WRIST-POSITION PARAMETERIZATION FOR FAST ON-LINE DETERMINATION OF GRASP CONFIGURATIONS *

Raúl Suárez, Gerrit Färber

Institut d'Organització i Control de Sistemes Industrials (IOC)
Universitat Politécnica de Catalunya (UPC), Barcelona, Spain
raul.suarez@upc.edu, gerrit_faerber@gmx.de

Abstract: Determining the grasp optimal configuration of a mechanical hand is a computational hard work, mainly due to the large number of degrees of freedom (fingers and wrist) and possible solutions, and to the constraints imposed by the object. The paper presents an approach to reduce this problem under certain conditions. The configurations of the hand to grasp rectangular parallelepipeds of different sizes are determined off-line using an optimality criterion. The wrist positions of these configurations are approximated with functions of the size of the object "inside" the hand. These functions are stored to be used for a fast on-line computation of a real grasp configuration. The approach has been implemented and numerical examples are presented. *Copyright © 2006 IFAC*

Keywords: Robot hands, grasping and manipulation

1. INTRODUCTION

The grasp of an object performed by a mechanical hand is subject to several considerations like dexterity, equilibrium, stability and dynamic behaviour (Shimoga, 1996). The grasp quality regarding these topics depends on the configuration of the mechanical hand. The use of anthropomorphic mechanical hands has, among others, the advantage of allowing redundant solutions, however, this has the drawback that infinite solutions are possible and finding a "good" solution is a computational hard work (finding a valid solution is already a hard work).

Although there exist methods which, in one way or another, consider the hand kinematics in the grasp computation, for instance Miller, *et al.* (2003) and Chillenato, *et al.* (2003), quite frequently the grasping problem is addressed without considering the mechanical hand and focused only on the object to be grasped, for instance Nguyen (1998), Ponce and Faverjon (1995), Li, *et al.* (2003) and Zhu and Wang (2003), among several others. This may result in an excellent set of finger contact points, ensuring a robust grasp, but due to the limitations imposed by the hand kinematics the grasp may be unreachable. Other works deal with the problem of positioning the hand given the set of grasping contact points, which is equivalent to the inverse kinematics problem. Nevertheless, since the hand kinematics was not considered in the search of the grasp contact points this may result in long iterative processes, first generating a set of contact points for the fingers and, then, trying to find a reachable hand configuration to perform the grasp. This may be a time consuming approach, so learning approaches were proposed to reduce the computation time during a real task execution. As an example, the use of neural nets was presented by Gorce and Rezzoug (2003), but strong assumptions are considered to provide robustness to the system (for instance, the initial error in the wrist orientation is smaller than 90º, so the hand is already "well" oriented, and the initial error in the wrist position is relatively small, just 10 cm). Finding a proper wrist position and orientation is a key point, because with this information solving the inverse kinematics of an anthropomorphic mechanical hand is relatively simple.

In this context, this work proposes a new approach to quickly find, during on-line grasp operations, a hand configuration close to the optimal for the given object according to some predefined criterion. As in the traditional learning approaches, the computational hard work is done off-line, with the difference that in this approach the knowledge is captured as explicit functions that provide the position of the wrist, given the

---

*This work is supported by the CICYT projects DPI2004-03104 and DPI2005-00112, and Acción Integrada HI2005-0290.

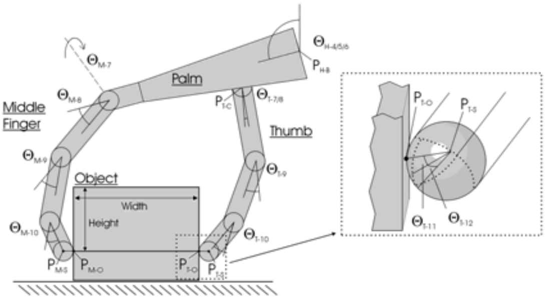

Fig. 1. Schematic of the hand grasping an object defined by its height and width (only two fingers are shown). The inset shows the two virtual articulations in the fingertips.

proper height and width of the object to be grasped. With this information, the on-line determination of the whole hand configuration is simple.

The proposed approach was implemented for the anthropomorphic mechanical hand MA-I, developed and built at the Robotics Lab of the IOC. Numerical examples are included in the paper.

