Contact Force Computation for Bimanual Grasps

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***Abstract*—This paper presents a method to compute contact forces for bimanual grasps. The method is based on the optimization of the force distribution of the hands and minimizing the force exerted by each finger, using two different cost functions. Both cost functions and the constrains of the optimization problem are formulated as functions of the joint torques based on the existing relation between the grasping forces, the hand-jacobian matrix and the torque of the joint fingers. Additionally, a bimanual grasp index is presented to measure the force distribution between the hands. The paper includes some application examples of the proposed approach.**

I. INTRODUCTION

The human grasping behaviour has been analyzed along many years, providing a better understanding of how humans manipulate objects with the hands. As result of years of research, a big amount of knowledge has been obtained and used in different domains, such as in medical rehabilitation with the development of anthropomorphic prosthesis, as well as in the robotic field with the design of mechanical hands and the formulation and implementation of theoretical strategies to compute suitable hand configurations to grasp different types of objects.

The use of complex mechanical hands to grasp and manipulate objects involves two main phases [1], the first one is the well-known grasp synthesis or grasp planning phase, which consists in finding the location of grasping points in the object surface that can be reached by the hand, accomplishing at the same time some basic constraints; the second phase can be called the grasp holding phase, it consists in keeping the grasp while the object is manipulated by computing the force that each finger of the hand must apply on the object contact points in order to resist external disturbances.

The grasp holding phase has been widely studied and some of the results are discussed bellow in the section II. However, the development of dual-arm systems equipped with anthropomorphic hands adds an extra complexity to finding bimanual grasps and to the computation of the contact forces, and a new variety of challenges arises, such as equilibrate the forces that each hand should exert while they lift the object or to balance an external perturbation acting on it.

In this work we present a method to compute contact forces for dual-arm robotic systems considering the torque limitation of the hand joints and an index that describes the degree of balance between the forces that each hand contributes.

The rest of the paper is structured as follows. A review of related works is present in Section II. Section III presents the problem statement. The proposed approach is introduced in Section IV. Experiments and results are shown in Section V and finally, Section VI summarizes the work and presents some topics deserving future work.

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II. RELATED WORKS

The determination of grasping forces to compensate external disturbances has been an active field of research and many approaches have been proposed to find them. One of them was the decomposition of the forces into two components: the manipulating and grasping forces, together with the introduction of the *Grasp Matrix* to find a solution [2], [3]. These two components are better known as the particular and homogeneous solutions of the static grasping force problem, which basically can be solved by determining the homogeneous component, also defined as the *internal forces* of the grasp. Some researchers have characterized these internal forces as a virtual linkage with virtual actuators representing the grasped object, such that when a force is applied at each grasping point it generates a joint force in the virtual mechanism [4], [5]. Many of the works that have adopted the internal forces' approach to determine the grasping forces have considered only fingertip grasps with full control of the joint torques of the fingers in order to simplify the grasp force distribution among the contacts [6], [7], [8]. However, has been shown that when enveloping or power grasp is used, there are some contacts that exerts force against the object that are not controllable [9], [10]. This fact has been analyzed by decomposing the space of the contact forces into four subspaces that represent the active and passive forces and the controllable and uncontrollable internal forces, which allow characterize grasps based on the size of such subspaces [11], [12], [13].

