Determining Compliant Motions for Planar Assembly Tasks in the Presence of Friction*

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Abstract: *The force-compliant control is an important aspect for performing assembly tasks with robots, since the geometric constraints of the task are usually used as guides. This paper presents the specification of compliant motions for planar assembly tasks (two degrees of freedom of translation and one of rotation), taking into account the effect of friction.*

1 Introduction

The automatic execution of assembly tasks with robots requires the generation of fine-motion plans in order to successfully perform the tasks despite the uncertainties that may affect them [5][10]. Fine-motion plans use the geometric constraints of the task to guide the mobile object towards the goal. Therefore it is necessary to specify compliant motions to be executed by the robot while the mobile object moves in contact with the fixed objects [6]. This paper deals with the specification of compliant motions for planar assembly tasks (two degrees of freedom of translation and one of rotation) taking into account the effect of friction.

The motion of an object in contact with fixed objects in the environment has been studied in different ways by several authors. Brost and Mason [1] use the dual representation of forces to represent the forces and the contact constraints as acceleration centers. By using geometric computations in the dual plane, they characterize the object contact modes by means of regions representing the sets of forces that are consistent with a set of motions. They consider planar assembly tasks with friction. Rajan et al. [8] develop an algebraic analysis that characterizes the contact modes of planar assembly tasks with friction. Each contact mode is defined by the directions of motion of each contact point in the manipulated object. Erdmann [2] analyzes of friction for planar assembly tasks by embedding into the Configuration Space the force constraints that define the classic Coulomb friction cone in the physical space. His method allows the determination of the possible motions of an object subjected to an applied force and torque. Other related works are Hirai and Asada's approach [3] which introduces the polyhedral convex cones (PCC) for the analysis of the kinematics and statics of manipulation in 6 d.o.f; there is also Paul and Ikeuchi's approach [7] that uses the dual representation and the PCC method for partitioning the contact space in a finite number of states for planar assembly tasks.

Section 2 of the paper analyzes the task constraints for contact situations involving a different number of basic contacts in a simple and graphical way [9]. Section 3 presents the specification of compliant motions for these contact situations and, finally, section 4 presents the conclusions of the work.

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2 Contact situations

Definition 1: *A contact situation* between two rigid polygonal objects is the simultaneous occurrence of a given set of basic contacts, which can be of two types: an edge of the mobile object against a vertex of a fixed object (type-A), or a vertex of the mobile object against an edge of a fixed object (type-B).

Let $\{W\}$ and $\{T\}$ be the reference frames attached to the workspace and to the mobile object, respectively. $\{T\}$ has the origin at the mobile object reference point, and an orientation ϕ with respect to $\{W\}$.

Definition 2: The *Configuration Space* of a mobile object is the space $\mathcal{C}$ of all the possible configurations of the object, a configuration being specified by the position and orientation of $\{T\}$ with respect to $\{W\}$ [4].

For movements in the plane with two degrees of freedom of translation and one of rotation, $\mathcal{C}$ is $\mathbf{R}^2 \times S^1_\rho$, where S^1_ρ is the circle of radius ρ, the gyration radius of the moving object. As a result, any configuration is described with three generalized coordinates (x, y, q), with $q = \rho\phi$, all having units of length.

