Independent contact regions for frictional grasps on 3D objects*

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Abstract— This paper presents an efficient algorithm to compute independent contact regions on the surface of complex 3D objects such that a finger contact anywhere inside each of these regions assures a force-closure grasp despite the exact contact position. Independent contact regions provide robustness in front of finger positioning errors during an object grasping, and give relevant information for finger repositioning during the object manipulation. The object is described with a mesh of surface points, so the procedure is applicable to objects of any arbitrary shape. The proposed approach uses information from the wrench space, and generates the independent regions by growing them around the contact points of a given starting grasp. A two-phase approach is also provided to find a locally optimum force-closure grasp that serves as starting grasp, considering as grasp quality measure the largest perturbation wrench that the grasp can resist with independence of the perturbation direction. The approach has been implemented and several examples are provided to illustrate its performance.

Index Terms— Frictional grasp, independent contact regions.

I. Introduction

Grasp synthesis for real world complex objects that assure the immobility of the object despite the influence of external disturbances has been a topic of great interest in grasping and manipulation of objects. These grasps satisfy the properties of form or force-closure [1]. In a form-closure grasp the position of the contacts ensure the object immobility; this property is mostly used in the fixture design for object inspection or to do some action on it, basically when the task requires a grasp that does not rely on friction. When the forces applied by the fingers ensure the object immobility, the object is in a force-closure grasp; this is commonly used in grasp and manipulation of objects with frictional contacts. The synthesis of force-closure grasps has been tackled mainly for precision grasps (i.e. grasps formed by a set of particular finger contact points on the object surface) in 2D polygonal [2] or non-polygonal objects [3], 3D polyhedral objects [4], objects with smooth curved surfaces [5] or 3D discretized objects [6].

In a real world execution, the actual and the theoretical grasp may differ due to finger positioning errors; to provide

robustness in front of these errors, the computation of independent contact regions (ICRs) on the object boundary was introduced [7]. Each finger can be positioned on an ICR assuring a force-closure (FC) grasp, with independence of the exact position of each finger. The determination of ICRs has been solved for 2D polygonal [8] and non-polygonal objects [9], and for 3D polyhedral objects [4] [10]. The ICRs have also been used to determine contact regions on 3D objects based on initial examples, although the results depend on the chosen example [11].

A previous work of the authors [12] presented an algorithmic approach to compute ICRs for frictionless contacts on 3D discrete objects; this paper extends the previous approach to determine independent contact regions on a 3D discrete object using any number n of frictional contacts (provided that $n \geq 3$). The proposed algorithm generates the ICRs by growing them from a starting FC grasp. In order to use a good starting FC grasp, a procedure to obtain a locally optimum one is also proposed. The optimization procedure is an oriented search that looks for the grasp that resists the largest perturbation wrench, with independence of the perturbation direction [13]. Then, the obtained ICRs assure a FC grasp with a controlled minimum quality. The approach does not take into account the kinematical constraints imposed by the mechanical hand or gripper.

The rest of the paper is organized as follows. Section II provides the required background on frictional grasps, including the force-closure test and the quality measure used in the paper. Section III presents the approach to compute a starting FC grasp, and the algorithm to compute the independent contact regions. Section IV shows the application of the approach on different objects. Finally, Section V summarizes the work and discusses some future applications.

II. Preliminaries

A. Assumptions

The following assumptions are considered to compute the independent contact regions for a frictional grasp on an arbitrary 3D object:

- The object surface is represented with a large set Ω of points, described by position vectors $\boldsymbol{p}_i$ measured with respect to a reference system located in the object's center of mass. Each point has an associated unitary normal direction $\hat{\boldsymbol{n}}_i$ pointing toward the interior of the object.

- The number of points in Ω is large enough to accurately represent the surface of the object; each point is connected with some neighboring points forming a mesh.