The computation of the contact forces using the internal forces' approach involve to solve a problem with a large number of variables and with an infinite number of possible solutions, arising the *Grasp Force Optimization* problem (noted GFO hereafter) whose goal is to find the optimal solution or at least a sub-optimal one. The GFO often has to deal with the nonlinearity of the contact friction models which can be expressed as the positive definiteness of a symmetric matrix [1], as linear matrix inequalities [14], [15] or modelling the whole problem so that it accomplish the requirements to be formulated using the dual method of linear programming [16], then, the GFO problem can be solved by choosing a suitable cost function. Furthermore, one of the main challenges of the GFO is to find optimal or sub-optimal solutions as faster as possible in order to be used in real time applications. Researchers have concluded that finding good initial conditions for the optimization problems in combination with gradient methods and convex optimizations, it is possible obtain a faster convergency with an optimal or sub-optimal solutions [14], [17], [18], [19]. Another method to fastly compute the grasping forces consist in transform the optimization problem into a non-constrained quadratic problem that can be solved analytically, reducing the time computation significantly [20]. The extension of the GFO approaches to bimanual systems is an interesting area of research that has not been deeply explored [21], [22], even when the use of binamual systems for cooperative task is increasing. For this reason, we propose here a new method to compute the contact forces for bimanual grasps. The proposed approach uses a linear optimization method based on the torque limitation of the joints of the hand fingers. Unlike the approaches cited above the proposed optimization method does not need to compute the internal forces on the grasped object to compute the contact forces. Moreover, two different cost functions for the optimization of the contact forces are proposed in order to distribute them uniformly between the hands and to minimize the finger forces, respectively. The use of any of them depends on the pursued objective. Furthermore, an index to measure the force distribution between the hands is also presented in this work.

III. PROBLEM STATEMENT

In a previous work we presented a grasp planner for bulky objects using a dual-arm robotic system [23]. The scope of that work included the search of the reachable contact points for two robotic hands using an object slicing method (See Fig. 1). In the real experimentation, the target contact points have been moved a certain distance towards the interior of the object following the normal direction to the object surface in order to grasp the object with large enough forces using a control position of the fingertips. With this approach it is possible that the force contributions of the hands are not balanced. The problem to be solved in this work is the determination of the suitable contact forces for the bimanual grasps in order to compensate external perturbations during the manipulation of the object with balanced force contributions of the hands.

The general assumptions taken into account in the proposed approach are the following:

- Only precision grasps with frictional hard contacts between the fingers and the object are considered (Nevertheless, the approach can be easily reformulated to use soft contacts).
- The frictional behaviour at the contact points is described by the Coulomb's friction model.
- Each hand uses the same number of fingers.
- All the fingers can exert the same maximum torque.
- Only the weight of the object is considered as the external perturbation to be counteracted.
- The weight of the object is known.

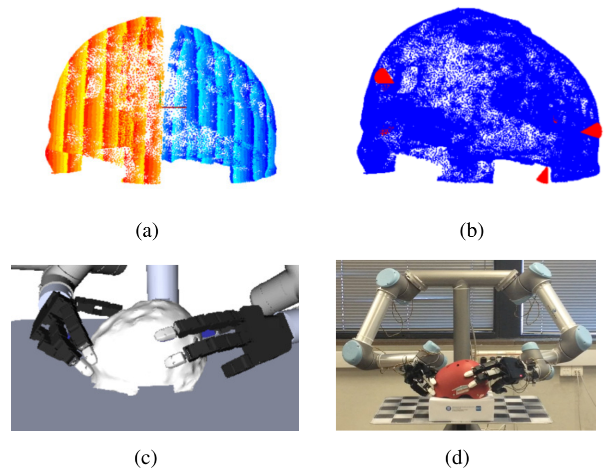


Fig. 1. Bimanual grasp planner based on an object slicing method. a) The object to be grasped segmented in slices. b) Location of the grasping points after searching through the object slices. c) Simulation of the bimanual grasp. d) Execution of the bimanual grasps in a real system.

IV. PROPOSED APPROACH

Given the weight of the object, it must be found the forces that the fingers must apply on the object to counterbalance it. The general equation to describe this force balance can be expressed as:

$$\boldsymbol{G}\boldsymbol{F} = -\boldsymbol{w_e} \tag{1}$$

where $\boldsymbol{F} = \{\boldsymbol{f}_1, \ldots, \boldsymbol{f}_n\}^T$ are the forces applied by the n fingers contacting on the object, $\boldsymbol{G}$ is the grasp matrix and $\boldsymbol{w_e}$ is the external wrench to be compensated produced by the object weight.

