Determining independent contacts regions to immobilize 2D articulated objects

Noé Alvarado[1], Raúl Suárez[1] and Máximo A. Roa[2]

***Abstract*— This paper deals with the problem of determining independent contacts regions (ICRs) on 2D articulated objects, such that a finger contact in each region guarantees a force-closure (FC) immobilization, independently of the exact position of the finger. These regions allow a robust finger or fixture placement on the links of the articulated object, despite of possible errors in the position of the contacts. The proposal defines a generalized wrench space for articulated objects and then computes the ICRs starting from an initial FC grasp, considering frictional contacts. The approach has been implemented, and some illustrative examples are provided.**

***Index Terms*— Fixturing, force-closure grasp, grasping, independent contact regions.**

I. INTRODUCTION

Immobilizing an object by using fingers or fixtures to constrain its degrees of freedom despite the possible existence of external perturbations has been an active research topic that still presents open problems [1]. The objects that can be grasped, manipulated or fixed by a robotic hand are of different shapes and sizes, and they can be either rigid or articulated. Articulated objects are formed by rigid links connected by some type of joint or hinges, such as a truck toy, staplers, or scissors (Fig. 1).

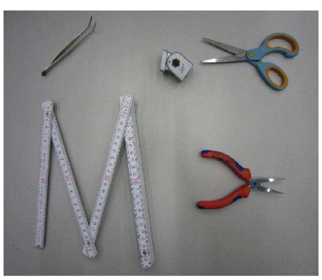

Fig. 1. Examples of articulated objects.

There are two properties commonly required for a grasp: force-closure or form-closure [2]. Both properties can be characterized in the object configuration space, that for a 2D rigid body has dimension $d = 3$. Any 2D rigid object can be always immobilized with $d + 1 = 4$ frictionless contacts or with 3 frictional contacts [3], [4].

[1] These authors are with the Institute of Industrial and Control Engineering (IOC) - Universitat Politècnica de Catalunya (UPC), Barcelona, Spain (noe.alvarado,raul.suarez@upc.edu).

[2] This author is with the Institute of Robotics and Mechatronics, German Aerospace Center (DLR), Wessling, Germany (maximo.roa@dlr.de).

This work was partially supported by the Spanish Government through the projects DPI2011-22471 and DPI2013-40882-P.

Several algorithms have been proposed to obtain precision grasps that satisfy the properties mentioned above, either for 2D or 3D objects. In 2D, grasp planning approaches have been proposed for polygonal objects with frictionless [5] and frictional contacts [6], [7], and for non polygonal objects with frictionless [8], [9] and frictional contacts [10], [11]. There are also works dealing with the grasp of 3D objects, either polyhedral [12], [13] or non polyhedral with frictionless and frictional contacts [14], [15], [16], [17].

In a real application, the actual grasp may differ from the expected one due to finger positioning errors. To deal with these errors and provide robustness to the grasp, the concept of independent contact regions (ICRs) was introduced [18], such that the fingers can be independently positioned inside their corresponding regions while ensuring a FC grasp, regardless of the exact position of the fingers. The computation of ICRs has been done in 2D for either polygonal [7], [19] or irregular objects [20] with frictional and frictionless contacts; and for the case of 3D objects, several works have dealt with polyhedrons [21] or objects with any shape considering any number of contacts [22], [23].

The work done in the area of robotic grasping has focused mainly on the search of FC grasps in both 2D and 3D single objects with different types and number of contacts.

However, few works have dealt with articulated objects, but nevertheless there are some relevant ones using different approaches, such as interactive perception [24], occlusion-aware systems [25], or even the modeling and static analysis of an articulated object with three-rigid links [26] for achieving a non-prehensile manipulation. Another relevant work [27] presents a systematic procedure to find a set of frictionless contact points that immobilizes a 2D serial chain with n polygons, based on second order effects. The lower bound of the number of contact points necessary to immobilize any chain of $n \neq 3$ hinged polygons without parallel edges was demonstrated to be $n+2$, while for $n = 3$ using 5 contacts allows only the immobilization of some chains with particular shapes of the polygons. In the general case, $n+3$ frictionless points are enough to immobilize any chain of n polygons.

Although these different approaches deal with finding precision FC grasps, we are not aware of any work dealing with the computation of ICRs on articulated objects. Note that for the case of articulated objects the ICRs must guarantee the immobilization of all the object internal degrees of freedom and the spatial immobilization of the object as a whole. This means that traditional procedures for ICRs computation cannot be directly applied to the case of articulated objects,

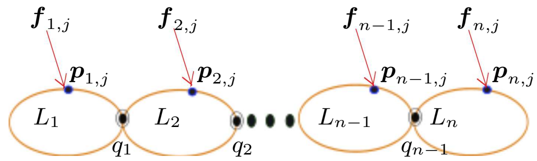

Fig. 2. Articulated object with n links (a generic force $\boldsymbol{f}_{i,j}$ acting on a point $\boldsymbol{p}_{i,j}$ is represented on each link i).

since the wrench space needs to be generalized to deal with the internal degrees of freedom provided by the object articulations. Therefore, the aim of this paper is the proposal of a procedure to compute ICRs on 2D articulated objects considering frictional contacts. The proposed approach has the following phases: a) find an initial FC grasp using the algorithm proposed in [28] extended to the case of n links and considering frictional contacts, and b) determine ICRs by extending the algorithm presented in [22] to the case of articulated objects. Note that the approach presented in [22] works in 3- or 6-dimensional wrench spaces for 2D and 3D objects respectively, and it is extended here to a generalized wrench space whose dimension will depend on the number of links of the articulated object (for a 2D serial articulated chain with n links, the dimension is $n+2$).