Coulomb's friction model is used in this work, stating that there is no slipping at the contact point if $\boldsymbol{f}_i^t \leq \mu \boldsymbol{f}_i^n$, with $\boldsymbol{f}_i^n$ and $\boldsymbol{f}_i^t$ being the tangential and normal components of the applied force, respectively, and μ being the friction coefficient. In the three-dimensional physical space this is a nonlinear model, defining a friction cone that includes all the possible grasp forces. To simplify the model, the cone is linearized with a m-side polyhedral convex cone (the more sides the better the approximation, but the greater the computational cost to deal with the linearized cone). The grasping force at the contact point is given by

$$\boldsymbol{f}_i = \sum_{j=1}^{m} \alpha_{ij} \boldsymbol{s}_{ij}, \ \alpha_{ij} \geq 0 \tag{1}$$

with $\boldsymbol{s}_{ij}$ representing the normalized vector of the j-th edge of the convex cone. The wrench produced by the force $\boldsymbol{f}_i$ is

$$\tilde{\boldsymbol{\omega}}_i = \sum_{j=1}^{m} \alpha_{ij} \boldsymbol{\omega}_{ij} , \ \boldsymbol{\omega}_{ij} = \begin{pmatrix} \boldsymbol{s}_{ij} \\ \boldsymbol{p_i} \times \boldsymbol{s}_{ij} \end{pmatrix} \tag{2}$$

where $\boldsymbol{\omega}_{ij}$ are called the primitive contact wrenches. Therefore, each contact point in the physical space has m associated points in the wrench space, one for each edge $\boldsymbol{s}_{ij}$ of the convex cone. Let $\boldsymbol{\omega}_i$ be the "normal contact wrench" for the force $\boldsymbol{f}_i$, i.e. the primitive contact wrench in case of a frictionless contact point, where the grasp forces can only be applied in the direction normal to the object surface. The relation between the normal contact wrench $\boldsymbol{\omega}_i$ and the primitive contact wrenches $\boldsymbol{\omega}_{ij}$ for the linearized friction cone in a particular contact point is:

$$\boldsymbol{\omega}_i = \frac{1}{m} \sum_{j=1}^{m} \boldsymbol{\omega}_{ij} \tag{3}$$

For a given grasp $G = \{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n\}$ the wrenches applied through the contact points on the object are grouped in a wrench set $W = \{\boldsymbol{\omega}_{11}, \ldots, \boldsymbol{\omega}_{1m}, \ldots, \boldsymbol{\omega}_{n1}, \ldots, \boldsymbol{\omega}_{nm}\}$. Each physical point $\boldsymbol{p}_i$ in Ω has a corresponding normal contact wrench $\boldsymbol{\omega}_i$ in the wrench space; when it is clear, both of them will be used to indicate a fixture constraint (in general, the same wrench can be produced at different contact points).

B. Force-closure condition

A necessary and sufficient condition for the existence of a FC grasp is that the origin of the wrench space lies strictly inside the convex hull (CH) of the primitive wrench set [14]. This condition is employed in this work using the following lemma.

Lemma 1: Let G be a grasp with a set W of contact wrenches, $\mathcal{I}$ the set of strictly interior points of $CH(W)$, and H a supporting hyperplane of $CH(W)$ (i.e. a hyperplane containing one of the facets of $CH(W)$). The origin O of the wrench space satisfies $O \in \mathcal{I}$ if and only if any $P \in \mathcal{I}$ and O lie in the same half-space for every H of $CH(W)$.
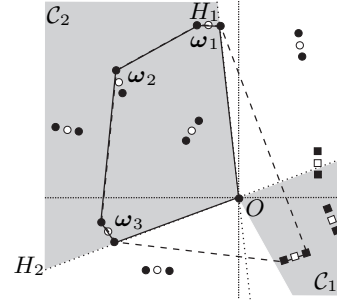

Fig. 1. Synthesis of a FC grasp. The convex hull for the grasp set $F = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3\} \cup \{\boldsymbol{0}\}$ (in continuous lines) defines the supporting hyperplanes H_1 and H_2 that contain the origin. The convex set $\mathcal{C}_1$ contains primitive wrenches corresponding to 3 points (depicted as white squares), thus the algorithm provides 3 FC grasps, one of them illustrated with the convex hull in discontinuous lines.