2. PROPOSED APROACH

Let the size of the object portion that the grasp must be able to accept inside the hand be defined by the "object height" and "object width" as shown in figure 1 (the length of the third dimension of the object is not relevant in this type of grasp). The proposed approach can be summarized as follows.

Steps to be done off-line:

1. Determination of the grasp configuration of the hand (wrist and fingers) for objects of different height and width within a desired range and according to some optimality criterion. This is done for a finite number of pairs [height, width] and several valid procedures can be used.
2. Determination of simple functions W_x, W_y, and W_z of the variables [height, width] that approximate, respectively, the X, Y and Z-coordinates of the wrist positions obtained in Step 1 up to a desired precision (a partition of the space height-width into different domains with different functions in each one can be considered).

Steps to be done on-line:

3. Determination of the height and width of the object, or its proper bounding box.
4. Use of the functions W_x, W_y, and W_z to obtain the X, Y and Z-coordinates of the wrist position (it may require the identification of the domain containing the height and width of the object).
5. Determination of the inverse kinematics of the fingers and the orientation of the palm for the wrist position obtained in the previous step.

This approach presents a good balance between the amount of information to be stored and the amount of computation to be done on-line, being both of them relatively small and quite acceptable. Moreover, from the research point of view, one particular advantage of the approach is that the information obtained off-line explicitly shows the behaviour of the hand when a grasp optimization criterion is applied in Step 1, allowing a better understanding of the grasp action and opening new opportunities for further improvements. This is not possible with other techniques where the knowledge is implicitly stored as, for instance, neural nets. The rest of the paper deals with the off-line steps of the approach. The on-line steps are straightforward and they are outside the scope of the paper.

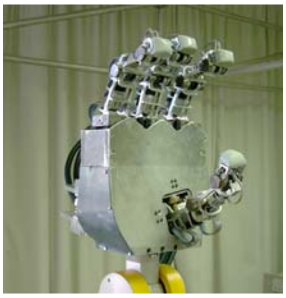

Fig.2. Mechanical Hand MA-I.

2.1 General Considerations

The following general assumptions are initially considered; nevertheless, some of them can be easily relaxed to obtain variations of the approach.

The hand is anthropomorphic with spherical fingertips, the contact with the object can take place at different points of the fingertips, and deformations of the fingertips are not modelled, i.e. "hard contacts" are considered (Nguyen, 1998). The effect of these assumptions is equivalent to consider two virtual degrees of freedom (d.o.f.) at each fingertip, i.e. a virtual joint with two revolute d.o.f. that fixes the contact point on the spherical fingertip (figure 1).

A grasp with any number of fingers can be considered to obtain the wrist information off-line; it depends on the desired type of grasp. In this work the most natural planar grasp is considered, i.e. the thumb pushing against the middle-finger and the index- and ring-fingers giving additional robustness with coplanar contact points. The object is considered to be a rectangular parallelepiped, which in any case is a simple bounding box for any other real object, allowing the detection of possible collisions between the object and the hand just from the object width and height as defined above. Note that there is no constraint in the length of the third dimension of the object. The index-, middle- and ring-finger are initially located on the same face; in the case of real objects that do not allow this condition the grasp will still be possible using only the thumb and the middle-finger, while the other two fingers are most likely to be positioned in acceptable reachable grasp positions.

The determination of the hand configuration (wrist and fingers) for objects of different width and height can be done using different optimality criteria (Mishra, 1995) and techniques, ranging from a guided search to the use of nonlinear programming. In this work, it was decided to use a guided search with the objective function (quality measure) reinforcing the hand configurations with the joint positions as close as possible to the middle of their ranges. This criterion generates natural grasps with relaxed hand configurations far from the mechanical joint limits, besides it is easy to be implemented.

The functions W_x, W_y, and W_z can also be determined in different ways. From experimentation, different "behaviours" of the wrist position for different ratios of the object height and width were clearly identified; thus, a partition of the height-width space was considered in order to generate different domains where simple second order surfaces were enough to approximate the wrist position coordinates with an acceptable precision. Even when a particular hand was used for experimentation, the same behaviours are expected for any other anthropomorphic hand.