The rest of the paper is structured as follows. Section II provides an overview of the problem, including the main assumptions. Section III presents a procedure to find the elements of the vector of generalized wrenches for an articulated object with n links. Section IV summarizes the algorithm to find a FC grasp. Section V presents the procedure to compute ICRs. Section VI shows illustrative examples of the proposed approach. Finally, Section VII presents some conclusions and future work.

II. Problem Statement and Assumptions

Consider a 2D serial articulated object with n links and with rotational joints, as illustrated in Fig. 2. The problems to be addressed are the following:

1) Search a set of contact points on the surface of the links that allows a FC grasp.
2) Compute the ICRs for a FC grasp on the surface of the links.

The following assumptions are considered in this work:

- The links are connected by rotational joints.
- The links can overlap each other when they rotate (i.e. the problem could be of dimension $2\frac{1}{2}$, treated as 2-dimensional for simplicity).
- The boundary of each link is represented with a large enough set of points Ω (i.e. the links can be of any shape, either polygonal or non-polygonal).
- The normal direction pointing towards the interior of the object at each boundary point is known.
- The contacts between the fingers and the object are frictional, and Coulomb's friction law is considered.
- The reachability of the contact points for a particular device is not considered, although the approach in this paper can be integrated in other algorithms that include such analysis [29], [30].

III. Generalized Wrenches for Articulated Objects

For a single 2D solid object and considering frictional contact points, the grasp force $\boldsymbol{f}_i$ applied at a contact point $\boldsymbol{p}_i$ can be decomposed in two components $\boldsymbol{f}_{i,n}$ and $\boldsymbol{f}_{i,t}$ which are respectively normal and tangent to the object boundary. To avoid slippage of the finger, Coulomb's law must be satisfied: $\boldsymbol{f}_{i,t} \leq \mu \boldsymbol{f}_{i,n}$, where μ is the friction coefficient. This implies that the force applied by the finger must lie inside a friction cone centered on the direction normal to the object boundary and limited by the so-called primitive forces, $\boldsymbol{f}_i^r$ and $\boldsymbol{f}_i^l$. $\boldsymbol{f}_i$ is a positive combination of $\boldsymbol{f}_i^r$ and $\boldsymbol{f}_i^l$, i.e. $\boldsymbol{f}_i = \alpha \boldsymbol{f}_i^l + \beta \boldsymbol{f}_i^r$ with $\alpha, \beta \geq 0$. The primitive forces produce torques, forming the primitive wrenches $\boldsymbol{w}_i^l = [\boldsymbol{f}_i^l \tau_i^l]^T$ and $\boldsymbol{w}_i^r = [\boldsymbol{f}_i^r \tau_i^r]^T$, and the wrench produced by $\boldsymbol{f}_i$ is $\boldsymbol{w}_i = \alpha \boldsymbol{w}_i^l + \beta \boldsymbol{w}_i^r$. Since each $\boldsymbol{p}_i$ is associated with the wrenches $\boldsymbol{w}_i$, $\boldsymbol{w}_i^r$ and $\boldsymbol{w}_i^l$, a grasp defined by a set of k frictional contacts, $G = \{\boldsymbol{p}_1, ..., \boldsymbol{p}_k\}$, is associated with the sets $W = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_k\}$ and $W_p = \{\boldsymbol{w}_1^l, \boldsymbol{w}_1^r, ..., \boldsymbol{w}_k^l, \boldsymbol{w}_k^r\}$.

This section describes the generalization to n links of the procedure developed in [28] to obtain generalized wrenches for a serial articulated object. The method considers a virtual robot of $n+2$ joints (Fig. 3) wherein the first and second joints are virtual ones and the other joints correspond to the articulated object to be immobilized. The introduction of the virtual links and corresponding joints is done to represent the degrees of freedom of the first real link. The following basic nomenclature will be used:

- L_i: Links of the virtual robot, $i = -1, \ldots, n$. L_{-1} and L_0 are virtual ones, and L_1 to L_n correspond to the real ones.
- q_i: Generalized joint coordinates for the virtual robot, $i = -2, \ldots, n-1$. Joints -2 to 0 are virtual ones, and joints 1 to $n-1$ correspond to the real ones.
- $\boldsymbol{Q}_i$: Position of the joint q_i, for $i = 0, \ldots, n-1$. For $i = n$, Q_n is the position of the final end of the link with respect to the base frame.
- $\boldsymbol{P}_{i,j}$: Contact point j on link L_i with respect to the base frame.
- $\boldsymbol{p}_{i,j}$: Position vector (in the base frame) of contact point j on link L_i as measured from $\boldsymbol{Q}_{i-1}$ (i.e. $\boldsymbol{p}_{i,j} = \boldsymbol{P}_{i,j} - \boldsymbol{Q}_{i-1}$), $i = 1, \ldots, n$, $j = 1, \ldots, k_i$, where k_i is the number of contact points on link L_i. The total number of contacts is $k = \sum_i k_i$.
- $\boldsymbol{r}_i$: Position vector of $\boldsymbol{Q}_i$ measured from $\boldsymbol{Q}_{i-1}$ (i.e. $\boldsymbol{r}_i = \boldsymbol{Q}_i - \boldsymbol{Q}_{i-1}$).
- $\boldsymbol{s}_{i,j}$: Position vector of contact point j on link L_i measured from $\boldsymbol{Q}_i$ (i.e. $\boldsymbol{s}_{i,j} = \boldsymbol{P}_{i,j} - \boldsymbol{Q}_i = \boldsymbol{p}_{i,j} - \boldsymbol{r}_i$).
- $\boldsymbol{f}_{i,j}$: Force j applied to link L_i at contact point $\boldsymbol{p}_{i,j}$.

A. Determination of the generalized wrenches

The Jacobian J_i for each link L_i ($i = -1, \ldots, n$) is computed to relate the external forces applied to each link L_i with the forces or torques required in each joint for

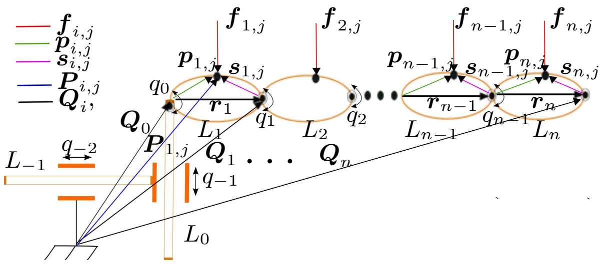

Fig. 3. General scheme of the virtual robot, $L_{-1}\dots L_n$ represent all the links of the virtual robot and $L_1\dots L_n$ are the articulated object's links.

an equilibrium condition. The total forces or torques τ_k $(k = -2, ..., n-1)$ to be applied at joints q_k are the components of a vector $\boldsymbol{\tau}$ given by:

$$\boldsymbol{\tau} = \sum_{i=-1}^{n}\sum_{j=1}^{k_i}\boldsymbol{\tau}_{i,j} = \sum_{i=-1}^{n}\sum_{j=1}^{k_i} J_i^T \boldsymbol{w}_{i,j} = \sum_{i=-1}^{n}\sum_{j=1}^{k_i} J_i^T \begin{bmatrix} f_{x_{i,j}} \\ f_{y_{i,j}} \\ M_{s_{i,j}} \end{bmatrix} \tag{1}$$

where $M_{s_{i,j}} = \boldsymbol{s}_{i,j} \times \boldsymbol{f}_{i,j}$.

Therefore, expanding eq. (1) the components τ_k $(k = -2, ..., n-1)$ are :

$$\begin{array}{rcll}
\tau_{-2} & = & \sum_j f_{x_{1,j}} + \sum_j f_{x_{2,j}} + ... + \sum_j f_{x_{n-1,j}} + \sum_j f_{x_{n,j}} & = 0 \\
\tau_{-1} & = & \sum_j f_{y_{1,j}} + \sum_j f_{y_{2,j}} + ... + \sum_j f_{y_{n-1,j}} + \sum_j f_{y_{n,j}} & = 0 \\
\tau_{0} & = & \sum_j \boldsymbol{p}_{1,j} \times \boldsymbol{f}_{1,j} + \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{2,j} + ... + \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{n-1,j} & \\
 & & + \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{n,j} & = 0 \\
\tau_{1} & = & 0 + \sum_j \boldsymbol{p}_{2,j} \times \boldsymbol{f}_{2,j} + ... + \sum_j \boldsymbol{r}_2 \times \boldsymbol{f}_{n-1,j} & \\
 & & + \sum_j \boldsymbol{r}_2 \times \boldsymbol{f}_{n,j} & = 0 \\
\vdots & & & \\
\tau_{n-2} & = & 0 + 0 + ... + \sum_j \boldsymbol{p}_{n-1,j} \times \boldsymbol{f}_{n-1,j} + \sum_j \boldsymbol{r}_{n-1} \times \boldsymbol{f}_{n,j} & = 0 \\
\tau_{n-1} & = & 0 + 0 + ... + 0 + \sum_j \boldsymbol{p}_{n,j} \times \boldsymbol{f}_{n,j} & = 0
\end{array} \tag{2}$$

