Grasp Analysis and Synthesis of 2D articulated objects with 2 and 3 links

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Abstract

This paper proposes a solution to the problem of grasp analysis and synthesis of 2D articulated objects with 2 and 3 links considering frictionless contacts. The boundary of each link of the object is represented with a finite set of points. The grasp analysis is carried out to verify whether a set of contact points on the object boundary allows a force-closure grasp. The grasp synthesis implies the determination of a set of contact points that allows a force-closure grasp. The paper describes how to find the elements of the generalized wrench vector generated by a force applied on any link of the articulated object. The approach have been implemented and some illustrative examples are included in the paper.

1 Introduction

The majority of robots used in industry, home, schools and research carry out activities or operations where it is necessary grasping, fixing or manipulating objects of different shapes and sizes. Furthermore, many of these objects may be articulated objects, e.g objects which consists of rigid links connected by joints or hinges, such as scissors, staplers, doors, laptop computers, pliers, truck toy and somes cell phones. The goal of a grasp is to constraint the object degrees of freedom despite the existence of some external force disturbances.

Typically, a grasp must satisfy one of the following properties: force-closure (hereafter FC, the forces applied by the fingers ensure the object immobility) or form-closure (the position of the fingers ensures the object immobility) [1]. Both properties can be characterized in the object configuration space. The configuration space of a 2D rigid body has dimension $m = 3$. Markenscoff et al. [2] and Mishra et al. [3] show that a 2D object can be immobilized with $m+1 = 4$ frictionless fingers. Several works have been developed dealing with FC grasps on 2D objects, either polygonal objects with three frictional contacts [4][5] and four frictionless contacts [6][7], or non-polygonal objects with four frictionless contacts [8], and objects with any shape [9]. Nevertheless, the majority of the work done in the area of object grasping with robotic hands is centered on the grasp of a single object as those mentioned above, either in 2D or 3D, while there are few works done on the grasp and manipulation of articulated objects composed by n links. Jain et al. [10] illustrated the use of spatial operator algebra algorithms for the modeling and dynamic analysis of multiple parallel manipulators grasping an articulated object. Katz and Brock [11] developed interactive perception skills for the manipulation of unknown objects that possess inherent degrees of freedom (such as scissors, pliers, but also door handles, drawers, etc.). In order to manipulate unknown articulated objects successfully the robot has to be able to acquire a model of the object's kinematic structure and then be able to use this model for a purposeful manipulation. Zyada et al. [12] presented the control of non-prehensile manipulation of a two-rigid-link object applying two cooperative arms. They presented the dynamic model for manipulating a two-rigid-link object, the analysis for the object static holding, and a control algorithm for lifting up the object in a plane.

Although the works mentioned above can grasp and manipulate articulated objects, they did not show a systematic procedure to find a set of points on the object boundary that allows a FC grasp nor a test to check whether a given grasp is FC, or, in same cases the manipulation of the object is non-prehensile [12].

Van der Stappent et al. [13][14][15] studied the immobilization with frictionless contact points of a serial chain of n polygons connected by $n-1$ rotational joints (hinges), i.e. a chain with a configuration space with dimension $m = n+1$. They determine the number of contacts needed to immobilize a serial chain and describe a systematic method to place the contact points on each link of the chain to obtain one particular solution. A chain of n polygons without parallel edges can be immobilized with $n+2$ contacts, and any chain with n polygons can be immobilized with $n+4$ contacts. Chains of polygons without parallel edges can be robustly immobilized (any contact can be perturbed slightly without destroying the immobilization) with $\frac{6}{5}(n+2)$ contacts, for chains with $n \geq 6$ they propose the chain division into sets of 5 polygons

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from one end of the chain until at most 5 polygons are left, these polygons are immobilized with a number of contacts that depends on the number of remaining polygons and each group of 5 polygons is immobilized with 6 contacts with the following arrangement (0,0,3,0,3), which means 0 contacts in the first, second and fourth polygons, and 3 contacts in the third and fifth polygons. On the other hand, $\frac{5}{4}(n+2)$ contacts are enough to immobilize robustly any chain with $n \neq 3$ polygons, in this case when the chain has $n \geq 6$ polygons, the chain is divided into sets of 4 polygons until at most 4 polygons are left, each group of 4 polygons is immobilized with 5 contacts using the arrangement (0,0,3,2), which means 0 contacts in the first and second polygons, 3 contacts in the third polygon and 2 contacts in fourth polygon, the remaining polygons (except if they are 3) are immobilized as it was described for chains of polygons without parallel edges, if three polygons are left, they are combined with the last quadruple and they are robustly immobilized with the contact arrangement (4,0,0,4,0,0,4), i.e. 4 contacts in the first, fourth and seventh polygons, and 0 contacts in the second, third, fifth and sixth polygons [13]. When the polygons have parallel edges the chain can be immobilized with $n+2$ contacts if n is even, and with $n+3$ if n is odd [14]. The most recent work [15] describes 2 approaches to immobilize articulated objects based on geometric effects of first and second order. Based on second-order effects, chains of $n \neq 3$ hinged polygons without parallel edges can be immobilized with $n+2$ fingers. Moreover, taking into account the effects of first order constraints, any chain of n polygons can be immobilized with $n+3$ fingers.

