Grasp Synthesis of 3D Articulated Objects with n Links

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***Abstract*—This paper addresses the problem of grasp synthesis with force-closure for 3D articulated objects consisting of n links and considering frictional and non-frictional contacts. The surface of each link is represented by a finite set of points. First, the article presents a methodology to represent the generalized wrench space for an articulated 3D object with n links. This wrench space is generated by the forces applied on the links of the articulated object. Second, the algorithm that finds the set of points which allow a force-closure grasp using the generalized wrench space is described. The approach has been implemented and some illustrative examples are included in the paper.**

I. Introduction

The problem of grasping and fixturing of objects using multifingered hands or fixture devices is one of the greatest fields of research in the robotic research [1]. The objects that can be handled and/or fixed by a robotic hand or fixturing machine can be of different size and shape, and they can be either rigid or articulated (rigid links joined by some type of joints) such as those shown in Fig. 1.

Fig. 1. Examples of articulated objects.

In the literature related to the area of grasping with multifingered robotic hands, there are two important properties that a grasp must satisfy: form-closure and force-closure (FC) [2][3]. Both properties can be characterized in the configuration space of the object. A 3D single rigid object has $d = 6$ degrees of freedom, and the number of contacts necessary in the general case for its immobilization is $k = d + 1 = 7$ frictionless contacts (except for objects with rotation symmetries) and 4 frictional contacts [4][5]. The search of grasps for any 3D object that satisfy the two aforementioned properties, and considering both types of contacts, is a problem that has been treated in a large number of works. For example, using 7 frictionless contacts on polyhedral object [6], or employing more than 4 frictional contacts for the same type of objects [7][8], for smooth objects [9] or objects of any shape [10][11], and for any object using both type of contacts [12][13]. However, there are works related to the search of grasps using a lower number of contacts with or without friction. For instance, considering the mobility theory of second order [14], polyhedral and smooth objects can be immobilized using 4 frictionless contacts [15] instead of 7, or using $k \geq 2$ frictional contact instead of 4 on polyhedral [16], smooth [17] and objects with arbitrary shape [18]. The surveys presented in [19][20] are recent works in which a large number of approaches related to the synthesis of grasp are summarized.

Manipulating objects is another relevant areas where grasping plays an important role in robotic research. For example, manipulating objects with more than one hand [21][22], or manipulating unknown objects based on tactile information [23].

However, few works have addressed the problem of grasping, fixing and manipulation articulated objects in comparison with those associated with simple rigid objects. These works used different approaches, such as interactive perception [24] [25] or occlusion-aware systems [26]. Rimon and Van der Stappen [27] present a significant work describing a procedure to find a set of frictionless contact points to immobilize an articulated 2D object with n polygonal links. For chains with $n \neq 3$ polygons without parallel edges or chains of $n = 3$ polygons with certain conditions they show that immobilization is possible with $n+2$ contacts, while, for the general case, $n+3$ contacts are needed to immobilize any articulated object with n polygons.

Although there are many works presenting approaches to find FC grasps for 3D rigid objects and some works for articulated objects, we are not aware of works addressing the systematic synthesis of FC grasps for 3D articulated objects. Therefore, the objective of this work is the proposal of a procedure that finds FC grasps for articulated 3D objects with n links considering frictionless and frictional contact. First, the proposed approach defines a vector of generalized forces produced by forces applied on each link of the object and the corresponding generalized force space, and, then, searches the contact points on the links for a set that allows a FC grasp.

The paper is structured as follows. Section 2 provides the problem description, including the considered assumptions. The methodology to find the elements of the generalized force vector is presented in Section 3. Section 4 describes

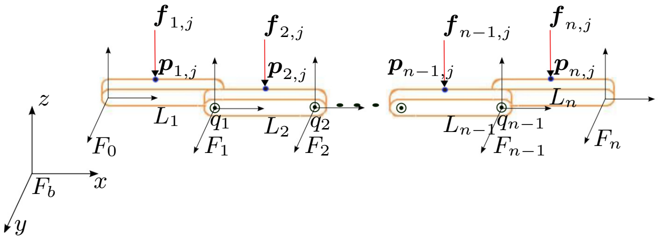

Fig. 2. Articulated object with n links (a generic force $\boldsymbol{f}_{i,j}$ acting on a point $\boldsymbol{p}_{i,j}$ is represented on each link i).

the algorithm to find a grasp with FC. Section 5 shows demonstrative examples of the proposed approach. Finally, conclusions and future work are presented in Section 6.