To find $\boldsymbol{F}$, it is proposed a contact force optimization strategy with two different cost functions used to minimize either the infinite norm (L_∞) of the quotient between the finger joint torques and the maximum torque of the joints, and the Least Square Norm (L_2) of the finger joint torques, in order to balance the forces between the hands and minimize the force exerted by the fingers respectively. Furthermore, an index to measure the force distribution between the hands are also presented.

A. Formulation of the Problem Constraints

When a grasp is redundant (i.e. when the number of grasping points is larger than the minimum needed to grasp an object), like in the case of bimanual grasps with more than two fingers per hand, there are infinite force combinations that could compensate an external perturbation, but not all these combinations can be exerted by the fingers.

In order to determine which forces can be exerted by each finger, consider a hypersphere in the joint torque space, expressed by

$$\boldsymbol{\tau}_i^T \boldsymbol{\tau}_i = 1 \tag{2}$$

where $\boldsymbol{\tau}_i \in \Re^m$ is the vector of the joint torque of the i-th finger with m joints. The relationship

$$\boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{f}_i \tag{3}$$

where $\boldsymbol{J}_i$ and $\boldsymbol{f}_i$ are the Jacobian and the force applied for the i-th finger, maps the torque unit sphere into an ellipsoid in the space of the fingertip forces,

$$\boldsymbol{\mathcal{F}}_i^T \boldsymbol{J}_i \boldsymbol{J}_i^T \boldsymbol{\mathcal{F}}_i = 1 \tag{4}$$

The ellipsoid represented in (4) indicates the forces $\boldsymbol{\mathcal{F}}_i = \{\boldsymbol{f}_{i,j}, \ldots, \boldsymbol{f}_{i,\infty}\}$ that each finger can exert given a set of joint torques $|\boldsymbol{\tau}_i| \leq 1$ for a given joint configuration (see Fig.2-a). Therefore, if the maximum torque of the finger joints is known, it is possible to compute the finger force $\boldsymbol{f}_i$ for a given joint configuration as:

$$\boldsymbol{f}_i = (\boldsymbol{J}_i^{\dagger})^T \boldsymbol{\tau}_i \tag{5}$$

where $\boldsymbol{J}_i^{\dagger}$ is the pseudo inverse of $\boldsymbol{J}_i$. Since $\boldsymbol{F}$ in eq. (1) represents the forces exerted by n fingers, eq. (5) can be extended as:

$$\boldsymbol{F} = (\boldsymbol{\mathcal{J}}^{\dagger})^T \boldsymbol{\mathcal{T}} \tag{6}$$

where $\boldsymbol{\mathcal{J}}^{\dagger} = diag(\boldsymbol{J}_1^{\dagger}, \ldots, \boldsymbol{J}_n^{\dagger})$ is the block diagonal matrix of the pseudo inverse jacobians of n fingers and $\boldsymbol{\mathcal{T}} = \{\boldsymbol{\tau}_{1,1}, \ldots, \boldsymbol{\tau}_{n,m}\}^T$ is the vector containing the joint torque of the n fingers.

If eq. (6) is replaced into eq. (1) it is possible to counter balance $\boldsymbol{w_e}$ considering the joint torque space:

$$\boldsymbol{G}(\boldsymbol{\mathcal{J}}^{\dagger})^T \boldsymbol{\mathcal{T}} = -\boldsymbol{w_e} \tag{7}$$

Additionally, the forces that the fingertips exert on the object must lie inside of the friction cone (see Fig. 2-b) defined by

$$\frac{\boldsymbol{f}_i}{||\boldsymbol{f}_i||} \cdot \hat{n}_i \geq \frac{1}{\sqrt{1+\mu^2}} \tag{8}$$

where $\hat{n}_i$ is the unitary normal at the i-th contact point and μ is the friction coefficient between the fingers and the grasped object. For convenience eq. (8) is manipulated to be ≤ 0 yielding:

$$\boldsymbol{f}_i^T (I - (1+\mu^2)\hat{n}_i \cdot \hat{n}_i^T) \boldsymbol{f}_i \leq 0 \tag{9}$$