The following subsections present the details of the approach, as well as the implementation for the mechanical hand MA-I (Suárez and Grosch, 2004). The hand MA-I has a palm and four fingers: thumb, index-, middle- and ring-finger, and is assembled to the wrist of a six d.o.f. robot arm (figure 2). The developments can be applied to any mechanical hand with the same kinematics structure (anthropomorphic) while the numerical results are, of course, for the actual dimensions of the used hand.

2.2 *Hand related nomenclature*

The following subindices are used along the paper:

H	Hand
F	Any finger, particularized as T, I, M, R for Thumb, Index-, Middle- and Ring-finger

Besides, the following basic nomenclature is used (positions are defined in a world reference system):

$P_{F\text{-}O}$	Contact point of finger F with the object
P^{i}_{F}	Base position of articulation i of finger F
$P_{F\text{-}S}$	Centre position of the sphere in fingertip of finger F (note that $P_{F\text{-}S}=P^{11}_{F}=P^{12}_{F}$)
$P_{F\text{-}C}$	Position of the connection finger-palm
$P_{H\text{-}B}$	Position of the hand-base
$\Theta_{F\text{-}i}$	Angle of articulation i of finger F
$D_{TS\text{-}MS}$	Distance between $P_{T\text{-}S}$ and $P_{M\text{-}S}$
$D_{TC\text{-}MS}$	Distance between $P_{T\text{-}C}$ and $P_{M\text{-}S}$
$D_{TC\text{-}TS}$	Distance between $P_{T\text{-}C}$ and $P_{T\text{-}S}$

In order for the explanation to be independent of the robot arm, it is considered that the base position of the hand, $P_{H\text{-}B}$, is determined by three parameters $X_{H\text{-}B}$, $Y_{H\text{-}B}$ and $Z_{H\text{-}B}$ and the hand orientation is determined by three virtual joints $\Theta_{H\text{-}4}$, $\Theta_{H\text{-}5}$ and $\Theta_{H\text{-}6}$. The values $X_{H\text{-}B}$, $Y_{H\text{-}B}$, $Z_{H\text{-}B}$ and $\Theta_{H\text{-}4}$, $\Theta_{H\text{-}5}$, $\Theta_{H\text{-}6}$ describe the robot wrist configuration (6 d.o.f.) and are obtained from the six real joints of the robot arm using the direct kinematics. $\Theta_{F\text{-}7}$ to $\Theta_{F\text{-}10}$ describe the angles of the finger phalanges, $\Theta_{F\text{-}7}$ being the abduction-adduction d.o.f. Finally, $\Theta_{F\text{-}11}$ and $\Theta_{F\text{-}12}$ represent virtual articulations, describing the position of the contact point on the spherical fingertip (figure 1).

The kinematics chain from $P_{T\text{-}O}$ to $P_{M\text{-}O}$ can be divided into two chains, one formed by the middle finger plus the palm and another formed only by the thumb. The common point of the two chains is $P_{T\text{-}C}$, where the thumb is connected with the palm (figure 1). $P_{T\text{-}C}$ serves as an auxiliary reference-point, and will be used instead of the actual wrist position as reference for the position of the hand.

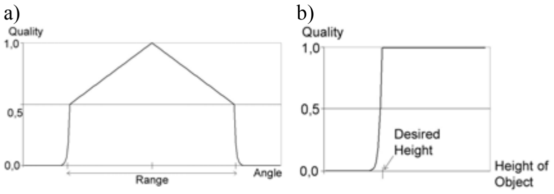

Fig. 3. Quality-function used for: a) the articulations of the hand; b) the object height allowed by the hand configuration.

2.3 *Quality measure of a grasp configuration*

The quality measure of a grasp configuration, Q, used to reinforce the configurations with the joint positions close to the middle of their ranges is computed as

$$Q = Q_{\text{Hand}} \cdot Q_{\text{Fingertips}} \cdot Q_{\text{Height}}$$

Q_{Hand} refers to the quality of the hand configuration considering the articulations of the fingers without the fingertips and is obtained as the product of the individual qualities of the articulations, defined each one such that inside the range of the articulation the quality decreases linearly from 1 at the range centre to 0.5 at the range bound, and outside the range it converges exponentially towards zero (figure 3a).