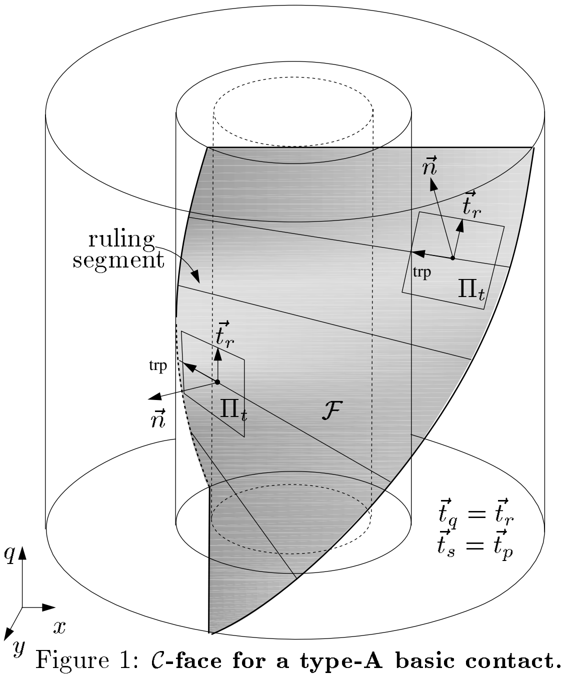

Figure 1: **$\mathcal{C}$-face for a type-A basic contact.**

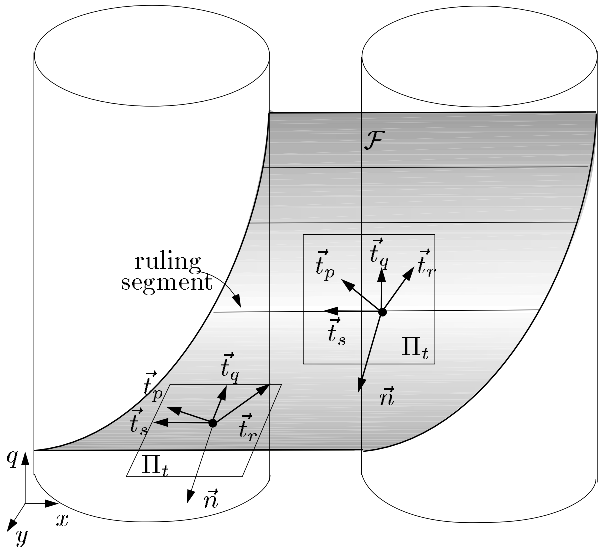

Figure 2: **$\mathcal{C}$-face for a type-B basic contact**

2.1 One basic contact

Definition 3: The $\mathcal{C}$*-face* $\mathcal{F}_i$ corresponding to a basic contact i is the set of configurations in $\mathcal{C}$ where the basic contact takes place, when only considering the constraints imposed by the edges and vertices involved in the basic contact, i.e. the contact edge and the edges adjacent to the contact vertex.

Proposition 1: *The $\mathcal{C}$-face corresponding to a given basic contact is a ruled surface whose ruling segments are parallel to the xy-plane, have the same length as the contact edge and have their extremes on two helices over two cylinders* [9].

Corollary: For a type-A basic contact the ruling segments are tangent to a cylinder concentrical with the supporting ones, and with radius equal to the distance from the reference point to the line containing the contact edge (figure 1).

Corollary: For a type-B basic contact the ruling segments are parallel (figure 2).

Let us define:

d_W,d_T: the distances in physical space from the straight line that contains the contact edge to the origins of $\{W\}$ and $\{T\}$, respectively.

ψ_W: the orientation with respect to $\{W\}$ of the normal to the contact edge. For type-B basic contacts, it is constant, and for type-A basic contacts it depends on the orientation of the mobile object ($\psi_W = \psi_T + \phi + \pi$, where ψ_T is the orientation with respect to $\{T\}$ of the normal to the contact edge).

D: the distance from the origin of $\{W\}$ to the projection on the xy-plane of the line that supports the ruling segment corresponding to a given mobile object orientation ϕ.

For a type-A basic contact D is given by (figure 3a):

$$D = x_m \cos\psi_W + y_m \sin\psi_W + d_T \tag{1}$$

where (x_m, y_m) are the coordinates of the contact vertex of the fixed object measured in $\{W\}$. For type-B basic contacts D is given by (figure 3b):

$$D = h\cos(\psi_W + \pi - \gamma - \phi) + d_W \tag{2}$$

where h and γ are respectively the module and orientation of the vector defining the contact vertex of the mobile object with respect to $\{T\}$.

Therefore, for a basic contact i the straigtht line that contains the ruling segment of the $\mathcal{C}$-face for a given orientation ϕ is:

$$\begin{aligned} x\cos(\psi_{Wi}) + y\sin(\psi_{Wi}) &= D_i \\ q &= \rho\phi \end{aligned} \tag{3}$$

D_i being defined by equation (1) or (2), depending on the type of basic contact.

2.2 Two basic contacts

Definition 4: The $\mathcal{C}$-edge $\mathcal{E}_{ij}$ corresponding to a contact situation involving two basic contacts i and j is the set of configurations in $\mathcal{C}$ where the contact situation takes place, only considering the constraints