If ineq. (9) is extended for n fingers, it results:

$$\boldsymbol{F}^T (I - (1+\mu^2)\boldsymbol{\mathcal{N}}) \boldsymbol{F} \leq 0 \tag{10}$$

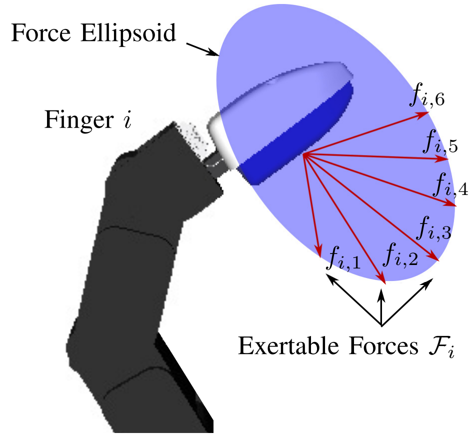
where $\boldsymbol{\mathcal{N}} = diag(\hat{n}_1 \cdot \hat{n}_1^T, \ldots, \hat{n}_n \cdot \hat{n}_n^T)$ is the block diagonal matrix of $\hat{n}_i \cdot \hat{n}_i^T$ of the n fingers.

Since the finger forces $\boldsymbol{F}$ can be expressed by eq. (6), eq. (10) can be reformulated as:

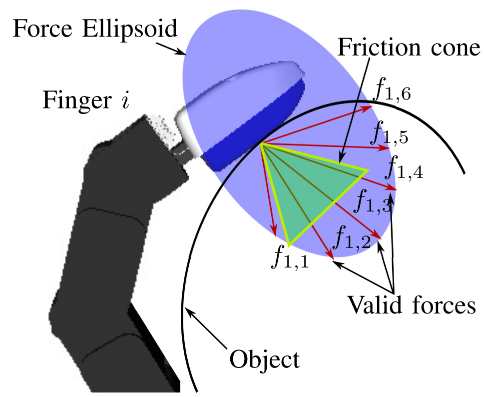
$$\boldsymbol{\mathcal{T}}^T \boldsymbol{B} \boldsymbol{\mathcal{T}} \leq 0 \tag{11}$$

where $\boldsymbol{B} = \boldsymbol{\mathcal{J}}^{\dagger}(I - (1+\mu^2)\boldsymbol{\mathcal{N}})(\boldsymbol{\mathcal{J}}^{\dagger})^T$.

If an optimization problem is formulated to balance the force contribution of each hand or to minimize the force exerted by the fingers as functions of the joint torques, eq. (7) and eq. (11) can be used as the constraints of the problem and the maximum torque τ_{max} that the finger joints can perform



(a)



(b)

Fig. 2. a) Illustrative example of the exertable forces belonging to the Force Ellipsoid of a finger for a given joint configuration and $|\boldsymbol{\tau}_i| \leq 1$. b) Valid exertable forces given a friction coefficient.

can be used to set a bound constraint, so that the optimization problem can be expressed as:

$$\begin{aligned} \underset{\boldsymbol{\mathcal{T}}}{\text{minimize}} \quad & \Phi \\ \text{subject to} \quad & -\tau_{max} \leq \boldsymbol{\mathcal{T}} \leq \tau_{max} \\ & \boldsymbol{\mathcal{T}}^T \boldsymbol{B} \boldsymbol{\mathcal{T}} \leq 0 \\ & \left|\left| \boldsymbol{G}(\boldsymbol{\mathcal{J}}^{\dagger})^T \boldsymbol{\mathcal{T}} + \boldsymbol{w_e} \right|\right|^2 = 0 \end{aligned} \tag{12}$$

where Φ is a generic representation of a cost function.

B. Cost Functions

Two different cost functions have been selected to perform the optimization of the contact forces. The first one is based on the minimization of the infinite norm (L_∞) of the torque vector $\boldsymbol{\mathcal{T}}$ and divided by the maximum torque of the joints τ_{max}, i.e.:

$$\Phi_0 = \left|\left| \frac{\boldsymbol{\mathcal{T}}}{\tau_{max}} \right|\right|_{L_\infty} \tag{13}$$

This cost function allows a uniform distribution of the forces between the fingers, which means that for a bimanual grasp the force contribution of each hand would be well balanced if each hand uses the same number of fingers.