$Q_{\text{Fingertips}}$ is obtained as the product of the individual qualities of the virtual articulations in the fingertips, defined each one such that inside the range the quality-function decreases squarely from 1 at the range centre to 0.5 at the range bound, and outside the range it converges exponentially towards zero. Small changes in the finger articulations can result in a very large change in the virtual articulations of the fingertips; this is why the function used for the finger joints is slightly modified for the fingertips.

Q_{Height} indicates whether the hand configuration allows enough room inside it for the desired object height. It could be enough to use a binary variable indicating whether the hand configuration allows the desired height or not, but the search for an optimal configuration described below requires a continuous function to ensure convergence. Therefore, $Q_{\text{Height}} = 1$ if the desired object height is reached and exponentially decreases towards 0 otherwise (figure 3b).

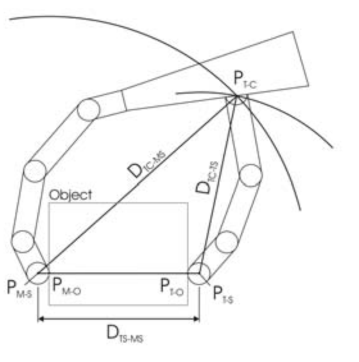

Fig.4. Initial position of the hand ($P_{T\text{-}C}$).

2.4 *Determination of the hand grasp configuration for a given object size*

For simplicity and without loss of generality, the grasp plane is assumed to be horizontal. For each pair [height, width] the hand grasp configuration is determined in two phases:

a) Determination of an initial hand configuration considering only the thumb, the middle-finger and the palm. Includes the determination of:
   (*i*) Initial position of $P_{T\text{-}C}$.
   (*ii*) Inverse kinematics of middle-finger and palm.
   (*iii)* Orientation of the wrist.
   (*iv*) Inverse kinematics of the thumb
   (*v*) Fingertip contact points of middle-finger and thumb: angles $\Theta_{F\text{-}11}$ and $\Theta_{F\text{-}12}$.
   (*vi*) Position of the hand-base $P_{H\text{-}B}$.
   (*vii*) Configuration of the index- and ring-fingers.

b) Guided search for the "optimal" position of $P_{T\text{-}C}$ according to the objective function Q. Includes:
   (*viii*) Search for an optimal configuration using Q.

Steps (*i*) to (*viii*) are solved as follows.

(*i*) *Initial position of* $P_{T\text{-}C}$. The positions of $P_{M\text{-}O}$ and $P_{T\text{-}O}$ are determined from the object width, and from them, $P_{M\text{-}S}$ and $P_{T\text{-}S}$ are directly obtained (figure 4). Then, $P_{T\text{-}C}$ is initially determined as the intersection of two circles orthogonal to the grasp plane centred on $P_{M\text{-}S}$ and $P_{T\text{-}S}$; the radius of the first one being $D_{TC\text{-}MS\_min}+(D_{TC\text{-}MS\_max}-D_{TC\text{-}MS\_min})\ D_{TS\text{-}MS}\ /\ D_{TS\text{-}MS\_max}$ and the radius of the second given by an equivalent expression replacing $D_{TC\text{-}MS}$ by $D_{TC\text{-}TS}$ (the subindices *max* and *min* refer to the maximum and minimum values of the variables). In this way both radius are proportional to the object width and the position of $P_{T\text{-}C}$ is properly adjusted. It must be noted that this is just an initial value for $P_{T\text{-}C}$, the ranges of the joints were not yet considered so trying to place the fingers on the desired object contact points may need some joints to be outside their reachable ranges (in particular $\Theta_{T\text{-}7}$ is likely to be out of range). Intersections of the hand with the object are also not yet considered. Fixing this point in advance results in a reduction of the computing time in the subsequent search.

(*ii) Inverse kinematics of the middle-finger and palm.* There are infinite solutions, thus, in order to consider the most natural hand configuration it is selected $\Theta_{M\text{-}7}=0$, i.e. the middle-finger aligned with the palm. The middle-finger joints $\Theta_{M\text{-}8}$, $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ are coplanar and then infinite solutions are possible again. Taking an initial solution with $\Theta_{M\text{-}8}=\Theta_{M\text{-}9}=\Theta_{M\text{-}10}$, it results that $\Theta_{M\text{-}8}$ is the unique variable in the Euclidean distance from $P_{T\text{-}C}$ to $P_{M\text{-}S}$, and even when $\Theta_{M\text{-}8}$ cannot be explicitly solved it can be approximated using a fifth-order polynomial function ($\Theta_{M\text{-}8}$ could be also obtained using the Newton's iterative method, however, this produces more iterative calculations). The obtained $\Theta_{M\text{-}8}$ is used to compute the exact values of $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ from the middle-finger inverse kinematics using simple trigonometric replacements.