In this work we consider general grasps of 2D serial articulated objects with $n=2$ and $n=3$ links, and $m=n+2$ degrees of freedom, considering the minimum number of frictionless contacts $k=m+1=n+3$. We deal, first, with the problem of determining whether a given set of contact points on an articulated object allows a FC grasp, and second, with the problem of finding FC grasps. The latter is done starting from a random initial grasp, if it not FC then the contact points are iteratively changed to search for a FC grasp. The algorithm developed here is based on the work done by Roa and Suárez [16] extending it to the case of articulated objects. Both in the grasp analysis and in the grasp synthesis, the generalized force vector plays a relevant role, thus, in this paper a procedure to find a proper representation of this vector is presented. The contribution of the work is a systematic procedure to analyze and synthesize FC grasps of articulated objects using the generalized wrench space. This work does not take into account the particular device used for the grasp, so it may happen that in the grasp synthesis the proposed contact points were nor reachable by a particular robotic hand, in any case they are always useful for object fixturing.

The rest of the paper is structured as follows. Section II provides an overview of the problem, including the main assumptions and basic background. Section III presents a procedure to find the elements of the vector of generalized wrenches for an articulated object with 2 and 3 links. Section IV discusses the analysis to determine whether a given set of points allow a FC grasp. Section V presents the algorithm to find a FC grasp. Section VI shows some illustrative examples of the proposed approach. Finally, Section VI presents some conclusions and the future work.

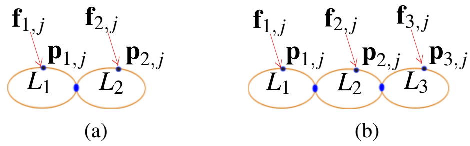


Figure 1. Articulated objects: a) with n=2 links, and b) with n=3 links.

2 Problem definition and assumptions

Consider a 2D serial articulated object with $n=2$ and $n=3$ links, as illustrated in Fig. 1. The problems to be addressed are the following:

- Determine whether a given a set of contact points on the surface of the links allows a FC grasp.
- Search for a set of contact points on the surface of the links that allows a FC grasp.

The following assumptions are considered in this work:

- The links are connected by rotational joints.
- The links can overlap each other, this does not generate any problem.
- The objects can be of any shape (either polygonal or non-polygonal).
- The boundary of each link is represented with a (large enough) set Ω of points described by position vectors $\mathbf{p}_{i,j}$.
- The normal direction $\hat{\mathbf{n}}_{i,j}$ pointing towards the interior of the object at each point $\mathbf{p}_{i,j}$ is known.
- The contact points between the fingers and the object are frictionless. This assures a worst case grasp, since the existence of friction in real cases will increase the robustness of the grasp.

3 Generalized wrenches for articulated objects

3.1 Generalized wrenches for a rigid body

Consider a coordinate system located at the center of mass (CM) of the object used to describe the positions $\mathbf{p}_i$ of the contact points and the forces applied on the object. A force $\mathbf{f}_i$ applied on the object at $\mathbf{p}_i$ generates a torque $\boldsymbol{\tau}_i = \mathbf{p}_i \times \mathbf{f}_i$ with respect to CM; $\mathbf{f}_i$ and

$boldsymbol\tau_i$ can be grouped together in a wrench vector $\mathbf{w}_i = (\mathbf{f}_i, \boldsymbol{\tau}_i)^T$. For frictionless grasps, the grasp forces can only be applied in the direction normals to the object boundary, thus the wrench vector is given by:

$$\mathbf{w}_i = \begin{bmatrix} \mathbf{f}_i \\ \boldsymbol{\tau}_i \end{bmatrix} = \mathbf{f}_i \begin{bmatrix} \hat{\mathbf{n}}_i \\ \mathbf{p}_i \times \hat{\mathbf{n}}_i \end{bmatrix} \quad (1)$$

A grasp defined by a set of k frictionless contacts, $G = \{\mathbf{p}_1, ..., \mathbf{p}_k\}$, is able to apply k wrenches $\mathbf{w}_i$ on the object, which can be grouped in a wrench set $W = \{\mathbf{w}_1, ..., \mathbf{w}_k\}$. The information in W is enough to analyze whether G allows or not a FC grasp.

A planar object has 3 degrees of freedom (dof), then the wrench vector is 3-dimensional and, in absence of rotational symmetries, four contacts are sufficient to assure the FC condition, i.e. $G = \{\mathbf{p}_1, ..., \mathbf{p}_4\}$ allowing a set of wrenches $W = \{\mathbf{w}_1, ..., \mathbf{w}_4\}$.