The second cost function is based on the minimization of the Least Square norm (L_2) of the torque vector $\boldsymbol{\mathcal{T}}$,

$$\Phi_1 = ||\boldsymbol{\mathcal{T}}||_{L_2} \tag{14}$$

This cost function allows to minimize the torque performed by each finger joint. However, unlike the first cost function, this one does not ensure a uniforme distribution of the forces between the hands.

To solve the optimization problem as fast as possible, a gradient-based algorithm [24] was used, which is implemented in the NLopt nonlinear-optimization package [25]. Therefore, it is necessary to formulate the gradient of the cost functions as well as the gradient of the constraints by computing their partial derivatives as a function of $\boldsymbol{\mathcal{T}}$.

The gradient of the cost function Φ_0 is:

$$\nabla\Phi_o = \begin{cases} \dfrac{\boldsymbol{\tau}_i}{\boldsymbol{\tau}_{max}}, & \text{if } \boldsymbol{\tau}_i = \boldsymbol{\tau}_{max_i}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

the gradient of the cost function Φ_1 is:

$$\nabla\Phi_1 = 2\boldsymbol{\mathcal{T}} \tag{16}$$

and the gradient of the constraints in eq. (12) are:

$$\begin{aligned} \nabla c_1 &= 2\boldsymbol{\mathcal{T}}^T\boldsymbol{B} \\ \nabla c_2 &= 2(\boldsymbol{G}(\boldsymbol{\mathcal{J}}^\dagger)^T\boldsymbol{\mathcal{T}} + \boldsymbol{w_e})^T\boldsymbol{G}(\boldsymbol{\mathcal{J}}^\dagger)^T \end{aligned} \tag{17}$$

C. Bimanual Force Index

Based on the cost functions formulated for the optimization problem, it is proposed the *Force Distribution Index* (FDI) to measure the force distribution between the hands:

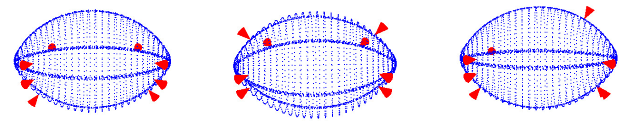
$$FDI = \frac{H_1^f}{H_2^f} \tag{18}$$

where $H_j^f = \sum_{i=0}^{n/2} ||\boldsymbol{f}_i||$ is the sum of the norms of the forces exerted by the fingers of the hand j, and $j \in \{1, 2\}$ with $j = 1$ corresponding to the hand with the smallest summatory, i.e $H_1^f < H_2^f$. FDI indicates how well distributed is the force contribution between the hands, it tends to 1 when the forces are well balanced and to zero otherwise.

V. EXPERIMENTS

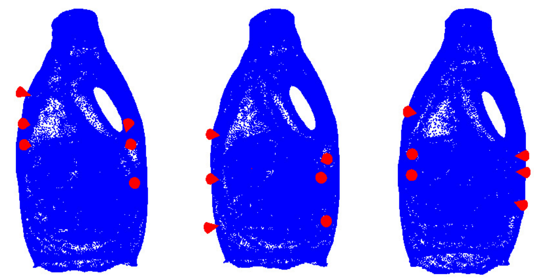
To demonstrate the performance of the proposed approach, we use two objects, a rugby ball and a detergent bottle. For each object three different bimanual grasps were computed (Fig. 3 and 4), then, for each grasp, the contact forces were computed 10 times optimizing the cost functions Φ_1 and Φ_2. At each time the object weight was incremented in 100 g starting from their original weights. The considered friction coefficient was 0.4.