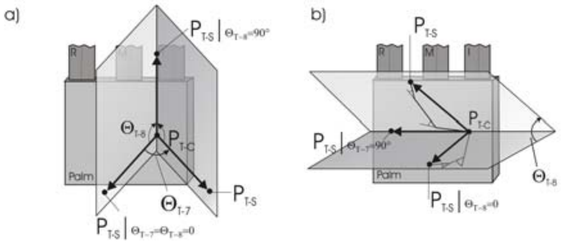

Fig. 5.Schematics showing: a) the two planes for the determination of $\Theta_{T\text{-}7}$; b) the determination of $\Theta_{T\text{-}8}$.

(*iii*) *Orientation of the wrist.* $\Theta_{H\text{-}4}$ and $\Theta_{H\text{-}5}$ must be determined according to the current location of $P_{T\text{-}C}$. The known data are $P_{M\text{-}S}$, $P_{T\text{-}C}$ and $P_{M\text{-}S}|_{\Theta_{H\text{-}4}=\Theta_{H\text{-}5}=0}$. Then $\Theta_{H\text{-}4}$ and $\Theta_{H\text{-}5}$ are obtained solving the system:

$$\left(P_{M-S}-P_{T-C}\right)=R_{\Theta_{H-4}}\cdot R_{\Theta_{H-5}}\cdot\left(P_{M-S}\big|_{\Theta_{H-4}=\Theta_{H-5}=0}-P_{T-C}\right)$$

where $R_{\Theta_{H\text{-}4}}$ and $R_{\Theta_{H\text{-}5}}$ are the rotation matrices for the rotating axis of $\Theta_{H\text{-}4}$ and $\Theta_{H\text{-}5}$.

(*iv*) *Inverse kinematics of the thumb.* The three angles of the thumb $\Theta_{T\text{-}8}$, $\Theta_{T\text{-}9}$ and $\Theta_{T\text{-}10}$ are computed as those of the middle-finger to agree with the distance $D_{TC\text{-}TS}$. $\Theta_{T\text{-}7}$ is the angle between the plane swept by the thumb when moving $\Theta_{T\text{-}8}$ for $\Theta_{T\text{-}7}=0$ and the plane defined by $P_{T\text{-}S}$ and the rotation axis of $\Theta_{T\text{-}7}$ (figure 5a). Then, analogously, $\Theta_{T\text{-}8}$ is recomputed as the angle between the plane swept by the thumb when moving $\Theta_{T\text{-}7}$ for $\Theta_{T\text{-}8}=0$ and the plane defined by $P_{T\text{-}S}$ and the rotation axis of $\Theta_{T\text{-}8}$ (figure 5b).

(*v*) *Fingertip contact points of middle-finger and thumb* (angles $\Theta_{F\text{-}11}$ and $\Theta_{F\text{-}12}$). The two virtual articulations describing the actual contact point on the fingertip spheres of the middle-finger and the thumb ($\Theta_{T\text{-}11}$, $\Theta_{T\text{-}12}$, $\Theta_{M\text{-}11}$, $\Theta_{M\text{-}12}$) can now be determined from the position and the orientation of the palm and the positions of the other joints of the middle-finger and the thumb. These angles are easily computed using trigonometric functions and the distances (see figure 6): D_1 from $P_{F\text{-}O}$ to the *xy*-plane, D_2 from $P_{F\text{-}O}$ to the *xz*-plane, and D_3 from $P_{F\text{-}O}$ to the *yz*-plane.

(*vi*) *Position of the hand-base.* The computation of the position of the hand base, $P_{H\text{-}B}$, is straightforward from the palm position and orientation given by $P_{T\text{-}C}$ from step (*i*), and $\Theta_{H\text{-}4}$ and $\Theta_{H\text{-}5}$ from step (*iii*).