3.2 Generalized wrenches for a serial articulated object

As it was mentioned in Section 1 serial articulated objects with $n = 2$ and $n = 3$ links has $m = n + 2$ dof and can be held with $k = m + 1 = n + 3$ frictionless contacts at contact points $G = \{\mathbf{p}_{i,j}, i = 1, ...n, j = 1, ..., k_i\}$, where k_i indicates the number of contact points on link i (note that $k = \sum_i k_i$). We will present in this section a procedure to obtain m-dimensional generalized wrenches $\mathbf{W}_{i,j}$ such that the set of k elements $\mathscr{W} = \{\mathbf{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ with $k = \sum_i k_i$ allows to determine whether the set G can produce a FC grasp or not with a procedure analogous to the one used for the case of a single object using $\mathbf{w}_i$ and W.

The generalized wrench vector generated by the application of a force on a link of the articulated object is deduced using a general analysis of open kinematic chains with $n+2$ links. Fig. 2 shows a virtual robot with $n+2$ links which contains the articulated object and the auxiliary elements used for the developments. The procedure proposed in this section has similarities to that described in [17], in the sense that they use virtual kinematic chains in order to generate a systematic constraint-based approach to specify complex tasks of general sensor-based robot systems consisting of rigid links and joints.

The following basic nomenclature is used:
L_i: Virtual robot link i, $i = 1, \ldots, n$.
q_i: Position of joint i (generalized coordinates).
$\mathbf{p}_{i,j}$: Contact point j on link L_i, $i = 1, \ldots, n$, $j = 1, \ldots, k_i$ where k_i is the number of contact points on link L_i
$\mathbf{f}_{i,j}$: Force j applied to the link L_i at the contact point $\mathbf{p}_{i,j}$
The vector of generalized wrenches is obtained as follows. Consider a virtual robot of n links, the first two links are virtual ones and the rest of them are equivalent to the articulated object to be grasped (see Fig. 2). The first three joints are therefore not real, but they are useful for the model development. The first and second joints are prismatic and the third one is revolute, the remaining joints are those of the articulated object.

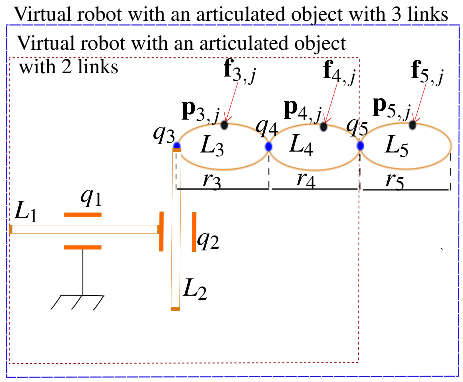


Figure 2. General scheme of the virtual robot, where links $L_1 \ldots L_5$ represent the total links of the virtual robot, and $L_3 \ldots L_5$ are the links of the articulated object.

Compute the Jacobian J_i for each link L_i of the virtual robot in order to relate forces applied on the link with those in the virtual robot joints under equilibrium conditions.

Since the first three joints are virtual, we can consider them positioned at the "zero" position, i.e. we can assume without lost of generality that $q_1 = q_2 = q_3 = 0$. These considerations allows the simplification of the Jacobians of the previous step.

Use the Jacobian to relate the external forces and moments applied to each link L_i with the torques and forces necessary in the joints for an equilibrium condition. Fig. 2 shows a general force $\mathbf{f}_{i,j}$ applied to each link L_i of the articulated object, wich can be expresed as

$$\mathbf{f}_{i,j} = \begin{bmatrix} f_{x_{i,j}} \\ f_{y_{i,j}} \end{bmatrix} \quad (2)$$

The force $\mathbf{f}_{i,j}$ and the moment that it produces are grouped into $\mathbf{w}_{i,j}$ as

$$\mathbf{w}_{i,j} = \begin{bmatrix} f_{x_{i,j}} \\ f_{y_{i,j}} \\ M_{f_{i,j}} \end{bmatrix} \quad (3)$$

where $M_{f_{i,j}} = \mathbf{p}_{i,j} \times \mathbf{f}_{i,j}$
The torque to be applied in joint k to balance the effect of a force $\mathbf{f}_{i,j}$ applied on link L_i is

$$\tau_{k,w_{i,j}} = V_{i,j}[k] \quad (4)$$

where $V_{i,j}[k]$ is the element k of the vector $\mathbf{V}_{i,j}$ given by

$$\mathbf{V}_{i,j} = J_i^T \mathbf{w}_{i,j} \quad (5)$$

The toque required in joint k to balance all the forces $\mathbf{f}_{i,j}$ applied on link L_i is

$$\tau_{k,w_i} = \sum_{j=1}^{k_i} V_{i,j}[k] \quad (6)$$

Finally, the total torque in joint k required to balance all the forces $\mathbf{f}_{i,j}$ acting on the virtual robot is

$$\tau_k = \sum_{i=1}^{n} \sum_{j=1}^{k_i} V_{i,j}[k] \tag{7}$$

The forces $\mathbf{f}_{i,j}$ acting on the virtual robot include the external perturbations forces and the forces applied by the grasping device. Since it is desired to immobilize the articulated object, the total torque in each joint must be null (considering perturbations and forces applied by the fingers), then it must be $\tau_k = 0$.