Now, it is possible to consider a generalized wrench space $\mathcal{W}$ defined by the base $\{\hat{\boldsymbol{\tau}}_{-2}, \hat{\boldsymbol{\tau}}_{-1}, \hat{\boldsymbol{\tau}}_0, \hat{\boldsymbol{\tau}}_1, \dots, \hat{\boldsymbol{\tau}}_2, \hat{\boldsymbol{\tau}}_{n-1}\}$ for the articulated object, where the generalized wrenches $\boldsymbol{W}_{i,j}$ generated respectively by forces $\boldsymbol{f}_{i,j}$ are

$$\boldsymbol{W}_{1,j} = \begin{bmatrix} f_{x_{1,j}} \\ f_{y_{1,j}} \\ \boldsymbol{p}_{1,j} \times \boldsymbol{f}_{1,j} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{W}_{2,j} = \begin{bmatrix} f_{x_{2,j}} \\ f_{y_{2,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{2,j} \\ \boldsymbol{p}_{2,j} \times \boldsymbol{f}_{2,j} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\boldsymbol{W}_{n-1,j} = \begin{bmatrix} f_{x_{n-1,j}} \\ f_{y_{n-1,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{n-1,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}_{n-1,j} \\ \vdots \\ \boldsymbol{p}_{n-1,j} \times \boldsymbol{f}_{n-1,j} \\ 0 \end{bmatrix} \quad \boldsymbol{W}_{n,j} = \begin{bmatrix} f_{x_{n,j}} \\ f_{y_{n,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{n,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}_{n,j} \\ \vdots \\ \boldsymbol{r}_{n-1} \times \boldsymbol{f}_{n,j} \\ \boldsymbol{p}_{n,j} \times \boldsymbol{f}_{n,j} \end{bmatrix} \tag{3}$$

Taking into account the above considerations, the primitive wrenches $\boldsymbol{W}^c_{i,j}$ (where $c \in \{l, r\}$ is used to represent the left and right boundaries of the friction cone) generated by the primitives forces $\boldsymbol{f}^c_{i,j}$ that constrain a force $\boldsymbol{f}_{i,j}$ to be inside a friction cone, are defined by

$$\boldsymbol{W}^c_{1,j} = \begin{bmatrix} f^c_{x_{1,j}} \\ f^c_{y_{1,j}} \\ \boldsymbol{p}_{1,j} \times \boldsymbol{f}^c_{1,j} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{W}^c_{2,j} = \begin{bmatrix} f^c_{x_{2,j}} \\ f^c_{y_{2,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}^c_{2,j} \\ \boldsymbol{p}_{2,j} \times \boldsymbol{f}^c_{2,j} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\boldsymbol{W}^c_{n-1,j} = \begin{bmatrix} f^c_{x_{n-1,j}} \\ f^c_{y_{n-1,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}^c_{n-1,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}^c_{n-1,j} \\ \vdots \\ \boldsymbol{p}_{n-1,j} \times \boldsymbol{f}^c_{n-1,j} \\ 0 \end{bmatrix} \quad \boldsymbol{W}^c_{n,j} = \begin{bmatrix} f^c_{x_{n,j}} \\ f^c_{y_{n,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}^c_{n,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}^c_{n,j} \\ \vdots \\ \boldsymbol{r}_{n-1} \times \boldsymbol{f}^c_{n,j} \\ \boldsymbol{p}_{n,j} \times \boldsymbol{f}^c_{n,j} \end{bmatrix} \tag{4}$$

The dimension of $\mathcal{W}$ is $n+2$, and therefore the generalized wrenches $\boldsymbol{W}_{i,j}$ and $\boldsymbol{W}^c_{i,j}$ have $n+2$ components. Note that this is different from the traditional wrench space for a rigid object, whose dimension is 3 for 2D objects and 6 for 3D objects. Moreover, note also that each generalized wrench $\boldsymbol{W}_{i,j}$ (or $\boldsymbol{W}^c_{i,j}$) has only three independent components, which come from the two independent parameters defining $f_{x_{i,j}}$ (or $f^c_{x_{i,j}}$) and $f_{y_{i,j}}$ (or $f^c_{y_{i,j}}$), and a third parameter defining the contact point $\boldsymbol{p}_{i,j}$ on the object boundary.

From the representation of generalized wrenches in eq. (4), since the last component depends only on the forces $\boldsymbol{f}_{n,j}$ applied on the last link L_n, it is straightforward that the forces in the link n must be able to produce positive and negative torques in order to counterbalance any perturbation. This in turn means that for frictionless contacts it must be $k_n \geq 2$, i.e. there must be at least two applied forces $\boldsymbol{f}_{n,1}$ and $\boldsymbol{f}_{n,2}$ on the last link in order to expand the whole space of τ_{n-1}, while for frictional contacts only one contact could be enough if the forces in the friction cone allow both positive and negative torques (which is not always allowed by the link shape). Since the virtual links can be added to any extreme of the articulated object, the same reasoning applies for the first link L_1.