II. Problem Statement and Assumptions

Consider a 3D serial articulated object with n links and rotational joints, as illustrated in Fig. 1. The problems to be addressed are as follows:

1) Representation of the generalized wrenches for a 3D articulated object.
2) Search for a set of contact points on the surface of the links that allows a FC grasp.

The following assumptions are considered:

1) The links are connected by rotational joints.
2) The boundary of each link is represented with a large enough set Ω of points (i.e. the links can be of any shape, either polyhedral or non-polyhedral).
3) The contacts between the fingers and the object can be either frictionless or frictional.

III. Generalized Wrenches for 3D Articulated Objects

A. Generalized wrenches for a rigid body

1) Frictionless contacts: A force $\boldsymbol{f}_i$ applied at point $\boldsymbol{p}_i$ generates a torque $\boldsymbol{\tau}_i = \boldsymbol{p}_i \times \boldsymbol{f}_i$; $\boldsymbol{f}_i$ and $\boldsymbol{\tau}_i$ are grouped into a wrench vector $\boldsymbol{w}_i = (\boldsymbol{f}_i, \boldsymbol{\tau}_i)^T$. Considering frictionless contacts points $\boldsymbol{w}_i$ are given by:

$$\boldsymbol{w}_i = \begin{bmatrix} \boldsymbol{f}_i \\ \boldsymbol{\tau}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_i \\ \boldsymbol{p}_i \times \boldsymbol{f}_i \end{bmatrix} = f_i \begin{bmatrix} \hat{\boldsymbol{n}}_i \\ \boldsymbol{p}_i \times \hat{\boldsymbol{n}}_i \end{bmatrix} \tag{1}$$

with f_i being the magnitude of $\boldsymbol{f}_i$ and $\hat{\boldsymbol{n}}_i$ the unitary vector normal to the object boundary at $\boldsymbol{p}_i$.

For 3D objects, 7 contacts are sufficient to assure the FC condition, i.e. a set of points $G = \{\boldsymbol{p}_1, ..., \boldsymbol{p}_7\}$ that allows an appropriate set of wrenches $W = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_7\}$.

2) Frictional contacts: A grasp force $\boldsymbol{f}_i$ applied at a contact point $\boldsymbol{p}_i$ can be decomposed in two components $\boldsymbol{f}_{i,n}$ and $\boldsymbol{f}_{i,t}$ (normal and tangent). To avoid slippage of the finger, Coulomb's law must be satisfied: $\boldsymbol{f}_{i,t} \leq \mu \boldsymbol{f}_{i,n}$ (μ being the friction coefficient). For 3D objects, the friction cone is usually linearised using an m-side polyhedral convex cone. The grasping force applied at the contact point is given by:

$$\boldsymbol{f}_i = \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{l}_{i,j}, \quad \alpha_{i,j} \geq 0 \tag{2}$$

where $\boldsymbol{l}_{i,j}$ represents the normalized vector defining the j-th edge of the convex cone. The wrench generated by $\boldsymbol{f}_i$ is

$$\boldsymbol{w}_i = \sum_{j=1}^{m} \alpha_{i,j} \boldsymbol{w}_{i,j}, \; \boldsymbol{w}_{i,j} = \begin{bmatrix} \boldsymbol{l}_{i,j} \\ \boldsymbol{p}_i \times \boldsymbol{l}_{i,j} \end{bmatrix} \tag{3}$$

where $\boldsymbol{w}_{i,j}$ are called primitive contact wrenches. Each $\boldsymbol{p}_i$ has m associated $\boldsymbol{w}_{i,j}$, one for each $\boldsymbol{l}_{i,j}$ of the convex cone.

For frictional grasps, 4 contact points are sufficient to assure the FC condition, i.e. a set of points $G = \{\boldsymbol{p}_1, ..., \boldsymbol{p}_4\}$ that allows an appropriate set of primitive contact wrenches $W = \{\boldsymbol{w}_{1,1}, ..., \boldsymbol{w}_{1,m}, ..., \boldsymbol{w}_{4,1}, ..., \boldsymbol{w}_{4,m}\}$.