The total torque τ_k is obtained from (7) considering the corresponding Jacobians, and making it equal zero for each joint, results:

- for the articulated object with 2 links,

$$\begin{array}{ccccc} \tau_1 & = & \sum_j fx_{3,j}+\sum_j fx_{4,j} & = & 0 \\ \tau_2 & = & \sum_j fy_{3,j}+\sum_j fy_{4,j} & = & 0 \\ \tau_3 & = & \sum_j M_{f_{3,j}} & = & 0 \\ \tau_4 & = & \sum_j M_{f_{4,j}} & = & 0 \end{array} \tag{8}$$

- and for the articulated object with 3 links,

$$\begin{array}{ccccc} \tau_1 & = & \sum_j fx_{3,j}+\sum_j fx_{4,j}+\sum_j fx_{5,j} & = & 0 \\ \tau_2 & = & \sum_j fy_{3,j}+\sum_j fy_{4,j}+\sum_j fy_{5,j} & = & 0 \\ \tau_3 & = & \sum_j M_{f_{3,j}} & = & 0 \\ \tau_4 & = & \sum_j M_{f_{4,j}} - Zx_4 + Zy_4 & = & 0 \\ \tau_5 & = & \sum_j M_{f_{5,j}} & = & 0 \end{array} \tag{9}$$

where
$Zx_4 = r_4 sin(q_4)(\sum_j fx_{4,j}+\sum_j fx_{5,j})$
$Zy_4 = r_4 cos(q_4)(\sum_j fy_{4,j}+\sum_j fy_{5,j})$

Now, it is possible to consider a generalized wrench space $\mathscr{W}$ defined by $\{\tau_1, \tau_2, \tau_3, \tau_4\}$ or by $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5\}$ for each articulated object respectively, where the generalized wrenches $\mathbf{W}_{3,j}, \mathbf{W}_{4,j}, \mathbf{W}_{5,j}$, generated respectively by forces $\mathbf{f}_{3,j}, \mathbf{f}_{4,j}, \mathbf{f}_{5,j}$, are defined as:

- wrenches on the articulated object with 2 links,

$$\mathbf{W}_{1,j} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{W}_{2,j} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{W}_{3,j} = \begin{bmatrix} fx_{3,j} \\ fy_{3,j} \\ M_{f_{3,j}} \\ 0 \end{bmatrix} \quad \mathbf{W}_{4,j} = \begin{bmatrix} fx_{4,j} \\ fy_{4,j} \\ 0 \\ M_{f_{4,j}} \end{bmatrix} \tag{10}$$

- wrenches on the articulated object with 3 links,

$$\mathbf{W}_{1,j} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{W}_{2,j} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

$$\mathbf{W}_{3,j} = \begin{bmatrix} fx_{3,j} \\ fy_{3,j} \\ M_{f_{3,j}} \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{W}_{4,j} = \begin{bmatrix} fx_{4,j} \\ fy_{4,j} \\ 0 \\ M_{f_{4,j}} - Zx_4 + Zy_4 \\ 0 \end{bmatrix} \quad \mathbf{W}_{3,j} = \begin{bmatrix} fx_{5,j} \\ fy_{5,j} \\ 0 \\ 0 \\ M_{f_{5,j}} \end{bmatrix}$$

Now, considering that the virtual joints do not really exist and the wrenches $\mathbf{W}_{1,j}$ and $\mathbf{W}_{2,j}$ on the virtual robot are always null, in order to simplify the nomenclature, considering only the links of the real object we can refer to them directly as L_1 and L_2 for the object with two links and L_1, L_2 and L_3 for the object with three links, and therefore the subscripts of the previous equations indicating the link are, in practice shifted in a value -2, i.e. a contact point $\mathbf{p}_{3,j}$ on the link 3 of the virtual robot, is now point $\mathbf{p}_{1,j}$ applied on the real articulated object (Fig. 1a and Fig. 1b), and a wrench $W_{3,j}$ applied on the link 3 of the virtual robot, is now the wrench $W_{1,j}$ applied on the real articulated object.

