_{\{i,k\}} \cup \mathcal{G}'_{\{j,k\}})$. Since the function test is only called when condition (6) is not satisfied it is already known that $\mathcal{U}' \cap \mathcal{G}'_{\{i,j\}} = \emptyset$, $\mathcal{U}' \cap \mathcal{G}'_{\{i,k\}} = \emptyset$ and $\mathcal{U}' \cap \mathcal{G}'_{\{j,k\}} = \emptyset$; therefore $\mathcal{U}'$ is either completely inside $\mathcal{E}_{\{i,j,k\}}$ or completely outside. As a consequence, it is only necessary to test if a point of $\mathcal{U}'$ is inside $\mathcal{E}_{\{i,j,k\}}$; then:

• *Condition (8).* It is tested as

$$V'_q \in \mathcal{H}(F'_i, F'_j, F'_k) \quad \text{for any given } q \tag{16}$$

• *Conditions (9) and (10).* They are never satisfied.

More Than Three Basic Contacts: From proposition 4, $\mathcal{E}'_S = \emptyset$, then the function test($\mathcal{E}'_S$, $\boldsymbol{g}_m$) always returns "false".

6. CONCLUSIONS

The dual force representation leads to an efficient methodology for determining the sets $\mathcal{G}$ of possible reaction force directions that can appear, in the presence of uncertainty, in any contact situation of an assembly task of polygonal objects moved on the plane.

With this methodology, a reaction force measured during the task execution by a force/torque sensor can be efficiently classified into one or more of the off-line

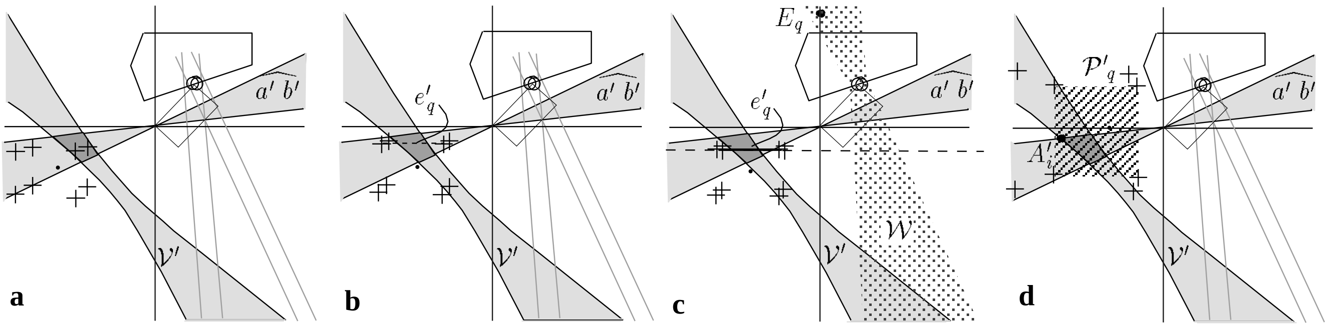

Fig. 6. Examples of the classification conditions for one basic contact.

computed sets $\mathcal{G}$. The classification algorithms proposed in the paper are, in fact, combinations of simple geometric tests, satisfying the real time requirements of the task execution. The algorithms have been implemented in C++ on a Silicon Graphics workstation (Crimson/ELAN). Typical classification times are about 50 μs for a one basic contact domain, and 2 ms for the 19 contact situations feasible during the positioning of a block into a corner with uncertainty.

This classification, based on force data, together with the classification based on configuration data, allows the fast on-line determination of the current contact situation during the assembly. This is a key aspect for the automatic planning and execution of assembly tasks.

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