B. Generalized wrenches for a serial articulated object

This section addresses the computation of a generalized wrench vector for a 3D articulated object with n links. The procedure outlined in this section is based on the procedure presented in [28][29] for a 2D articulated object, and used to compute optimal grasps [30] and independent contact regions in [31] for 2D articulated object. The methodology considers a virtual robot that contains the articulated object and other auxiliary elements (Fig. 3).

The following basic nomenclature will be used:

- n: Number of links of the articulated object.
- L_i: Link i of the virtual robot, $i = -4, \ldots, n$. Note that links $-4, \ldots, 0$ are virtual ones, and links 1 to n correspond to the real articulated object.
- q_i: Joint i of the virtual robot, $i = -5, \ldots, n-1$ (generalized coordinates). Note that joints $-5, \ldots, 0$ are virtual ones, while joints 1 to $n-1$ correspond to the joints of real articulated object.
- F_b: Base reference frame for the virtual robot.
- F_i: Reference frame attached to link L_i.
- $\boldsymbol{O}_i$: Position of the origin of reference frame F_i, $i = -4, \ldots, n$ respect to the base frame.
- $\boldsymbol{r}_i$: Position of the origin of frame F_i respect F_{i-1} (i.e. $\boldsymbol{r}_i = \boldsymbol{O}_i - \boldsymbol{O}_{i-1}$).
- $\boldsymbol{p}_{i,j}$: Contact point j on link L_i represented with respect to the frame F_{i-1}, $i = 1, \ldots, n$, $j = 1, \ldots, k_i$, where k_i is the number of contact points on link L_i. Note that the total number of contacts is $k = \sum_i k_i$.
- $\boldsymbol{s}_{i,j}$: Contact point j on link L_i respect to F_i (i.e. $\boldsymbol{s}_{i,j} = \boldsymbol{p}_{i,j} - \boldsymbol{r}_i$).
- $\boldsymbol{f}_{i,j}$: Force j applied to link L_i at contact point $\boldsymbol{p}_{i,j}$.
- $\boldsymbol{W}_{i,j}$: Generalized wrench produced by force $\boldsymbol{f}_{i,j}$ applied on $\boldsymbol{p}_{i,j}$.
- $\boldsymbol{S}_{i,j,l}$: Normalized vector of the l-th edge of the linearised friction cone, $l = 1, \ldots, m$.
- $\boldsymbol{W}_{i,j,l}$: Primitive contact wrench produced by a primitive face along $\boldsymbol{S}_{i,j,l}$. Therefore each $\boldsymbol{p}_{i,j}$ has l associated $\boldsymbol{W}_{i,j,l}$, one for each edge of the linearised friction cone.
- J_i: Jacobian for each L_i.

Algortim 1 computes the generalized wrench vector for articulated 3D objects with n links. The procedure is based

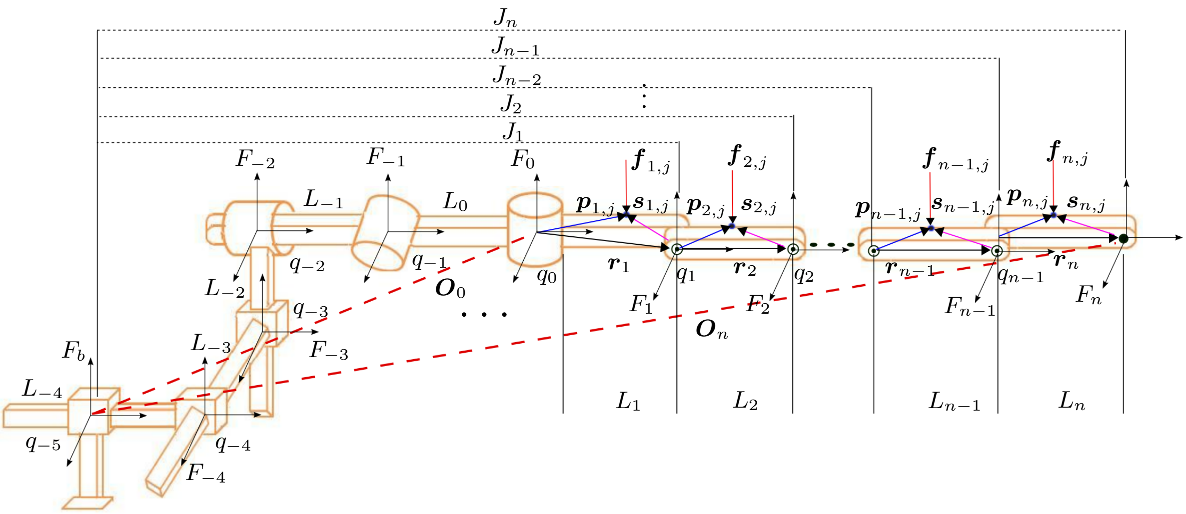

Fig. 3. Schematic diagram of the virtual robot, where links $L_{-4} \ldots L_n$ and joints $q_{-5} \ldots q_{n-1}$ represent the total links and joint of the virtual robot, while $L_1 \ldots L_n$ and $q_1 \ldots q_{n-1}$ are the links and joints of the articulated object. Also, the frames F_i are depicted.