It is worth observing that the dimension of $\mathscr{W}$ is $n+2$ and a generalized wrench $\mathbf{W}_{i,j}$ has therefore $n+2$ components; nevertheless, each $\mathbf{W}_{i,j}$ has only 3 independent components. Besides, thinking on FC grasps, in order to expand the whole $\mathscr{W}$ it is necessary that at least two forces were applied on the first and last link of the articulated object.

In summary, the dimension of the generalized wrench is equal to the number of degrees of freedom of the articulated object, i.e. $n+2$, but their independent components are always three, which are derived from the three independent components of the applied force.

3.3 Force-closure Test

The necessary and sufficient condition for the existence of a FC grasp is that the origin of the wrench space lies inside the convex hull of primitive contact wrenches [18] [3]. This condition could be verified with a linear programming problem using a ray-shooting technique [19] or using a geometric reasoning based on the following Lemma, which is the test used in this work.

Lemma. Let G be a grasp with a set $W = \{\mathbf{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ of primitive contacts wrenches for an articulated object with $m = k-1$ dof. Let H_l, $l = 1, ..., k$, be each of the hyperplanes defined in the wrench space by the set of points $W - \{\mathbf{W}_{i,j}\}$, with $i = 1, ..., n, j = 1, ..., k_i$, and P the centroid of W. P and the origin of the wrench space O must lie on the same side of each hyperplane H_l in order for the grasp G to be FC. ⋄

The Lemma is illustrated in an hypothetical 2D wrench space as shown in Fig. 3. Given a grasp with a wrench set $W = \{\mathbf{W}_{1,1}, \mathbf{W}_{2,1}, \mathbf{W}_{3,1}\}$, the hyperplanes H_l are defined by the following sets of points:

H_1 defined by $W - \{\mathbf{W}_{1,1}\} = \{\mathbf{W}_{2,1}, \mathbf{W}_{3,1}\}$
H_2 defined by $W - \{\mathbf{W}_{2,1}\} = \{\mathbf{W}_{1,1}, \mathbf{W}_{3,1}\}$
H_3 defined by $W - \{\mathbf{W}_{3,1}\} = \{\mathbf{W}_{1,1}, \mathbf{W}_{2,1}\}$

The lemma is fulfilled for the grasp shown in Fig. 3a because the origin O and P are on the same side of each hyperplanes H_l. However, the grasp shown in Fig. 3b fails, because the origin O and P are on different sides of the hyperplane H_3.

4 Grasp analysis

Using the results of the previous section, it is possible to analyze in a systematic way whether a given set of con-

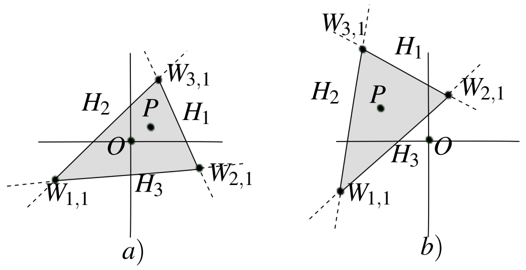


Figure 3. Illustration of the FC test in a hypothetical 2D space, a) grasp with FC , b) grasp without FC.

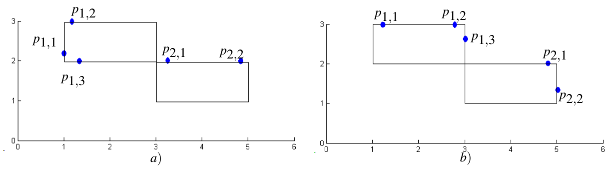


Figure 4. Illustration of two non FC grasps.

tact points on the articulated object allows a FC grasp.

Just as illustrative examples, consider an articulated object with two links as shown in Fig. 4 with the grasping points $G = \{\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{1,3}, \mathbf{p}_{2,1}, \mathbf{p}_{2,2}\}$. The grasp of Fig. 4a is not FC since contact points in the second link generate moments with the same sign. Therefore there may be external disturbances on the second link that produce moments that can not be counter balanced. The articulated object of Fig. 4b shall not be immobilized with the given grasp, because the contact points on the both links can not counterbalance any external force pushing the object down. The test described in Lemma 1 will not be fulfilled in the two cases.

If the set G allows a FC grasp means that the ConvexHull(W) contains the origin of the generalized wrench space. Note that each generalized wrench $\mathbf{W}_{i,j}$ will have a number of independent components smaller than the dimension of the wrench space (which ones depend on the link where the contact point is located), and this imposes additional constrains on the distribution of the forces applied on the object boundary, requiring them to be properly distributed on the object links (for instance, in the simple case of an object with two links and one joint, it is straightforward to see that at least two contact points are necessary on each of the links).

Note that the approach is a generalization of those already applied for the grasp analysis of 2D and 3D rigid bodies using respectively 3-dimensional and 6-dimensional wrench spaces, in this case the approach is applied to 2D articulated objects of 2 and 3 links using respectively 4-dimensional and 5-dimensional wrench spaces, even when each generalized wrench $\mathbf{W}_{i,j}$ has only three independent components.