From *Lemma 1*, checking whether a given point $P \in \mathcal{I}$ and the origin O lie in the same half-space defined by each supporting hyperplane H is enough to prove whether O lies inside $CH(W)$, i.e. to prove the FC property for the grasp G. P is chosen as the centroid of the primitive contact wrenches, which is always an interior point of $CH(W)$; therefore, the FC test checks whether the centroid P and the origin O lie on the same side for all the supporting hyperplanes of $CH(W)$.

C. Grasp quality measure

Several grasp quality measures have been proposed in the literature [15]; this work uses as a quality measure the largest perturbation wrench that the grasp can resist, with independence of the perturbation direction [13]; this is one of the most popular grasp quality measures. Geometrically, this quality is the radius of the largest ball centered at the origin O of the wrench space and fully contained in $CH(W)$, i.e. it is the distance from O to the closest facet of $CH(W)$.

III. COMPUTATION OF INDEPENDENT CONTACT REGIONS

A. Starting grasp for the ICR computation

The synthesis of a starting FC to be used for the search of the ICRs is performed using two algorithms, the first one generates an initial grasp with uncontrolled quality and the second one uses it to generate a grasp with locally optimum quality for the ICRs search. The initial FC grasp is obtained using an algorithm presented in a previous work [16]. This algorithm randomly choses $n-1$ points from Ω, and the convex hull $CH(W)$ of the primitive wrenches of the selected points plus the origin O is computed, as illustrated in Fig. 1 for a hypothetical 2D wrench space (the actual wrench space is 6-dimensional). Two regions, $\mathcal{C}_1$ and $\mathcal{C}_2$, are defined by the intersection of the half-spaces determined by the supporting hyperplanes of $CH(W)$ that contain the origin. If there is at least one primitive wrench lying in $\mathcal{C}_1$, then the corresponding grasp point is added to the set F^1, the conditions of the *Lemma 1* are fulfilled and a FC grasp is provided. If $\mathcal{C}_1$ is empty, the algorithm iteratively replaces
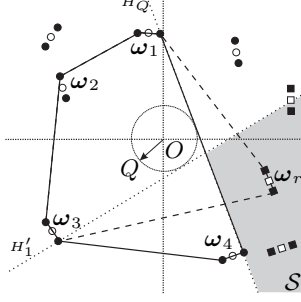
Fig. 2. Optimization procedure. The set $\mathcal{C}$ of wrenches that improve the actual quality (depicted as white squares in the gray area $\mathcal{S}$) is defined by the hyperplanes H'_1 and $H'_2 = H_Q$. The grasp for the next iteration cycle, $F^k = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3, \boldsymbol{\omega}_r\}$ is also shown.
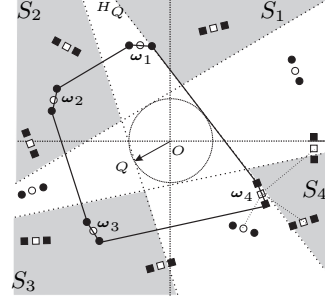

Fig. 3. Search of the independent contact regions. The search zones S_i for each grasp point are depicted in gray, and the neighbor points within each ICR are depicted as white squares.

one of the points in F^1 and performs another search of points in the new $\mathcal{C}_1$, until it contains at least one primitive wrench, i.e. until it finds at least one FC grasp. Further details and discussion on the completeness and advantages of the algorithm are provided in [16].

Algorithm 1: Search of an initial FC grasp

1) Generate a random set $F^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_{n-1}\}$, $k = 1$
2) Build $W^k = \{\boldsymbol{\omega}_{11}, \ldots, \boldsymbol{\omega}_{1m}, \ldots, \boldsymbol{\omega}_{n-1\,1}, \ldots, \boldsymbol{\omega}_{n-1\,m}\}$ $\cup \{O\}$
3) Compute $CH(W^k)$
4) Find $\mathcal{C}_1 = \{\boldsymbol{\omega}_i \mid \boldsymbol{\omega}_i \vee \boldsymbol{\omega}_{i1} \vee \ldots \vee \boldsymbol{\omega}_{im} \in \bigcap H_l^+\}$ and $\mathcal{C}_2 = \{\boldsymbol{\omega}_j \mid \boldsymbol{\omega}_j \wedge \boldsymbol{\omega}_{j1} \wedge \ldots \wedge \boldsymbol{\omega}_{jm} \in \bigcap H_l^-\}$
5) If $\mathcal{C}_1 \neq \emptyset$ then return $G = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_c\}$, with a randomly chosen $\boldsymbol{\omega}_c \in \mathcal{C}_1$
   Else
   Pick up a $\boldsymbol{\omega}_l \notin \mathcal{C}_1 \cup \mathcal{C}_2$
   Form F^{k+1} by replacing a $\boldsymbol{\omega}_i \in F^k$ such that $d_{L_2}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_l)$ be a minimum. Proceed to Step 2
   Endif