Table I and table II show the resultant grasping forces to counteract the weight $\boldsymbol{w_e}^{Rug} = [0, 0, -2.13858, 0, 0, 0]^T$ and $\boldsymbol{w_e}^{Det} = [0, 0, -2.923, 0, 0, 0]^T$ of the rugby ball and the detergent bottle respectively for the grasping points shown in Fig. 3-a and 4-a. The difference between the external forces



(a) (b) (c)

Fig. 3. Three different sets of grasping points for a rugby ball. a) Symmetrical grasp. b) Semisymetrical grasp. c) Irregular grasp.



(a) (b) (c)

Fig. 4. Three different sets of grasping points for a detergent bottle. a) - c) Irregular grasps.

and the sum of the computed grasping forces $(\Sigma\boldsymbol{f}_i)_{Rug}$ and $(\Sigma\boldsymbol{f}_i)_{Det}$ is small enough and therefore negligible.

The average time to compute the contact forces is 150 and 1500 ms, respectively for each object.

Fig. 5 depicts the force contribution of each hand as well as the Force Distribution Index (FDI) optimizing Φ_0 and Φ_1 for the three grasps of both the rugby ball and the detergent bottle. Fig. 5 a) - c), show the resultant force contribution of each hand for both cost functions for the three grasps of the rugby ball. Each picture shows four curves: two continuous ones colored in red and blue depicting the force contribution of the left and right hand respectively obtained by optimizing the cost function Φ_0 and two dotted ones colored in magenta and cyan depicting the force contribution of the left and right hand respectively by optimizing the cost function Φ_1. As can be seen, the continuous curves have a similar pattern and magnitude at each iteration, indicating an uniform distribution of the force contribution between the hands. On the other hand,

TABLE I

RESULTANT CONTACT FORCES TO COMPENSATE THE EXTERNAL PERTURBATION $\boldsymbol{w_e} = [0, 0, -2.13858, 0, 0, 0]^T$ ACTING ON THE RUGBY BALL

$\boldsymbol{f}^T$	x	y	z	m_x	m_y	m_z
$\boldsymbol{f}_1^T$	1.682	-1.824	-0.903	-0.009	-0.040	0.063
$\boldsymbol{f}_2^T$	1.326	0.204	0.061	0.005	-0.013	-0.060
$\boldsymbol{f}_3^T$	-0.444	5.552	0.594	0.347	0.057	-0.270
$\boldsymbol{f}_4^T$	-0.504	-3.376	1.043	-0.223	0.068	0.112
$\boldsymbol{f}_5^T$	0.979	-1.378	1.945	0.132	-0.055	-0.105
$\boldsymbol{f}_6^T$	-1.435	1.792	-0.182	-0.058	-0.030	0.158
$\boldsymbol{f}_7^T$	-2.023	2.605	-0.950	-0.035	0.060	0.239
$\boldsymbol{f}_8^T$	0.419	-3.575	0.531	-0.164	-0.040	-0.143
$(\Sigma\boldsymbol{f})_{Rug}^T$	0.000	0.000	2.139	-0.004	0.006	-0.006
$\boldsymbol{w_e}^{Rug}$	0.000	0.000	-2.138	0.000	0.000	0.000

TABLE II
RESULTANT CONTACT FORCES TO COMPENSATE THE EXTERNAL PERTURBATION $\boldsymbol{w_e} = [0, 0, -2.923, 0, 0, 0]^T$ ACTING ON THE DETERGENT BOTTLE

$\boldsymbol{f}^T$	x	y	z	m_x	m_y	m_z
$\boldsymbol{f}_1^T$	1.838	-3.307	-0.798	0.000	-0.054	0.224
$\boldsymbol{f}_2^T$	0.355	2.364	0.267	0.047	0.019	-0.234
$\boldsymbol{f}_3^T$	0.471	0.108	1.220	0.008	0.086	-0.010
$\boldsymbol{f}_4^T$	-1.320	0.304	0.622	-0.038	0.043	-0.102
$\boldsymbol{f}_5^T$	0.531	2.321	-0.357	-0.047	0.036	0.168
$\boldsymbol{f}_6^T$	-1.282	0.130	0.572	0.030	-0.032	0.076
$\boldsymbol{f}_7^T$	-0.448	-0.183	1.238	0.012	-0.082	-0.007
$\boldsymbol{f}_8^T$	-0.144	-1.737	0.158	-0.010	-0.010	-0.120
$(\Sigma \boldsymbol{f})_{Det}^T$	0.001	-0.001	2.923	0.003	0.007	-0.005
$\boldsymbol{w_e}^{Det}$	0.000	0.000	-2.923	0.000	0.000	0.000

the dotted curves show a significant disparity between them, but the magnitudes at each iteration are smaller in comparison with the continuous lines.