5 Grasp Synthesis

Following the developments above, the main idea of the algorithm described in this section is a generalization of that presented in [16] considering here that the wrench space may be of dimension 4 or 5 other than 3 and 6 for 2D and 3D solid objects respectively. The algorithm generates a grasp G^1 by selecting k random points from the set Ω that describes the object boundary, then computes the corresponding set W^1 of generalized wrenches and verifies if the points allow a FC grasp. If G^1 is not a FC grasp, then a search of new contact points is done, based on separating hyperplanes in the wrench space that define candidate points to replace one of the current points in G^1. This procedure is iteratively repeated until a FC grasp is found. The algorithm is as follows:

Algorithm: Grasp Synthesis

1. Generate a random initial grasp G^k, k=1, and compute the corresponding set of wrenches $W = \{\mathbf{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$.
2. While G^k is not a FC grasp DO
   (a) Determine a subset G_R^k of grasp points on G^k to be replaced.
   (b) Generate a subset Ω_C^k with candidate points to replace one of the points in G_R^k.
   (c) Obtain an auxiliary grasp G_{aux} replacing a point in G_R^k with one point from Ω_C^k.
   (d) Update the counter $k = k + 1$.
   (e) G^k=G_{aux}.
3. RETURN(G^k).

If grasp G^k fails the FC-test, the search procedure (steps (2a) to (2e) of the algorithm) iteratively tries to improve the grasp by changing one of the points in G^k.

In Step (2a) the subset G_R^k is formed by all the wrenches in W that simultaneously belong to all the hyperplanes H_i that produce the FC failure, i.e. those that do not satisfy the condition in Lemma 1. If there is only one critical hyperplane then G_R^k includes all the points producing the wrenches that define such hyperplane. Fig. 5 shows a hypothetical example in 2D (note that the real wrench space is m-dimensional), the grasp G producing wrenches $W = \{\mathbf{W}_{1,1}, \mathbf{W}_{2,1}, \mathbf{W}_{3,1}\}$ is not force-closure, being H_1 and H_2 the hyperplanes that produce the FC-test failure, then the set of possible points to be replaced G_R^k includes the point $\mathbf{p}_{2,1}$ that produces the wrench $\mathbf{W}_{2,1}$.

In Step (2b) the subset Ω_C^k with candidate points to replace one of the points in G_R^k is determined using hyperplanes $H_i^{'}$ passing through the origin and parallel to the critical hyperplanes H_i. Candidate points to be used for the replacement are those that simultaneously lies on the opposite side of the point P with respect to the hyperplanes $H_i^{'}$. In Fig. 5, the points that produce the wrenches

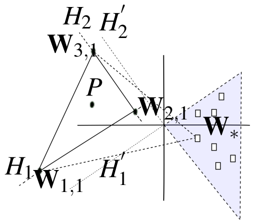


Figure 5. Illustration of the search procedure to find one FC grasp in a hypothetical 2D wrench space.

that lie in the gray area determined by the hyperplanes H_1' and H_2' belong to Ω_C^k.

In Step (2c) one of the points in G_R^k is replaced with the point producing a wrench W_* randomly taken from Ω_C^k. An auxiliary grasp G_{aux} is obtained with the replacement producing the set of wrenches $\{\mathbf{W}_{1,1}, \mathbf{W}_*, \mathbf{W}_{3,1}\}$, as shown in Fig. 5. For the candidate grasps, the centroid P^* and the distance $\left|\overline{P^*O}\right|$ are computed. If the relation $\left|\overline{P^*O}\right| < \left|\overline{P^kO}\right|$ is satisfied then the auxiliary grasp is selected, and the corresponding W_* is used for the replacement. If all the points in G_R^k were verified and none of them reduces the distance $\left|\overline{P^kO}\right|$, the selection is the candidate G^* that has the smaller distance $\left|\overline{P^*O}\right|$.

Finally in steps (2d) to (2e), the counter k is updated and the selected point is included in the new grasp G^k=G_{aux}.

6 Numerical Examples

In this section, we present some numerical examples illustrating the analysis and synthesis of grasps of articulated objects with 2 and 3 links. The proposed approach has been implemented using Matlab and C++ on a Intel Core2 Duo 2.0 GHz computer.