B. Force-closure Test

Considering the set $G = \{\boldsymbol{p}_{i,j}, i = 1, ...n, j = 1, ..., k_i\}$ of $k = \sum_i k_i$ contact points (with k_i is the number of contact points on link L_i), and a force $\boldsymbol{f}_{i,j}$ applied at each $\boldsymbol{p}_{i,j}$, two sets $W = \{\boldsymbol{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ and $W_p = \{\boldsymbol{W}^c_{i,j}, i = 1, ..., n, j = 1, ..., k_i, c \in \{l, r\}\}$ are obtained. The necessary and sufficient condition for the existence of a FC grasp is that the origin of the generalized wrench space lies inside the convex hull CH(W_p) of the contact wrenches W_p [4], [31]. This guarantees that the grasp can generate appropriate wrenches to counteract perturbation wrenches in any direction, i.e. to counterbalance any force(s) $\boldsymbol{f}_{i,j}$ applied on any link L_i of the articulated object. Note that this test is a generalization of the traditional FC test for objects without internal degrees of freedom. The test used in this work to verify this condition is derived from [16] for the case of a single rigid object and then extended in [28] for an articulated 2D object. Let P be the centroid of the primitives wrenches, O the origin of the wrench space and H_i a boundary hyperplane of CH(W_p): in order for a grasp G to be FC, P and O must lie on the same side of H_i $\forall i$.

IV. Finding an Initial FC Grasp

The algorithm described in this section is the extension to frictional contacts of the algorithm presented in [28] for frictionless contacts. The procedure generates an initial grasp G^m, $m = 1$, by selecting k random points from the set Ω that describes the object boundary, then computes the corresponding set W^m when frictionless contact points are considered, and W^m_p with primitives contact wrenches for frictional contacts. The next step is to check whether the points in G^m lead to a FC grasp. If G^m does not

Algorithm 1 Search of an initial FC grasp

Ensure: : Grasp G^m with FC
1: Generate a random initial grasp G^m, $m = 1$.
2: **while** G^m is not a FC grasp **do**
3: Form the corresponding set of wrenches W^m and primitives wrenches W_p^m
4: Determine a subset G_R^m of grasp points on G^m to be replaced.
5: Generate a subset Ω_C^m with candidate points to replace one of the points in G_R^m.
6: Obtain an auxiliary grasp G_{aux} replacing a point in G_R^m with one point from Ω_C^m.
7: Update the counter $m = m + 1$.
8: G^m=G_{aux}.
9: **end while**
10: **return** (G^m)

provide a FC grasp, then a search of new contact points is done, based on separating hyperplanes in the wrench space that define candidate points to replace one of the current points in G^m to obtain another grasp G^{m+1}. This is iteratively repeated until a FC grasp is found. The procedure is detailed in Algorithm 1 and explained below.

If grasp G^m fails the FC-test mentioned in Section III-B, the search procedure, Steps (3) to (8), iteratively tries to improve the grasp by changing one of the points in G^m.

In Step (4) a subset $G_R^m \subset G^m$ is generated with the points of the wrench space that simultaneously define all the critical hyperplanes H defining the boundary of CH(W) that produce a failure of the FC-test (i.e. P and O lie on different sides of the plane).

In Step (5) a subset Ω_C^m with candidate points to replace one point in G_R^m is determined by hyperplanes $H^{'}$ passing through the origin and parallel to the critical hyperplanes H. The replacement candidate points are those that simultaneously lie on the opposite side of the point P with respect to all the hyperplanes $H^{'}$.

In Step (6) one of the points in G_R^m is replaced by a point producing a wrench $\boldsymbol{W}_*$ randomly taken from Ω_C^m, $\boldsymbol{W}_*$ replaces the closest point in G_R^m, generating an auxiliary grasp G_{aux}. The centroid P^* and the distance $\left|\overline{P^*O}\right|$ are computed for the wrenches of the auxiliary grasp G_{aux}. Let P^m be the centroid of the set of wrenches W in the iteration m. If the relation $\left|\overline{P^*O}\right| < \left|\overline{P^mO}\right|$ is satisfied then the auxiliary grasp G_{aux} is selected as new grasp. If all the points in G_R^m were replaced and none of them reduces the distance $\left|\overline{P^mO}\right|$, the selection is the candidate G^* that has the smaller distance $\left|\overline{P^*O}\right|$. When frictional points are considered, the subset Ω_C^m is built using the generalized wrenches $\boldsymbol{W}_{i,j}$.The grasp G^m generated in each iteration is saved so it is not taken into account in subsequent iterations. This consideration avoids falling in local minima and allows the exploration of wrench space to continue until a FC grasp is found (if there is one).