From the initial FC grasp a locally optimum one is obtained to be used as the starting grasp of the ICR search algorithm; it is done by looking for the grasp that resists the largest perturbation wrench with independence of its direction (Section II-C), using the following procedure:

Algorithm 2: Search of a locally optimum grasp

1) Find an initial FC grasp $G^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_n\}$, $k = 1$, with the corresponding wrench set W
2) Compute $CH(W)$ and determine H_Q such that the distance D to the origin is a minimum. The current grasp quality is $Q^k = D_Q$
3) Build $\mathcal{T} = \{\tilde{\boldsymbol{\omega}}_j, \mid \|\tilde{\boldsymbol{\omega}}_1\| \leq \ldots \leq \|\tilde{\boldsymbol{\omega}}_j\|\}$, $j = 1 \ldots J$ ($J \leq n$) such that at least one $\tilde{\boldsymbol{\omega}}_{jm}$ lie on H_Q
4) Initialize $j = 1$. For the hyperplanes H_l of $CH(W)$ containing at least one primitive wrench of $\tilde{\boldsymbol{\omega}}_j$, build the hyperplanes H'_l containing all the primitive wrenches not belonging to $\tilde{\boldsymbol{\omega}}_j$ and lying to a distance Q^k from the origin O
5) Let $\mathcal{S} = \bigcap H'^{+}_l$, with H'^{+}_l the half-space such that $O \notin H'^{+}_l$. Find $\mathcal{C} = \{\boldsymbol{\omega}_i \mid \boldsymbol{\omega}_i \vee \boldsymbol{\omega}_{i1} \vee \ldots \vee \boldsymbol{\omega}_{im} \in S\}$

6) If $\mathcal{C} = \emptyset$ and $j \neq J$
   Let $j = j + 1$. Proceed to Step 4
   Elseif $\mathcal{C} = \emptyset$ and $j = J$
   A local maximum has been reached; return G^k
   Elseif $\mathcal{C} \neq \emptyset$
   Replace $\tilde{\boldsymbol{\omega}}_j$ with a random $\boldsymbol{\omega}_r \in \mathcal{C}$. Let $k = k+1$. Go to Step 2
   Endif

Fig. 2 illustrates the procedure in a hypothetical 2D wrench space. Step 3 looks for the grasp points that contribute with at least one primitive wrench to the facet F_Q defining the current grasp quality, and sorts them according to its norm. Step 4 builds the hyperplanes required to find the points that improve the actual grasp quality; the parameters of H'_l are computed from a set of linear equations (all the primitive wrenches not belonging to the actual $\tilde{\boldsymbol{\omega}}_j$ must lie on H'_l) and one non linear equation (distance of H'_l to the origin equal to Q^k). The set of equations admit 2 solutions; the hyperplane required is that one leaving O and $\tilde{\boldsymbol{\omega}}_j$ in different half-spaces. Step 5 looks for the points that have at least one primitive wrench lying in $\mathcal{S}$; one of these points will be a new grasp point. The procedure is followed until finding a local maximum, which implies that there are no more points that improve the actual quality Q^k.

Note that Steps 3 to 6 do not involve an explicit FC test; the procedure is based on pure geometric reasoning that avoids such test, thus reducing the computational complexity when compared to previous works [16]. The total number of iterations required to reach the local maximum depends directly on the number of local maximums in the wrench space, i.e. it is directly related with the object to be grasped.