Figure 5 d) - f) show the Force Distribution Index (FDI) for both cost functions, being the red line the results obtained by Φ_0 and the blue line the results obtained by Φ_1. The red line is always over 0.8 indicating that the force contribution is well distributed between the hands, which is coherent with the results in graphics a) - c). However, the blue line is below of 0.5 in almost all cases, indicating a poor balance of the force contribution of the hands.

Figure 5 g) - i) , show the resultant force contribution of each hand for both cost functions for the three grasps of the Detergent bottle. As in grasps a) - c) the two continuous lines colored in red and blue depict the force contribution of the left and right hand respectively obtained by optimizing the cost function Φ_0 and the two dotted lines colored in magenta and cyan depict the force contribution of the left and right hand respectively by optimizing the cost function Φ_1. Since the hand configurations depends of the location of the contact points, the forces that each hand contribute can vary considerably. On the other hand, the dotted lines show a force magnitude smaller in comparison with the continuous lines but less evident than in the case of the rugby ball grasps.

Figure 5 j) - l) show the Force Distribution Index (FDI) for both cost functions, being the red line the results obtained by Φ_0 and the blue line the results obtained by Φ_1. The results for both cost functions are always over 0.65, indicating that they produce an acceptable force distribution between the hands

VI. CONCLUSIONS AND FUTURE WORKS

This work has presented a method to optimize the contact forces for bimanual grasps as a function of the joint torques. The method is based on the optimization of the force distribution between the hands and the minimization of the forces exerted by the fingers. The optimization was performed using two different cost functions. The first one minimizes the infinite norm of the quotient between the torque vector of the fingers and the maximum torque of the joints in order to obtain a balanced force distribution between the hands in a bimanual grasp. The second one minimizes the torque performed by each joint, hence, minimizing the force exerted by each finger. The user can choose any of the two cost functions depending on the desired goal. The experiments shown good results using any of the two cost functions, with an average computation time of 150 and 1500 ms for each object.

Furthermore, a bimanual force index FDI, to measure how well balanced are the force contribution between the hands has been presented. FDI has shown that the cost function Φ_0 effectively allows a good distribution of the contact forces between the hand. On the other hand, using the cost function Φ_1, FDI shows a poor distribution of the contact forces but the force exerted by each finger is smaller than those resulting with Φ_0.

One thing that is worths to remark is that when the grasping points of each hand are symmetric, the FDI could easily reach high values. However, when the bimanual grasp is not symmetric the contribution could not be totally balanced, but exists a uniform behaviour in the force contribution of each hand, namely, the force contribution of each hand increases or decreases proportionally according to the external force acting on the object.

A future work is the implementation of the proposed approach together with a hand controller in order to perform real experimentation and the generalization to cases where each hand uses different number of fingers.