Algorithm 1 Representation of the generalized wrench vector

1: Define a virtual serial robot containing the articulated object.
2: Compute the position and orientation of each frame F_i respect to the base frame F_b.
3: Compute of the geometric Jacobian J_i for each link L_i.
4: Obtain the torques and forces τ_k at each joint of the virtual robot produced by forces $\boldsymbol{f}_{i,j}$ applied in each link L_i.
5: Obtain the generalized wrench vector $\boldsymbol{W}_{i,j}$ from τ_k.

on widely used concepts in the general analysis of an open kinematic chains [32]. The main steps of the procedure are explained below.

In Step (1), a schema of a virtual serial robot is generated. This robot has a relevant roll in the procedure. Its first six joints represent the six degrees of freedom of the first link of the articulated object, which forms part of this robot. The first six joints $q_{-5}, \ldots, q_0$ are not real, but they are useful for the model development, while each real joint q_1 to q_{n-1} represents one of the internal degrees of freedom of the object. $q_{-5}, \ldots, q_{-3}$ are prismatic joints and $q_{-2}, \ldots, q_0$ are revolute joints. Furthermore, the links of the virtual robot are L_{-4} to L_n, where the links L_1 to L_n correspond to the articulated object. The virtual robot supports forces $\boldsymbol{f}_{i,j}$ applied on the contact points $\boldsymbol{p}_{i,j}$ on link L_i of the articulated object.

In Step (2), the forward kinematic is used to compute the position and orientation of each frame F_i with respect to the base frame F_b. These frames are associated to the links L_i where forces $\boldsymbol{f}_{i,j}$ are applied. These frames will be used in the subsequent step to compute the Jacobians J_i. The well-known procedure of Denavit-Hartenberg [32] is used in this step.

In Step (3), the geometric Jacobian J_i, $i = 1, ..., n$), is computed for each link L_i of the virtual robot where a force $\boldsymbol{f}_{i,j}$ can be applied, i.e the real links $L_i, i = 1, \ldots, n$.

Step (4) is related with the computation of torques and forces τ_k at each link of the virtual robot produced by forces $\boldsymbol{f}_{i,j}$ applied at each link L_i. The relation between the vector $\boldsymbol{\tau}$ of torques τ_k and a force $\boldsymbol{F}$ at the end effector is [32],

$$\boldsymbol{\tau} = J^T(q)\boldsymbol{F} \tag{4}$$

Considering eq. (4):

- The vector $\boldsymbol{\tau}_{i,j}$ of torques $\boldsymbol{\tau}_{k_{i,j}}$ at joints q_k necessary to balance the effect of a wrench $\boldsymbol{w}_{i,j}$ produced by a force $\boldsymbol{f}_{i,j}$ applied on the link L_i is obtained as

$$\boldsymbol{\tau}_{i,j} = \left[\tau_{-5_{i,j}}, ..., \tau_{0_{i,j}}, \tau_{1_{i,j}}, ..., \tau_{(n-1)_{i,j}}\right]^T = J_i^T \boldsymbol{w}_{i,j} \tag{5}$$

where:

$$\boldsymbol{w}_{i,j} = \left[\, f_{x_{i,j}} f_{y_{i,j}} f_{z_{i,j}} Ms_{x_{i,j}} Ms_{y_{i,j}} Ms_{z_{i,j}} \,\right]^T$$

- The vector $\boldsymbol{\tau}_i$ of torques τ_{k_i} in joints q_k necessary to balance all the forces $\boldsymbol{f}_{i,j}$ applied to L_i results in

$$\begin{aligned}\boldsymbol{\tau}_i = \left[\tau_{-5_i}, ..., \tau_{0_i}, \tau_{1_i}, ..., \tau_{(n-1)_i}\right]^T &= \sum_{j=1}^{k_i} \boldsymbol{\tau}_{i,j} \\ &= \sum_{j=1}^{k_i} J_i^T \boldsymbol{w}_{i,j}\end{aligned} \tag{6}$$