Fig. 4 shows an example with frictional contacts in a hypothetical 2-dimensional wrench space, thus it can be graphically represented (remember that the dimension of the real wrench space is $n+2$). The grasp G^m producing wrenches $W^m = \{\boldsymbol{W}_{1,1}, \boldsymbol{W}_{2,1}, \boldsymbol{W}_{3,1}\}$ and $W_p^m = \left\{\boldsymbol{W}_{1,1}^l, \boldsymbol{W}_{1,1}^r, ..., \boldsymbol{W}_{3,1}^l, \boldsymbol{W}_{3,1}^r\right\}$ is not force-closure, being H_3 the hyperplane that produces the FC-test failure. Then, the set of possible points to be replaced is $G_R^m = \left\{\boldsymbol{p}_{1,1}, \boldsymbol{p}_{2,1}\right\}$, i.e. the points producing the wrenches $\boldsymbol{W}_{1,1}$ and $\boldsymbol{W}_{2,1}$ and its corresponding primitive wrenches, some of which define H_3. The contact points that produce wrenches lying in the gray area determined by the hyperplane $H_3^{'}$ belong to Ω_C^m. The auxiliary grasp G_{aux} with $\boldsymbol{W}_*$ replacing $\boldsymbol{W}_{2,1}$, i.e. with $W^{m+1} = \{\boldsymbol{W}_{1,1}, \boldsymbol{W}_*, \boldsymbol{W}_{3,1}\}$ and $W_p^{m+1} = \left\{\boldsymbol{W}_{1,1}^l, \boldsymbol{W}_{1,1}^r, \boldsymbol{W}_*^l, \boldsymbol{W}_*^r, \boldsymbol{W}_{3,1}^l, \boldsymbol{W}_{3,1}^r\right\}$, is FC.

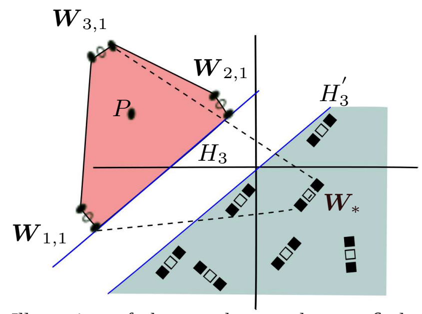

Fig. 4. Illustration of the search procedure to find one FC grasp in a hypothetical 2D wrench space using frictional contacts. The gray zone contains the candidate points.

V. Determining the ICRs

This section presents the algorithm to compute the ICRs, such that if a contact is located inside each region the resulting grasp is always FC. The algorithm works as follows.

For a given FC grasp, the grasp quality Q_g is fixed by the facet F_Q of the convex hull closest to the origin. Let F_v denote a facet of CH(W) that contains at least one primitive wrench $\boldsymbol{W}_{i,j}^c$ for a particular grasp point $\boldsymbol{p}_{i,j}$. Several hyperplanes $H_v^{'}$ parallel to each facet F_v are built at a distance Q_g from the origin (i.e. tangent to the hypersphere of radius Q_g). The role of these hyperplanes is determining regions $S_{i,j}$ of the wrench space where new wrenches (associated with new contact points) can generate FC grasps with equal or greater quality. The regions $S_{i,j}$ are the intersection of the half-spaces $H_v^{'+}$ that do not contain the origin O. The ICRs are determined by the set of neighbor points of $\boldsymbol{p}_{i,j}$ such that at least one of its primitive wrenches falls into the corresponding search zone $S_{i,j}$.

The procedure to determine the ICRs is given in Algorithm 2 and illustrated in Fig. 5 for, again, a hypothetical 2-dimensional wrench space. An initial FC grasp $G = \left\{\boldsymbol{p}_{1,1}, \boldsymbol{p}_{2,1}, \boldsymbol{p}_{3,1}\right\}$ generating the set of wrenches $W = \{\boldsymbol{W}_{1,1}, \boldsymbol{W}_{2,1}, \boldsymbol{W}_{3,1}\}$ and $W_p = \left\{\boldsymbol{W}_{1,1}^l, \boldsymbol{W}_{1,1}^r, ..., \boldsymbol{W}_{3,1}^l, \boldsymbol{W}_{3,1}^r\right\}$ was obtained using Algorithm 1; its initial grasp quality Q_g and the closest facet to the origin F_Q are also shown in Fig. 5. Now, in order to determine the search region associated with the point $\boldsymbol{p}_{2,1}$ with primitives wrenches $\boldsymbol{W}_{2,1}^l$ and $\boldsymbol{W}_{2,1}^r$, the hyperplane $H_1^{'}$ parallel to H_1 at a distance Q_g from O is built (note that at least one $\boldsymbol{W}_{2,1}^c$, $c \in \{l, r\}$, must belong to H_1, which in this case is $\boldsymbol{W}_{2,1}^l$). $H_1^{'}$ and F_Q define the region $S_{2,1}$ in the wrench space where, in the example, the wrenches corresponding to two neighboring points of $\boldsymbol{p}_{2,1}$ are located, and therefore selected for the corresponding ICR. The search zones $S_{i,j}$ for each grasp point are depicted in different color, and the wrenches associated with neighboring points within each ICR are depicted with squares (black ones representing $\boldsymbol{W}_{i,j}^c$). The core of Algorithm 2 are Step (6) and the loop starting in Step (9). In Step (6) the hyperplanes $H_v^{'}$ parallel to F_v and at a distance Q_g from O, as well as $H_v^{'+}$ are computed. Step 9 is the most costly one because it is necessary to check whether the primitive wrenches $\boldsymbol{W}_{o,j}^c$ of an unknown number of points $\boldsymbol{p}_{o,j}$ belong to each half-space $H_v^{'+}$.