6.1 Grasp Analysis Examples

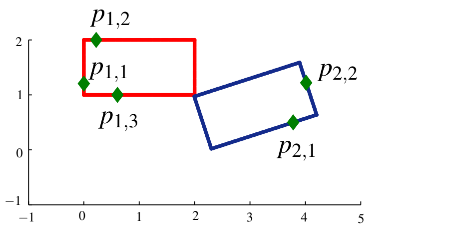
Example 1. Consider the articulated object to be grasped and immobilized composed of two rectangular links linked by one of the vertices and the frictionless contact points $\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{1,3}, \mathbf{p}_{2,1}, \mathbf{p}_{2,2}$ shown in Fig. 6a. The corresponding contact wrenches are:

$$\mathbf{W}_{1,1} = [0\ -1\ 1.8\ 0]^T$$
$$\mathbf{W}_{1,2} = [1\ 0\ -0.2\ 0]^T$$
$$\mathbf{W}_{1,3} = [0\ 1\ -1.4\ 0]^T$$
$$\mathbf{W}_{2,1} = [-0.57\ -0.81\ 0\ -0.3]^T$$
$$\mathbf{W}_{2,2} = [-0.5\ 0.86\ 0\ 1.8]^T$$

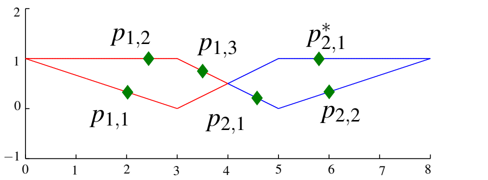
The convex hull of the wrenches contains the origin of the wrench space, and consequently the grasp is FC.

Example 2. Consider the articulated object composed of two links without parallel edges and the frictionless contact points $\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{1,3}, \mathbf{p}_{2,1}, \mathbf{p}_{2,2}$ shown in Fig. 6b. The corresponding contact wrenches are:

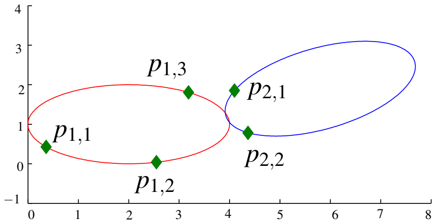
$$\mathbf{W}_{1,1} = [0\ -1\ 1.4\ 0]^T$$



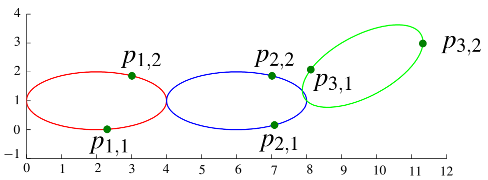
(a) Example 1



(b) Example 2



(c) Example 3



(d) Example 4

Figure 6. Grasp analysis examples.

$$\mathbf{W}_{1,2} = [0.70\ 0.70\ -1.2\ 0]^T$$
$$\mathbf{W}_{1,3} = [-0.70\ -0.70\ 0.5\ 0]^T$$
$$\mathbf{W}_{2,1} = [0.70\ 0.70\ 0\ 0.5]^T$$
$$\mathbf{W}_{2,2} = [-0.70\ 0.70\ 0\ 1]^T$$

The wrenches $\mathbf{W}_{2,1}$ and $\mathbf{W}_{2,2}$ on the second link have moments with the same sign (positive), this means that the object can not be immobilized because any external wrench applied to the second link with positive moment will move the object. In order to immobilize the object, the wrench $\mathbf{W}_{2,1}^*$ must have a moment opposite to $\mathbf{W}_{2,2}$. Replacing $\mathbf{p}_{2,1}$ by $\mathbf{p}_{2,1}^*$ that generates the wrench $\mathbf{W}_{2,1}^* = [0\ -1\ 0\ -1.8]^T$ produces a FC grasp.

Example 3. Consider the articulated object composed of two ellipses and the frictionless contact points $\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{1,3}, \mathbf{p}_{2,1}, \mathbf{p}_{2,2}$ shown in Fig. 6c. The corresponding contact wrenches are:

$$\mathbf{W}_{1,1} = [0.58\ 0.81\ -2.62\ 0]^T$$
$$\mathbf{W}_{1,2} = [-0.14\ 0.98\ -1.57\ 0]^T$$
$$\mathbf{W}_{1,3} = [-0.34\ -0.93\ 1.04\ 0]^T$$
$$\mathbf{W}_{2,1} = [0.82\ -0.56\ 0\ -0.75]^T$$
$$\mathbf{W}_{2,2} = [0.33\ 0.94\ 0\ 0.41]^T$$

The convex hull of the wrenches contains the origin of the wrench space, and consequently the grasp is FC.

Example 4. Consider the articulated object composed of three ellipses and the frictionless contact points $\mathbf{p}_{1,1}, \mathbf{p}_{1,2}, \mathbf{p}_{2,1}, \mathbf{p}_{2,2}, \mathbf{p}_{3,1}, \mathbf{p}_{3,2}$ shown in Fig. 6d. The corresponding contact wrenches are:

$$\mathbf{W}_{1,1} = [-0.07\ 0.99\ -1.76\ 0\ 0]^T$$
$$\mathbf{W}_{1,2} = [-0.28\ -0.95\ 1.19\ 0\ 0]^T$$
$$\mathbf{W}_{2,1} = [-0.28\ -0.95\ 0\ -4.35\ 0]^T$$
$$\mathbf{W}_{2,2} = [-0.30\ 0.95\ 0\ 0.96\ 0]^T$$
$$\mathbf{W}_{3,1} = [0.70\ -0.70\ 0\ 0\ -0.88]^T$$
$$\mathbf{W}_{3,2} = [-0.95\ 0.28\ 0\ 0\ 2.33]^T$$

The convex hull of the wrenches contains the origin of the wrench space, and consequently the grasp is FC.