- The torques τ_k in joints q_k necessary to balance all the forces $\boldsymbol{f}_{i,j}$ applied to all the links L_i is given by

$$\begin{aligned}\boldsymbol{\tau} = \left[\tau_{-5}, ..., \tau_0, \tau_1, ..., \tau_{n-1}\right]^T &= \sum_{i=-4}^{n} \sum_{j=1}^{k_i} \boldsymbol{\tau}_{i,j} \\ &= \sum_{i=-4}^{n} \sum_{j=1}^{k_i} J_i^T \boldsymbol{w}_{i,j}\end{aligned} \tag{7}$$

In step (5) the equations (5), (6) and (7) are used to consider a generalized wrench space $\mathcal{W}$ defined by the base $\{\hat{\tau}_{-5}, ..., \hat{\tau}_0, \hat{\tau}_1, ..., \hat{\tau}_{n-1}\}$, where "ˆ" indicates a unitary vector. The generalized wrenches

$\boldsymbol{W}_{1,j}, \boldsymbol{W}_{2,j}, \ldots, \boldsymbol{W}_{n-1,j}, \boldsymbol{W}_{n,j}$ and the primitive wrenches $\boldsymbol{W}_{1,j,l}, \boldsymbol{W}_{2,j,l}, \ldots, \boldsymbol{W}_{n-1,j,l}, \boldsymbol{W}_{n,j,l}$ generated, respectively, by forces $\boldsymbol{f}_{1,j}, \boldsymbol{f}_{2,j}, \ldots, \boldsymbol{f}_{n-1,j}, \boldsymbol{f}_{n,j}$ can be represented in $\mathcal{W}$. Worth mentioning that the dimension of $\mathcal{W}$ is $d = n+5$, then $\boldsymbol{W}_{i,j}$ and $\boldsymbol{W}_{i,j,l}$ have d components.

C. Force closure test

Considering the set $G = \{\boldsymbol{p}_{i,j}, i = 1, ...n, j = 1, ..., k_i\}$ of $k = \sum_i k_i$ contact points (with k_i being the number of contact points on link L_i), and forces $\boldsymbol{f}_{i,j}$ applied at each $\boldsymbol{p}_{i,j}$, two sets of wrenches $W = \{\boldsymbol{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ and $W_p = \{\boldsymbol{W}_{i,j,l}, i = 1, ..., n, j = 1, ..., k_i, l = 1, ..., m\}$ are obtained. The necessary and sufficient condition for the existence of a FC grasp is that the origin of the generalized wrench space lies inside the convex hull CH(W_p) of the contact wrenches W_p [33]. This guarantees that the grasp can generate appropriate wrenches to counteract perturbation wrenches in any direction, i.e. to counterbalance any force(s) $\boldsymbol{f}_{i,j}$ applied on any link L_i of the articulated object. Note that this test is a generalization of the traditional FC test for objects without internal degrees of freedom. The test used in this work is derived from [34] for the case of a single rigid object and then extended in [29] for an articulated 2D object. Let P be the centroid of the primitives wrenches, O the origin of the wrench space and H_i a boundary hyperplane of CH(W_p): in order for a grasp G to be FC, P and O must lie on the same side of H_i $\forall i$.

IV. Grasp Synthesis

This section describes the algorithm for the synthesis of FC grasps for 3D articulated objects. This algorithm is the extension of the algorithm presented in [29] for the case of 2D articulated objects. Note that the synthesis is carried out in a wrench space with dimension $d = n+5$, which is greater than the one considered for 2D and 3D rigid objects. The procedure generates an initial grasp G^m, $m = 1$, with k points selected randomly from the set Ω that describes the object boundary, then computes the corresponding set W^m when frictionless contact points are considered, or W_p^m with primitives contact wrenches for frictional contacts. The next step consists in checking if the points in G^m allow a FC grasp. If G^m does not allow a FC grasp, then a search of new contact points is done, based on separating hyperplanes in the wrench space that define candidate points to replace one of the current points in G^m to obtain another grasp G^{m+1}. This is iteratively repeated until a FC grasp is found. The procedure is detailed in Algorithm 2 and explained below.