B. Computation of the independent contact regions

The computation of the independent contact regions (ICRs) ensuring a minimum grasp quality Q is based on an arbitrary starting grasp fulfilling the FC property. In this work a locally optimum grasp, obtained with the procedure described above, is used as the starting grasp.

For a given FC grasp, the grasp quality Q is fixed by the facet F_Q of the convex hull closest to the origin. Let F_k denote a facet of $CH(W)$ which contains at least one primitive wrench for a particular grasp point $\boldsymbol{p}_i$. The

proposed approach builds several hyperplanes H_k'' parallel to each facet F_k, but lying at a distance $D = Q$ of the origin of the wrench space (i.e. they are tangent to the hypersphere of radius $r = Q$), as illustrated in Fig. 3 for a hypothetical 2D wrench space. These hyperplanes define S_i, the search zone containing the ICR for the grasp point $\boldsymbol{p}_i$; S_i is the intersection of the half-spaces $H_k''^+$ which do not contain the origin O. The ICR is determined by the set of neighbor points of $\boldsymbol{p}_i$ such that at least one of its primitive wrenches falls into the corresponding search zone S_i. The steps in the algorithm are:

Algorithm 3: Search of the independent contact regions
1) Find a locally optimum FC grasp, $G = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_n\}$
2) Fix the minimum acceptable quality Q
3) Build the hyperplanes H_k'' such that $D_{H_k''} = Q$
4) Let $S_i = \bigcap H_k''^+$ with $H_k''^+$ the half-space such that $O \notin H_k''^+$ (i.e. $\boldsymbol{\omega}_i \vee \boldsymbol{\omega}_{i1} \vee \ldots \vee \boldsymbol{\omega}_{im} \in S_i$)
5) Initialize $I_i = \{\boldsymbol{\omega}_i\}$. Label the points in each I_i as open
6) Check the neighbor points $\boldsymbol{\omega}_p$ of every open point $\boldsymbol{\omega}_j \in I_i$
  If $\boldsymbol{\omega}_p \vee \boldsymbol{\omega}_{p1} \vee \ldots \vee \boldsymbol{\omega}_{pm} \in S_i$
   $I_i = I_i \cup \{\boldsymbol{\omega}_p\}$; label $\boldsymbol{\omega}_p$ as open
  Endif
  Label $\boldsymbol{\omega}_j$ as closed
7) If there are open points in I_i, go to Step 6. Otherwise, the algorithm returns the set of points I_i, i.e. the ICR for the contact point $\boldsymbol{p}_i$. Steps 3 to 7 are repeated for the rest of the contact points, $i = 1, \ldots, n$.

Note that algorithm 3 is computationally very simple. In Step 3, the hyperplanes H_k'' are computed for the corresponding facets F_k of $CH(W)$. Let H_k be the hyperplane containing the facet F_k, described as

$$\boldsymbol{e}_k \cdot \boldsymbol{x} = e_{0k} \qquad (4)$$

The hyperplane H_k'' parallel to H_k but lying to a distance $D = Q$ from the origin is

$$\boldsymbol{e}_k \cdot \boldsymbol{x} = e_{0k}', \quad \text{with} \quad e_{0k}' = Q\,\|\boldsymbol{e}_k\| \qquad (5)$$

Therefore, only the computation of the scalar value e_{0k}' is required to build each hyperplane H_k''. Step 4 only identifies for every hyperplane the closed half-space $H_k''^+$ that does not contain the origin, and forms the search zones S_i; note that the selection of any arbitrary point from each S_i always generates a FC grasp. Step 6 is the more complex step in the algorithm; every checked point involves its classification with respect to the number of hyperplanes H_k that contain at least one primitive wrench for the contact point $\boldsymbol{p}_i$.