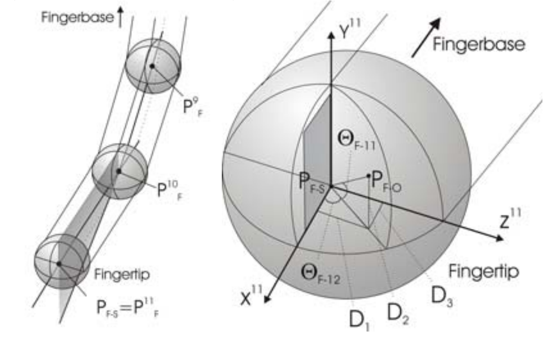

Fig. 6. a) Plane formed by the last two phalanges of the finger F. b) Schematics showing the distances D_1, D_2 and D_1 necessary for the determination of $\Theta_{F\text{-}11}$ and $\Theta_{F\text{-}12}$.

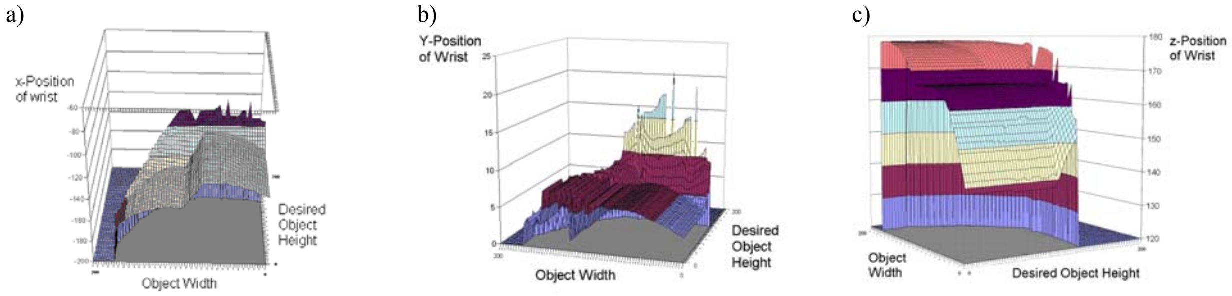

Fig. 7. a-c) X, Y and Z-coordinates of the wrist position.

(*vii*) *Configuration of the index- and ring-fingers.* Once the configuration of the middle-finger and the thumb have been completely determined, the angles $\Theta_{I\text{-}8}$ to $\Theta_{I\text{-}12}$ and $\Theta_{R\text{-}8}$ to $\Theta_{R\text{-}12}$ of the index- and ring-fingers can be determined in a similar way as it was described for the middle-finger. The only difference now is that the distances of interest $D_{IB\text{-}IS}$ and $D_{RB\text{-}RS}$ are the distances from $P_{I\text{-}S}$ and $P_{R\text{-}S}$ to the base of the corresponding finger $P^8{}_I$ and $P^8{}_R$.

(*viii*) *Search for an optimal configuration.* As mentioned before, the heuristic method used to determine the initial value of $P_{T\text{-}C}$ from step (*i*) does not necessarily result in a reachable hand configuration with all the joint angles inside their real ranges. Nevertheless, it is a very good start position for a guided search of the optimal configuration using the quality function from Subsection 2.3. The search is done with a simple greedy algorithm that iteratively compares the quality of the solution for the current $P_{T\text{-}C}$ with those for values of $P_{T\text{-}C}$ in the neighbourhood; the best solution is selected in each iteration, improving the position of the $P_{T\text{-}C}$ and the resulting hand configuration until no further improvement is found (theoretically, local maxima could be reached, but this effect was not detected in the real executions).