The algorithm starts with a set of points $G^m, m = 1$, randomly selected from the set Ω. Then, the set of wrenches W^m or W_p^m and the corresponding convex hull, CH(W^m) when the contacts are frictionless or CH(W_p^m) for frictional contacts, is computed.

If the grasp G^m does not satisfy the FC-test mentioned in Section III-C, the search procedure, Steps (3) to (8), iteratively tries to improve the grasp by changing one of the points in G^m.

Algorithm 2 Search of an initial FC grasp

Ensure: : Grasp G^m with FC
1: Generate a random initial grasp G^m, $m = 1$, and build the set W^m or W_p^m and the convex hull CH(W^m) or CH(W_p^m).
2: **while** G^m is not a FC grasp **do**
3: Form the corresponding set of wrenches W^m and primitives wrenches W_p^m, and generate the correponding convex hull.
4: Determine a subset G_R^m of grasp points on G^m to be replaced.
5: Generate a subset Ω_C^m with candidate points to replace one of the points in G_R^m.
6: Obtain an auxiliary grasp G_{aux} replacing a point in G_R^m with one point from Ω_C^m.
7: Update the counter $m = m+1$.
8: G^m=G_{aux}.
9: **end while**
10: **return** (G^m)

In Step (4) a subset $G_R^m \subset G^m$ is generated with the points of the wrench space that simultaneously define all the critical hyperplanes H defining the boundary of CH(W) or CH(W_p), i.e those hyperplanes producing a failure of the FC-test (i.e. P and O lie on different sides of the hyperplane).

In Step (5) a subset $\Omega_C^m \subset \Omega$ with candidate points to replace one point in G_R^m is determined by hyperplanes $H^{'}$ passing through the origin and parallel to the critical hyperplanes H. The replacement candidate points are those that simultaneously lie on the opposite side of the point P with respect to all the hyperplanes $H^{'}$.

In Step (6) one of the points in G_R^m is replaced by a point randomly taken from Ω_C^m producing a wrench $\boldsymbol{W}_*$. The wrench $\boldsymbol{W}_*$ replaces the closest point in G_R^m, generating an auxiliary grasp G_{aux}. The centroid P^* and the distance $\left|\overline{P^*O}\right|$ are computed for the wrenches of the auxiliary grasp G_{aux}. Let P^m be the centroid of the set of wrenches W in the iteration m. If the relation $\left|\overline{P^*O}\right| < \left|\overline{P^mO}\right|$ is satisfied then the auxiliary grasp G_{aux} is selected as new grasp. If all the points in G_R^m were replaced and none of them reduces the distance $\left|\overline{P^mO}\right|$, the selection is the candidate G^* that has the smallest distance $\left|\overline{P^*O}\right|$. When frictional points are considered, the subset Ω_C^m is built using the generalized wrenches $\boldsymbol{W}_{i,j}$. The grasp G^m generated in each iteration is saved so it is not taken into account in subsequent iterations. This consideration avoids falling in local minima and allows the exploration of wrench space to continue until a FC grasp is found (if there is one).

Fig. 4 shows an example with frictional contacts in a hypothetical 2-dimensional wrench space, thus it can be graphically represented (remember that the dimension of the real wrench space is $n+5$). The grasp G^m producing wrenches $W^m = \{\boldsymbol{W}_{1,1}, \boldsymbol{W}_{2,1}, \boldsymbol{W}_{3,1}\}$ and $W_p^m = \{\boldsymbol{W}_{1,1,1}, \boldsymbol{W}_{1,1,2}, ..., \boldsymbol{W}_{3,1,1}, \boldsymbol{W}_{3,1,2}\}$ is not FC, being H_2 and H_3 the hyperplanes that produce the FC-test failure. Then, the set of possible points to be replaced is $G_R^m = \{\boldsymbol{p}_{1,1}\}$, i.e. the points producing the wrenches $\boldsymbol{W}_{1,1}$ and its corresponding primitive wrenches, some of which define H_2 and H_3. The contact points that produce wrenches lying in the gray area determined by the hyperplanes $H_2^{'}$