The procedure can also be applied to generate ICRs with contact points that produce a lower grasp quality $Q_r = \alpha Q$, with $0 < \alpha < 1$ and Q the quality of the starting grasp. This is achieved considering a hypershpere of radius Q_r instead of Q in the procedure described above. When $\alpha \to 0$, the ICRs contain FC grasps without a lower limit on the grasp quality. In fact $\alpha = 0$ is a forbidden value, as it does not assure that any $CH(W)$ will strictly contain the origin O.
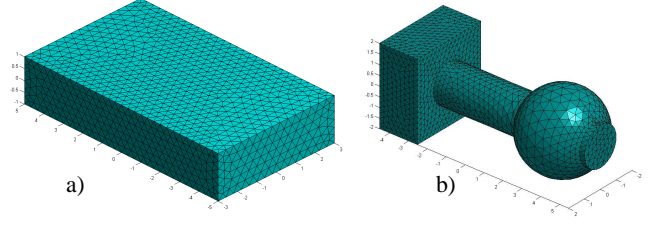

Fig. 4. Objects used to illustrate the approach: a) Parallelepiped discretized with a mesh of 3422 triangles, b) Workpiece discretized with a mesh of 3946 triangles.

The number of points in every ICR may be different for each $\boldsymbol{p}_i$, depending on factors such as the level of detail in the representation of the object surface and the smoothness of the surface, i.e. the rate of change in the normal vectors around the contact location. Finally, considering the ICRs for each finger, several grasps can be formed when each finger is placed in a different position inside its ICR; the geometrical procedure assures that all these grasps satisfy $O \in CH(W)$ and have a quality $Q > Q_r$. However, the obtained ICRs depend on the starting grasp; the search of the optimal ICRs is not addressed in this paper, but it is an interesting issue to explore in the future.

IV. APPLICATIONS

The algorithms presented above have been implemented in Matlab on a Pentium IV 3.2 GHz computer, and the performance is illustrated using the two objects shown in Fig. 4, whose boundary is described by a triangular mesh. The contact points $\boldsymbol{p}_i$ are the centroids of the triangles in the mesh, and the corresponding surface normal directions are the directions normal to the triangles. Two points are considered neighbors if its corresponding triangles share an edge.

The first object is a parallelepiped described with a mesh of 3422 triangles; the frictional grasps are computed considering 4 fingers and a friction coefficient of $\mu = 0.2$, and the friction cones have been linearized with an 8-side polyhedral convex cone. Fig. 5 shows an instance of the results obtained with the proposed approach. Algorithm 1 provides the first FC grasp (Fig. 5a) in 2.2 seconds and 0 iterations, plus other 5 possible FC grasps (corresponding to the points whose primitive wrenches fall in the set $\mathcal{C}_1$). Algorithm 2 optimizes this grasp to get the locally optimum FC grasp (Fig. 5b) in 110 seconds and 25 iterations. Fig. 6 plots the evolution of the grasp quality in the optimization phase; the quality always increases monotonically until it finds the locally optimum grasp. The local optimum depends on the initial grasp; in this example, the initial grasp quality is 0.015, and the locally optimum grasp quality is 0.185; the improvement factor, i.e. the ratio between the quality of the optimized grasp and the initial FC grasp is 12.3.

Algorithm 3 provides the corresponding independent contact regions (Fig. 5c) in 70 seconds, using as minimum quality $Q_r = 0.139$ ($\alpha = 0.75$). The points within the ICRs may be combined to obtain 45000 different grasps; Fig. 7
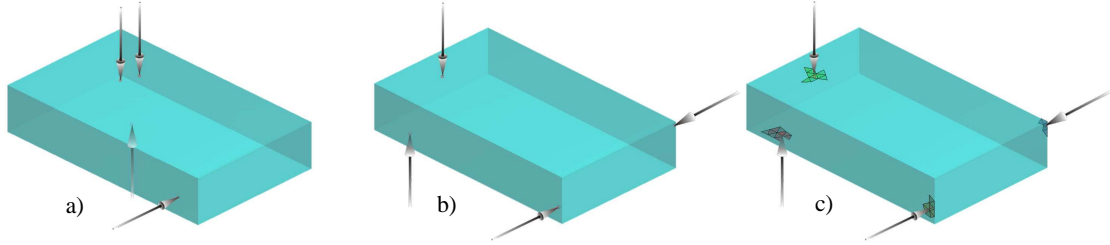
Fig. 5. Computation of ICRs on a parallelepiped: a) Initial FC grasp, $Q = 0.015$ (Algorithm 1), b) Locally optimum FC grasp, $Q = 0.185$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.139$ (Algorithm 3).
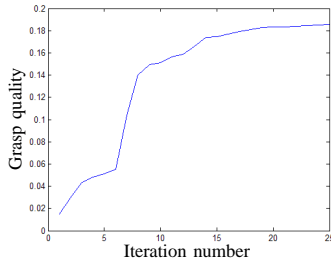