VI. Examples

In this section the approach proposed for the computation of the ICRs is illustrated with examples for articulated objects with 2, 3 and 4 links. The considered friction coefficient was $\mu = 0.5$. The implementation was done using Matlab and C++ on an Intel Core2 Duo 2.0 GHz computer. The library Qhull [32] was used to compute the convex hulls. The figures of the examples show: a) the initial randomly generated grasp, which in general is a non-FC grasp, and, b) the obtained FC Grasp and corresponding ICRs.

Algorithm 2 Computation of ICRs

Ensure: : Independent contact regions ICRs.

Find an initial FC grasp G using Algorithm 1.
Compute the initial quality Q_g.
Compute CH(W_p).
for $i = 1$ to n **do**
  for $j = 1$ to k_i (i.e. for each contact point $p_{i,j} \in G$) **do**
    For each facet F_v of CH(W_p) with at least one vertex $W^c_{i,j}$, build the hyperplane H'_v parallel to F_v and at distance Q_g from the origin O, leaving O and F_v in different half-spaces. Let H'^{+}_v be the open half-space such that $W^c_{i,j} \in H'^{+}_v$. The search regions $S_{i,j}$ are determined by intersecting the half-spaces H'^{+}_v, i.e $S_{i,j} = \cap_v H'^{+}_v$
    Initialize ICR$_{i,j} = \{p_{i,j}\}$.
    Label $p_{i,j}$ as open
    while there are open points $p_{i,u} \in$ ICR$_{i,j}$ **do**
      for all the neighboring points $p_{i,s}$ of $p_{i,u}$ **do**
        if $\exists c$ such that $w^c_{i,s} \in S_{i,j}$ **then**
          ICR$_{i,j}$ = ICR$_{i,j} \cup \{p_{i,s}\}$
          Label $p_{i,s}$ as open
        end if
      end for
      Label $p_{i,u}$ as closed
    end while
  end for
end for
return ICRs = $\{$ICR$_{i,j}, i = 1, ..., n$ and $j = 1, ..., k_i\}$

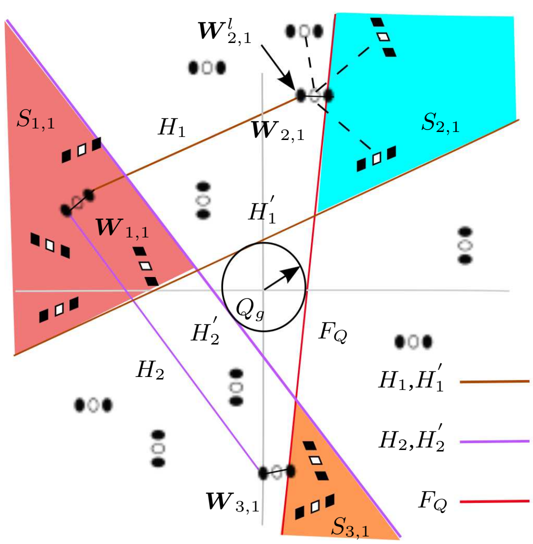

Fig. 5. Search for ICRs ensuring a minimum grasp quality. Search zones $S_{i,j}$ for each grasping point are depicted in color, and wrenches associated with neighboring points within each ICR are depicted as squares.

Example 1: Articulated object with 2 links shown in Fig. 6. The initial FC grasp was obtained after 2 iterations in 1 s, and the ICRs were obtained in 0.6 s.

Example 2: Articulated object with 3 links shown in Fig. 7. The initial FC grasp was obtained after 4 iterations in 4 s, and the ICRs were obtained in 1.3 s.

Example 3: Articulated object with 4 links shown in Fig. 8. The initial FC grasp was obtained after 14 iterations in 17 s, and the ICRs were obtained in 1.6 s.

Example 4: Articulated object with 4 links shown in Fig. 9. The initial FC grasp was obtained after 6 iterations in 11 s, and the ICRs were obtained in 2.1 s. Note that in this case there are no contacts on the third link, which does not prevent the FC grasp.

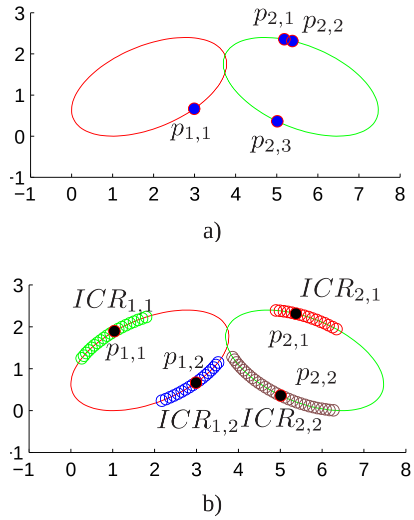

Fig. 6. Example 1: a) Non FC grasp, b) FC grasp and ICRs.