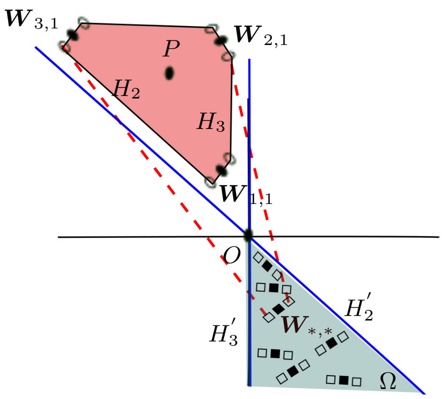

Fig. 4. Illustration of the search procedure to find one FC grasp in a hypothetical 2D wrench space using frictional contacts. The gray zone contains the candidate points.

and H_3' belong to Ω_C^m. The auxiliary grasp G_{aux} with $\boldsymbol{W}_{*,*}$ replacing $\boldsymbol{W}_{1,1}$, i.e. with $W^{m+1} = \{\boldsymbol{W}_{*,*}, \boldsymbol{W}_{2,1}, \boldsymbol{W}_{3,1}\}$ and $W_p^{m+1} = \{\boldsymbol{W}_{*,*,1}, \boldsymbol{W}_{*,*,2}, \boldsymbol{W}_{2,1,1}, \boldsymbol{W}_{2,1,2}, \boldsymbol{W}_{3,1,1}, \boldsymbol{W}_{3,1,2}\}$, is FC.

V. Examples

This section provides examples of the synthesis of FC grasps for a 3D articulated object with 3 links. Each link surface was represented using 1504 triangles, i.e. a total of 4512 triangles per object. The friction coefficient considered was $\mu = 0.2$ and the friction cone was linearized using a polyhedral convex cone with $m = 8$ sides. The implementation was done using Matlab and C ++ on a computer with Intel Core2 Duo 2.0 GHz processor. The articulated objects were generated in SolidWorks and Qhull [35] library was used to compute the convex hull.

Using Algorithm 1, the generalized wrench space $\mathcal{W}$ is defined by the base $\{\hat{\tau}_{-5}, ..., \hat{\tau}_0, \hat{\tau}_1, ..., \hat{\tau}_2\}$ and the contributions of forces $\boldsymbol{f}_{1,j}$, $\boldsymbol{f}_{2,j}$ and $\boldsymbol{f}_{3,j}$ define the generalized wrenches $\boldsymbol{W}_{1,j}$, $\boldsymbol{W}_{2,j}$,$\boldsymbol{W}_{3,j}$ as

$$\begin{array}{rcccccc}
\tau_{-5} = & f_{x_{1,j}} & + & f_{x_{2,j}} & + & f_{x_{3,j}} \\
\tau_{-4} = & f_{y_{1,j}} & + & f_{y_{2,j}} & + & f_{y_{3,j}} \\
\tau_{-3} = & f_{z_{1,j}} & + & f_{z_{2,j}} & + & f_{z_{3,j}} \\
\tau_{-2} = & Mx_{p_{1,j}} & + & (Mx_{p_{2,j}}+Mx_{r_1}) & + & (Mx_{p_{3,j}}+Mx_{r_2}+Mx_{r_1}) \\
\tau_{-1} = & My_{p_{1,j}} & + & My_{r_1} & + & My_{r_1} \\
\tau_0 = & Mz_{p_{1,j}} & + & (Mz_{p_{2,j}}+Mz_{r_1}) & + & (Mz_{r_2}+Mz_{r_1}) \\
\tau_1 = & 0 & + & My_{p_{2,j}} & + & (My_{p_{3,j}}+My_{r_2}) \\
\tau_2 = & 0 & + & 0 & + & Mz_{p_{3,j}} \\
& \underbrace{\phantom{Mx_{p_{1,j}}}}_{\boldsymbol{W}_{1,j}} & & \underbrace{\phantom{Mx_{p_{2,j}}}}_{\boldsymbol{W}_{2,j}} & & \underbrace{\phantom{Mx_{p_{3,j}}}}_{\boldsymbol{W}_{3,j}}
\end{array}$$

where, $Mx_{p_{i,j}}, My_{p_{i,j}}, Mz_{p_{i,j}}, Mx_{r_i}, My_{r_i}, Mz_{r_i}$ represent the moments generated by the forces $f_{i,j}$ with respect to the frame F_{i-1}.

Note that the dimension of $\mathcal{W}$ is $d = 3 + 5 = 8$, the generalized wrenches $\boldsymbol{W}_{i,j}$ and the primitive wrenches $\boldsymbol{W}_{i,j,l}$ have 8 components. In the examples we use the minimum necessary number of contacts $k = d+1 = 9$ for the frictionless case, and a conservative number of contacts $k = d - 1 = 7$ for the frictional case.