Fig. 6. Performance in the optimization phase (Algorithm 2) for the parallelepiped: increase in the grasp quality.
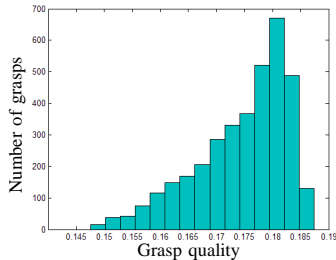

Fig. 7. Grasp quality distribution for all the possible grasps within the ICRs on the parallelepiped for $Q_r = 0.139$ ($\alpha = 0.75$).
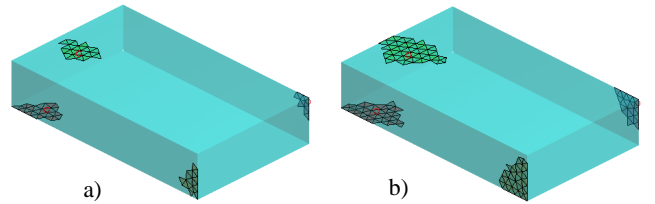

Fig. 8. Independent contact regions on the parallelepiped with different minimum quality: a) $Q_r = 0.093$ ($\alpha = 0.5$), b) $Q_r \approx 0$ ($\alpha = 10^{-5}$).
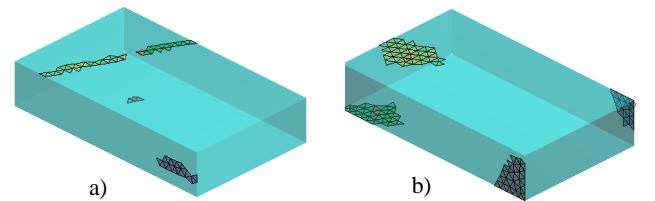

Fig. 9. Independent contact regions on the parallelepiped with the same minimum quality, $Q_r = 0.005$, but computed from: a) the initial grasp, b) the locally optimum grasp.

shows the quality distribution for all these possible grasps.

Obviously, for lower minimum grasp qualities the size of each ICR grows; Fig. 8 shows the ICRs for other two different minimum grasp qualities given by $\alpha = 0.5$ and $\alpha = 10^{-5} \approx 0$ (in the last case without a limit in the lower grasp quality). Finally, Fig. 9 shows a comparison between the ICRs generated for the same minimum quality, but computed from the initial and the locally optimum FC grasp; despite that the ICRs assure the same minimum quality for every possible grasp inside them, the size of the ICRs is larger for the second case, thus justifying the use of the optimization in the proposed approach.

The second object is a workpiece proposed in [5], discretized with 3946 triangles (Fig. 4b); the approach is initially applied for $n = 4$ fingers, $\mu = 0.2$, and frictional cones linearized with $m = 8$ sides. Fig. 10 shows the results for an ICR search on the workpiece. Algorithm 1 provides the first FC grasp with no iterations in 2.3 seconds, and it is optimized with Algorithm 2 after 29 iterations in 114 seconds. The grasp qualities are 0.035 and 0.174 for the initial and locally optimum FC grasps, respectively,

with an improvement factor of 5. Algorithm 3 provides the corresponding ICRs, with $Q_r = 0.131$ ($\alpha = 0.75$), in 66 seconds. The points within the ICRs allow 320 different grasps; Fig. 11 shows the quality distribution for all these possible grasps. Fig. 12 shows the ICRs for two additional quality ratios: $\alpha = 0.5$ and $\alpha = 10^{-5}$.