6.2 Grasp Synthesis Examples

Example 1: Articulated object with 2 ellipses. First, the grasp shown in Fig. 7a is obtained randomly, which is not a FC grasp. Then, the search process is executed and the FC grasp shown in Fig. 7b is obtained after 4 iterations.

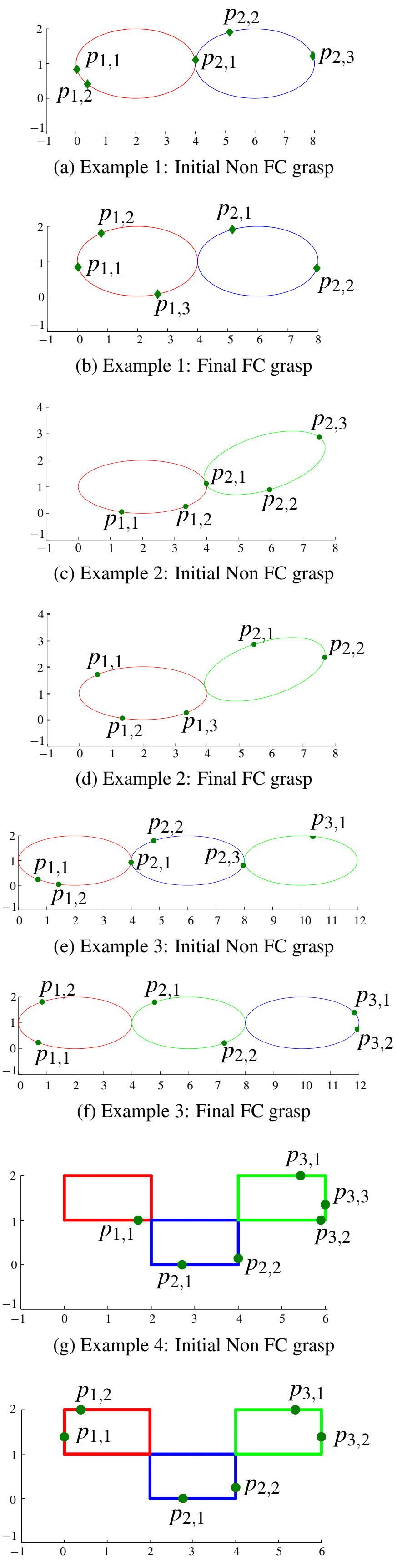
Example 2: Articulated object with 2 unaligned ellipses. First, the initial grasp shown in Fig. 7c is obtained randomly, since this is not a FC grasp, the search process is executed, obtaining the FC grasp of Fig. 7d after 4 iterations.

Example 3: Articulated object with 3 ellipses. Fig. 7e shows the initial non-FC grasp. The FC grasp shown in fig. 7f is achieved after 9 iterations.

Example 4: Articulated object with 3 rectangles. Fig. 7g shows the initial non-FC grasp. The FC grasp shown in fig. 7h is achieved after 50 iterations.

7 Conclusion

This paper proposes a systematic procedure to analyze and synthesize FC grasps of 2D articulated objects considering frictionless contacts as well as a procedure to model the corresponding the generalized wrench space. The approach was presented for the case of articulated objects with 2 and 3 links (either polygonals or non polygonals). The dimension of the generalized wrenches is always equal to the number of degrees of freedom of the articulated object ($n+2$ for objects with n links), but their independent components are always three, which are derived from the three independent components of the applied forces. Using the proposed generalized wrench space it is possible to analyze in a general and systematic way whether a given set of contact points on the links of an articulated object can generate a FC grasp or not. Besides, the proposed generalized wrench space can also be used to find FC grasps of articulated objects by means of a generalization of a procedure developed for rigid objects. Future work includes the generalization of the approach to the cases of articulated objects with n links, 3D articulated objects with frictionless and friction contacts and to articulated objects with close loops. Another point of interest is the synthesis of optimal grasps with respect to specific quality criteria.



(a) Example 1: Initial Non FC grasp

(b) Example 1: Final FC grasp

(c) Example 2: Initial Non FC grasp

(d) Example 2: Final FC grasp

(e) Example 3: Initial Non FC grasp

(f) Example 3: Final FC grasp

(g) Example 4: Initial Non FC grasp

(h) Example 4: Final FC grasp

Figure 7. Grasp synthesis examples.

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