Figure 5 and 6 show graphically an example for frictionless and frictional contacts, respectively.

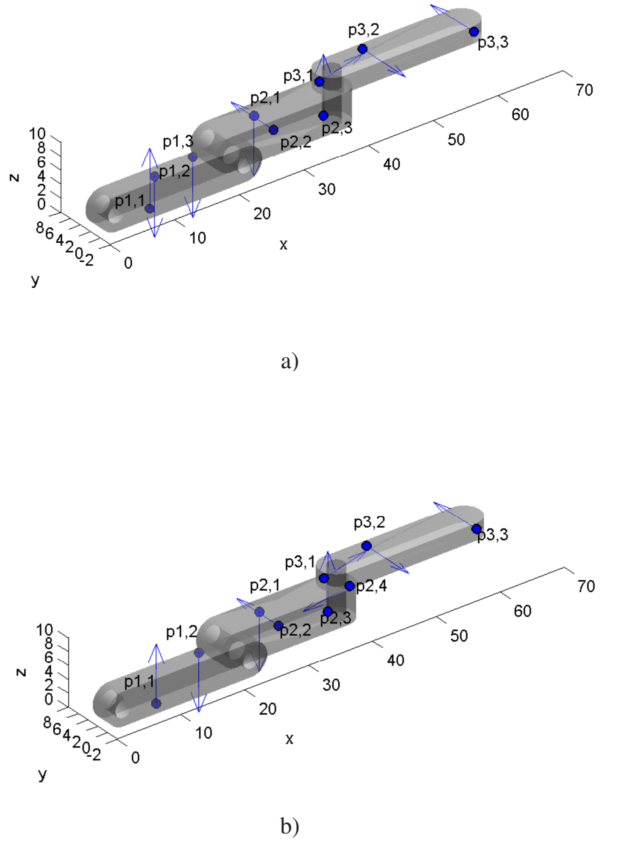

Fig. 5. Frictionless example a) Random Initial Non-FC grasp b) Final FC grasp after 4 iterations in 6s.

VI. Summary

This paper has proposed a systematic procedure to find FC grasps of 3D articulated objects with n links considering frictionless and frictional contacts. The approach has two main stages: the first one defines the wrench space and the elements contained in the wrench vectors generated by forces applied on each link of the articulated object. The second stage carries out the synthesis of FC grasps using the wrench space defined in the first stage. The dimension d of the resulting wrench space is equal to the number of degrees of freedom of the articulated object, i.e $d = n + 5$.

Future work includes the search for optimal grasps considering a specific quality measure and the computation of independent contact regions on the surface of the object links such that a grasp with a contact point in each of these regions assures a FC grasp. The implementation of the approach for close kinematic chains is another topic of interest.

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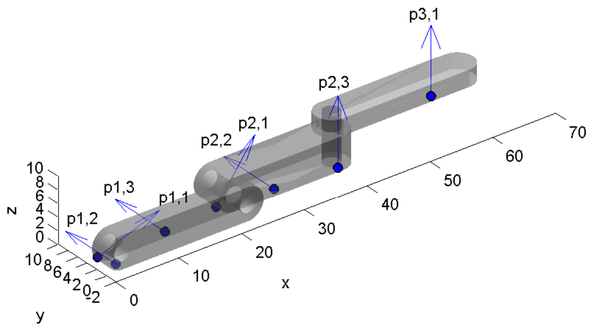
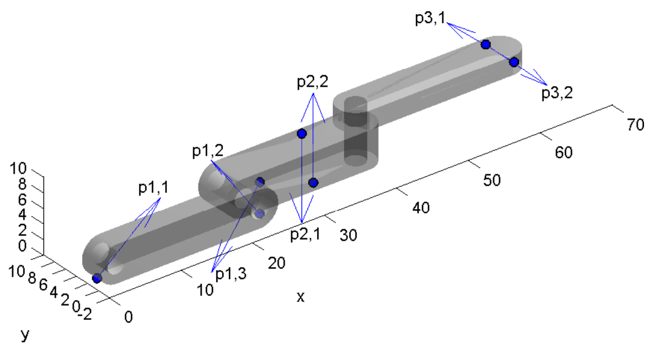


a)



b)

Fig. 6. Frictional example a) Random Initial Non-FC grasp b) Final FC grasp after 22 iterations in 86s.

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