2.5 Determination of the functions W_x, W_y, and W_z

The functions W_x, W_y, and W_z of the variables [height, width] that approximate, respectively, the *X*, *Y* and *Z*-coordinates of $P_{T\text{-}C}$ (the hand reference position) are obtained as follows. Once the optimal configuration of the hand has been determined for a number of pairs [width, height], the obtained *X*, *Y* and *Z* coordinates of $P_{T\text{-}C}$ are stored as three sets of points [width, height, *X*], [width, height, *Y*] and [width, height, *Z*]. Then, these sets of points are approximated with second order functions as,

$$X_{approx} = \alpha_6 \cdot Height^2 + \alpha_5 \cdot Width^2 + \alpha_4 \cdot Height \cdot Width + \alpha_3 \cdot Height + \alpha_2 \cdot Width + \alpha_1$$

for the *X* coordinate, and equivalent expressions for *Y* and *Z*. The second order surface is estimated from the corresponding sets of points using least squares. This is done solving the over-determined system of equations $B \cdot A = \alpha$, where *A* is the $n \times 6$ matrix

$$A = \begin{bmatrix} Height_1^2 & & Height_i^2 & & Height_n^2 \\ Width_1^2 & & Width_i^2 & & Width_n^2 \\ Height_1 Width_1 & \cdots & Height_i Width_i & \cdots & Height_n Width_n \\ Height_1 & & Height_i & & Height_n \\ Width_1 & & Width_i & & Width_n \\ 1 & & 1 & & 1 \end{bmatrix}$$

with *i* being the index of the *n* pairs [width$_i$, height$_i$], *B* is the column vector of the known values of each coordinate (e.g. [X_1 … X_i…X_n] for *X*), and α is the vector $[\alpha_6 \ \dots \ \alpha_1]^T$ of the second order equation parameters. These systems can be solved using the backslash operator of Matlab, so the vectors α describing each second order surface are easily computed for each coordinate *X*, *Y* and *Z*.

In order to get a good approximation with second order surfaces, the width-height space is partitioned into regions using the gradient method so that:

- The error in the approximated function with respect to each known point becomes smaller than a given threshold;
- The *X*, *Y* and *Z* coordinates of position $P_{T\text{-}C}$ approximated by the second order function allows a kinematics solution for all considered points.

The result is a set of regions with their own second order functions for the *X*, *Y* and *Z* coordinates of $P_{T\text{-}C}$. With this partition, the second order surfaces were found to be good enough for a useful approximation of the hand reference position.

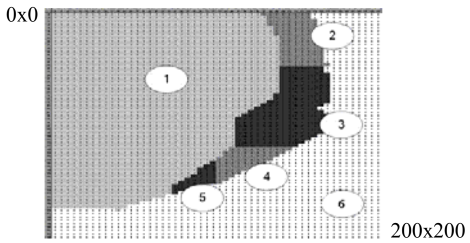

Fig. 8. Division of the width-height space into five approximation regions.

3. NUMERICAL RESULTS

The proposed methodology was implemented in C++ and applied to the mechanical hand MA-I, considering object sizes up to 200×200 mm with samples on a regular mesh with a resolution of 1×1mm, i.e. 40000 samples. Figure 7a to 7c show respectively the *X*, *Y* and *Z* coordinates of $P_{T\text{-}C}$ of the obtained grasps for the given height and width ranges. The partition of the width-height space generated six approximation regions (figure 8). Figure 9a shows the quality *Q* of the optimal grasp configuration for each pair [height, width], and figure 9b the quality *Q* of the grasps obtained using the approximated $P_{T\text{-}C}$, (i.e. the grasps obtained on-line using the corresponding functions W_x, W_y, and W_z). Three parts can be distinguished. Part 1 with *Q*=0 indicates that in this region no solution is found, i.e. an object with the given width can not be grasped allowing the given height inside the hand. Part 2 shows that Q is constrained by fulfilling the desired height of the object and the quality decreases rapidly, meaning that the hand approaches more extreme configurations for large widths or heights. In part 3 Q is constant for a given width regardless of the given height, meaning that the optimal grasp configuration of the hand remains the same for objects with a height smaller than an extreme value, i.e. the space allowed inside the hand for these optimal grasp configurations is larger than necessary.

As a graphical example, figure 10 shows the obtained optimal configuration of the hand for an object with width = 80mm and height = 100mm (the third dimension of the object is not relevant); the optimal grasp actually allows an object height of 102.64mm (this case belongs to part 3 in figure 9).