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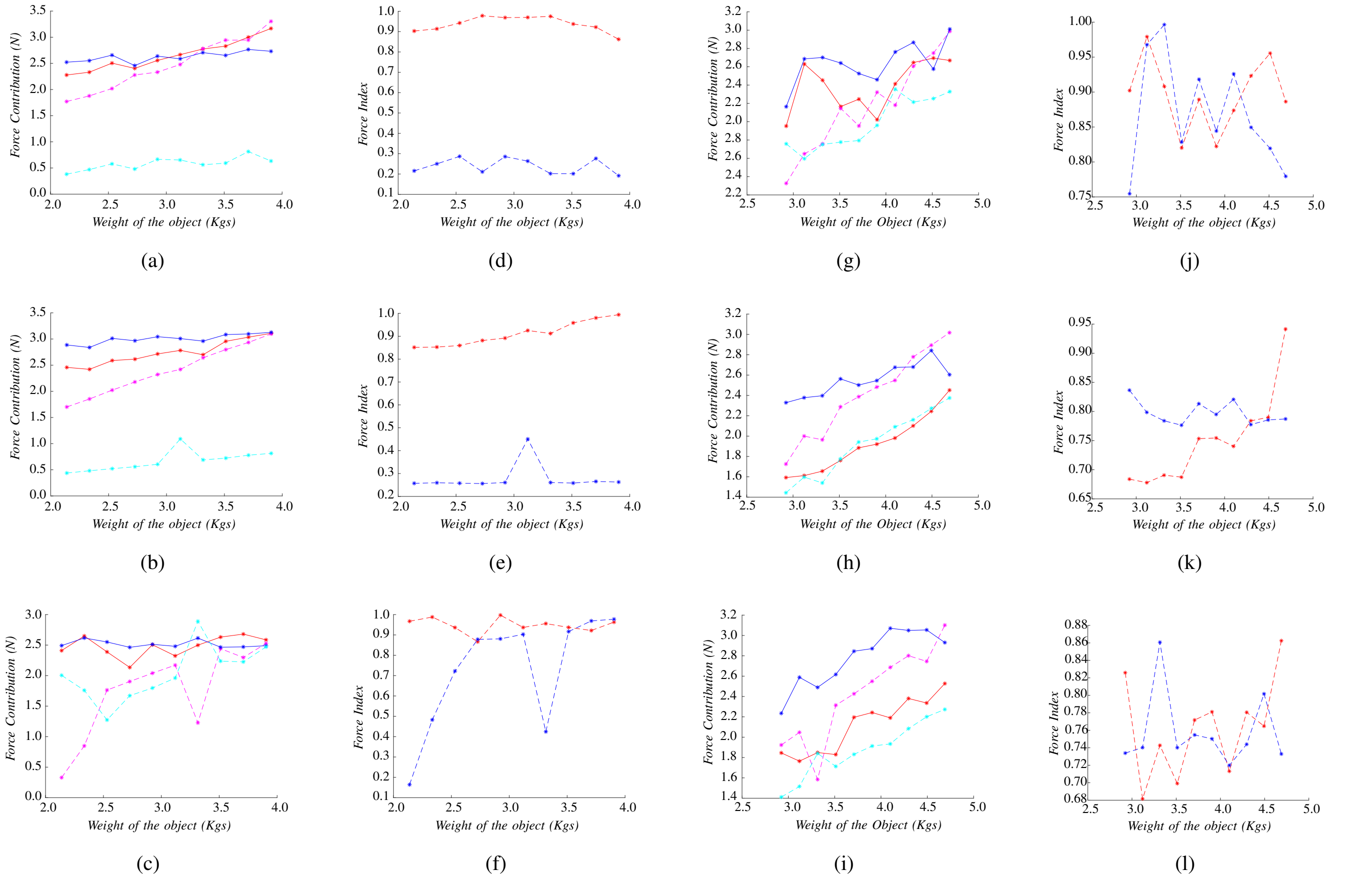

Fig. 5. Graphics of the results of the optimization method using the cost functions Φ_0 and Φ_1. a) - c) Force contribution of each hand for the three grasp of the rugby ball: Continuous lines colored in red and blue depict the force contribution of each hand by optimizing Φ_0 and dotted lines colored in magenta and cyan depict the force contribution of each hand by optimizing Φ_1. d) - f) Force Contribution Index (FCI). The red line show the results obtained by optimizing Φ_0 and blue line show the results obtained by optimizing Φ_1. g) - i) Force distribution of each hand for the three grasp of the detergent bottle. The meaning of the lines is the same as in graphs a) - c). j) - l) Force Contribution Index (FCI). The red line show the results obtained by optimizing Φ_0 and blue line show the results obtained by optimizing Φ_1.

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