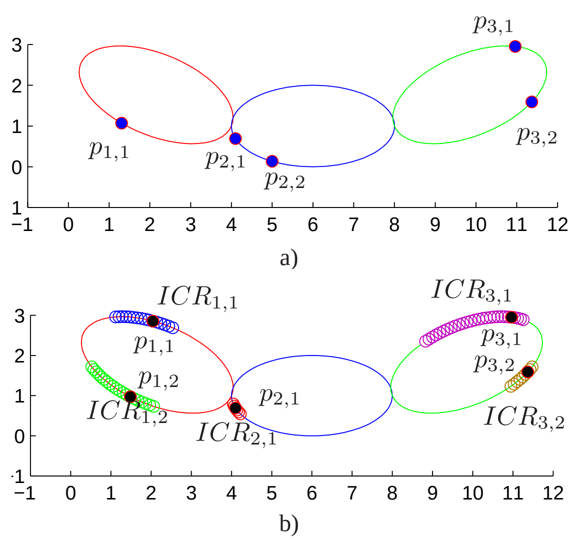

Fig. 7. Example 2: a) Non FC grasp, b) FC grasp and ICRs.

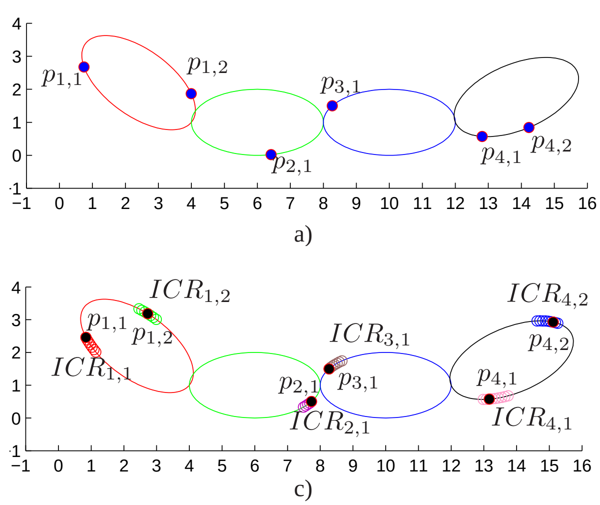

Fig. 8. Example 3: a) Non-FC grasp, b) FC grasp and ICRs.

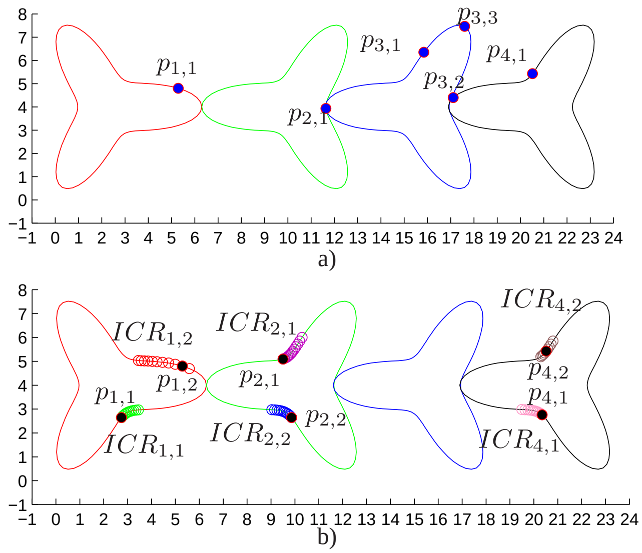

Fig. 9. Example 4: a) Non-FC grasp, b) FC grasp and ICRs.

VII. Conclusions

In this paper we proposed an approach to obtain independent contact regions (ICRs) for 2D articulated objects with n links considering frictional contacts. The approach has two stages: the first one performs the synthesis of a FC grasp, and the second one computes the ICRs around the contact points of the FC grasp. The algorithms were implemented and examples for articulated objects with two, three and four links were presented.

As future work we consider the extension to the computation of ICRs for objects with both rotational and prismatic joints; in this case the analysis could be directly done using the proposed approach based on the appropriate Jacobian matrices. Another future work is the generalization of the approach for 3D articulated objects considering frictionless and frictional contacts. 3D objects imply more degrees of freedom and therefore higher dimensional spaces; however, the algorithms were already running on wrench spaces of any dimension, since the proposed approach is valid for any number of links of a 2D articulated object. Thus, the complexity of the generalization for 3D objects could be determined by the development of the proper model for the generalized wrenches. Finally, another future development is the consideration of branched articulated objects and closed kinematic chains, for both 2D and 3D articulated objects.

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