4. DISCUSSION

Determining the hand configuration to perform a grasp is a hard work due to the high number of involved degrees of freedom with a tree distribution. Including any optimization makes it even harder. Dealing with this problem, this paper proposed a method to off-line parameterize the hand position as a function of the size of the portion of the object to be "inside" the hand. The position of a hand reference point is stored without consuming a lot of memory, and, when necessary, the on-line determination of the hand configuration can be done using this information without intensive computation, just knowing the actual object size. The advantage of the proposed approach compared to other ways of learning and storing information, like for instance neural nets, is that the behaviour of the hand reference point becomes explicit for a given type of grasp, object size or grasp optimality criterion. This knowledge could be used to improve the hand motion planning to perform the grasp and to plan more complex grasps, considering the hand kinematics. These are still open topics with very few relevant contributions. Even when the off-line grasp analysis was done considering a polyhedral object, due to the natural position of the hand with the finger joints close to the middle of their ranges, the obtained reference position of the hand can be used to on-line determine the hand configuration for other different practical cases (e.g., grasps with non-coplanar contact points). Of course, the validity of the solution depends on how different the intended grasp is from the nominal ones used in the off-line analysis. However, due to the complexity of the hand kinematics and the infinite number of possibilities, a formal quantization of this validation is not simple and was not yet done, remaining as future work.

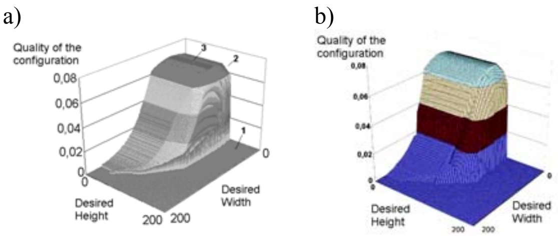

Fig. 9.a) Quality of the optimal grasp configurations: (1) no solution found, (2) the solution is dominated by the object height, (3) the solution is dominated by the object width; b) Quality of the grasp configurations obtained using the approximation of $P_{T\text{-}C}$.

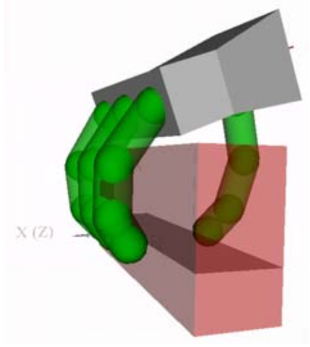

Fig. 10. Graphical example of an optimal grasp.

The proposed approach was implemented for the anthropomorphic mechanical hand MA-I, considering the 16 degrees of freedom of the fingers plus 8 virtual joints at the fingertips and the 6 degrees of freedom of the wrist. Numerical results of the implementation were included in the paper.

REFERENCES

Chillenato, Fisher, Morales and Del Pobil (2003). Ranking planar grasps configurations for a three-finger hand. *IEEE ICRA*, Taipei, pp.1133-1138.

Gorce, P. and N. Rezzoug (2003). A method to learn hand grasping posture from noisy sensing information. *Robotics*, **22**, pp.309-318.

Li, J.W., H. Liu and H.G. Cai (2003). On Computing Three-Finger Force-Closure Grasps of 2-D and 3-D Objects. *Trans. on Robotics and Automation*, **19**(1), pp.155-161.

Miller, A.T., S. Knoop, H.I. Christensen and P. Allen (2003). Automatic Grasp Planning Using Shape Primitives. *IEEE ICRA*, Taipei, pp.1824-1829

Mishra, B. (1995). Grasp Metrics, Optimality and Complexity. *Algorithmic Foundations of Robotics*, pp. 137-166, Wellesley, MA.

Nguyen, V.D. (1998). Constructing Force-Closure Grasps. *Int. Journal Robotic Research*, **7**(3), 3-16.

Ponce, J. and B. Faverjon (1995). On Computing Three Finger Force-Closure Grasp of Polygonal Objects. *IEEE Trans. on Robotics and Automation*, **11**(6), pp.868-881.

Shimoga, K.B. (1996). Robot Grasp Synthesis Algorithms: A Survey. *Int. Journal of Robotics Research*, **15**(3), 230-266.

Suárez, R. and P. Grosch (2004). Dexterous Robotic Hand MA-I, Software and Hardware Architecture. *Intelligent Manipulation & Grasping Int.Conf.*, Genova, pp91-96.

Zhu, X. and J. Wang (2003). Synthesis of Force-Closure Grasps on 3-D Objects Based on the Q Distance. *Trans. on Robotics and Automation*, **19**(4), pp.669-679.