Fig. 13 shows the obtained ICRs for the same optimum FC grasp in Fig. 10b, but using a different version of Algorithm 3 which considers all the wrenches inside the search zone $\mathcal{S}_i$ to be part of the ICR for each finger, i.e. it does not consider neighbors in the search of the ICRs, and the ICRs may be composed of non contiguous contact points. In the example of Fig. 13 one finger has an ICR composed by two disjoint zones (compare with Fig. 12b), because the wrenches of the points are neighbors in the wrench space, although physically the points are not neighbors at all on the object boundary. Finally, Fig. 14 shows another instance of ICR computation, now for a 5-finger grasp and $\mu = 0.1$.

V. SUMMARY

The computation of independent contact regions for frictional contacts has been tackled with an approach that includes two parts, the search of a starting grasp (obtained as the optimization of an initial FC grasp) and the computation
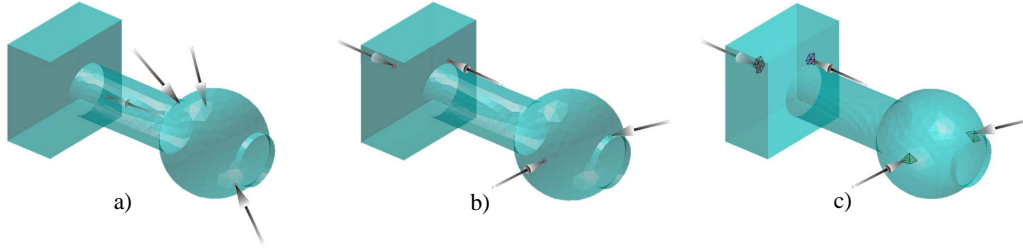
Fig. 10. Example on a workpiece: a) Initial FC grasp, $Q = 0.035$ (Algorithm 1), b) Locally optimum FC grasp, $Q = 0.174$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.131$ (Algorithm 3).
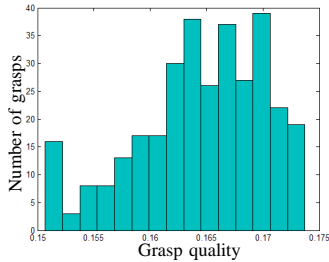

Fig. 11. Grasp quality distribution for all the possible grasps within the ICRs on the workpiece for $Q_r = 0.131$ ($\alpha = 0.75$).

Fig. 13. Independent contact regions on the workpiece with $Q_r \approx 0$, and dropping the condition of neighbor wrenches in Algorithm 3.
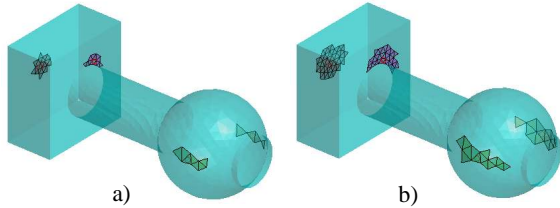

Fig. 12. Independent contact regions on the workpiece with different minimum quality: a) $Q_r = 0.087$ ($\alpha = 0.5$), b) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

Fig. 14. Independent contact regions on the workpiece for a five-finger grasp with $\mu = 0.1$ and $Q_r = 0.1$.

of the ICRs for the locally optimum grasp. The algorithms were implemented and the execution results, as the examples shown in the paper, illustrate the relevance and efficiency of the approach.

The presented approach can be applied to search ICRs starting from any provided FC grasp, and the proposed algorithm ensures a controlled minimum quality for any number of fingers ($n \geq 3$). Future works include the determination of ICRs for frictional contacts when k contact locations are fixed beforehand, and the application of such algorithm in manipulation tasks (these issues are currently under development), and the consideration of fingers with a finite contact area.

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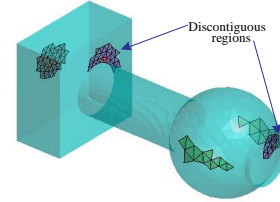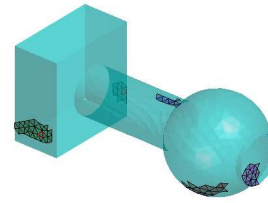[16] M. Roa and R. Suarez, "Geometrical approach for grasp synthesis on discretized 3D objects," in *Proc. IEEE/RSJ IROS 2007*, 2007, pp. 3283–3288.