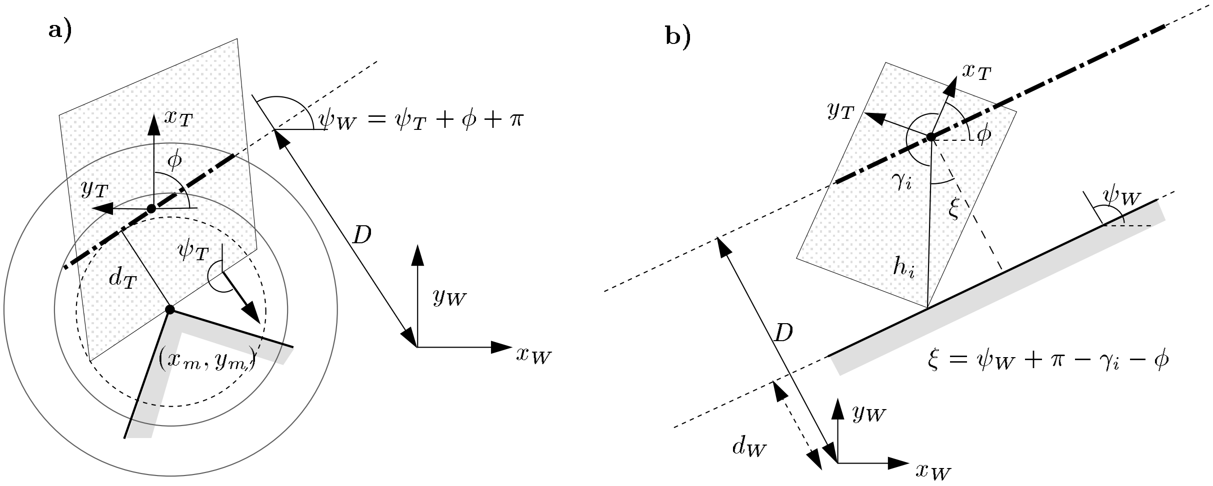

Figure 3: **Distance D to the origin of $\{W\}$ of the projection on the xy-plane of the line that supports the ruling segment for a given orientation corresponding to a type-A basic contact (a) and to a type-B basic contact (b).**

imposed by the edges and vertices involved in both contacts.

Proposition 2: *Since $\mathcal{E}_{ij}$ lies on the intersection of the straight lines containing the ruling segments (equation (3)), then $\mathcal{E}_{ij}$ is a segment of the curve $\mathcal{L}_{ij}$ given by:*

$$\begin{aligned} x &= \frac{D_i \sin\psi_{Wj} - D_j \sin\psi_{Wi}}{\sin(\psi_{Wj} - \psi_{Wi})} \\ y &= -\frac{D_i \cos\psi_{Wj} - D_j \cos\psi_{Wi}}{\sin(\psi_{Wj} - \psi_{Wi})} \\ q &= \rho\phi \end{aligned} \tag{4}$$

if $\sin(\psi_{Wj} - \psi_{Wi}) \neq 0$, otherwise it is a segment of the straight line defined by equation (3) for the orientation that satisfies $D_i = D_j$[9].

3 Determining motion commands

3.1 Force-compliant control

In order to be able to automatically execute assembly tasks with robots in the presence of uncertainty, fine-motion planners make use of active compliance strategies based on sensorial feedback: when contact exists between the mobile object and the fixed objects, the robot complies with the reaction force while moving along the projection of the commanded direction into the subspace orthogonal to the reaction force [6]. In this work, the force-compliant control based on the generalized damping model, represented by the following equation, is assumed:

$$\vec{f}(t) = B(\vec{v}_c(t) - \vec{v}_o(t)) \tag{5}$$

where $\vec{f}(t)$ is the actual reaction force exerted by the fixed objects on the mobile object at time t, B is the damping matrix, $\vec{v}_o(t)$ the actual velocity of the mobile object, and $\vec{v}_c(t)$ the commanded velocity.

By using the generalized coordinates introduced in section 2 to describe the configuration of the mobile object, the force $\vec{f} = (Fx, Fy)$ and the torque τ acting on the object can be represented by the generalized force vector $\vec{g} = (F_x, F_y, F_q)$, with $F_q = \tau/\rho$.

Since ρ is the radius of gyration, the inner product defining orthogonality is the same as the one defining kinetic energy, thus ensuring that orthogonality makes physical sense [2].

3.2 Directions of motion

Definition 6: The *tangent plane* Π_t associated to a given contact configuration c_o of a basic contact i, is the plane tangent to $\mathcal{F}_i$ at c_o.

Π_t is defined by the direction $\vec{n}$ normal to the $\mathcal{C}$-face at the contact point given by:

$$\vec{n} = \frac{1}{\Delta_n}(n_x, n_y, n_q/\rho) \tag{6}$$

where

$$\begin{aligned} n_q &= n_x r_y - n_y r_x \\ \Delta_n &= \sqrt{1 + (\frac{n_q}{\rho})^2} \end{aligned}$$

with $(n_x, n_y) = (cos\psi_W, sin\psi_W)$ being the normal to the contact edge, and (r_x, r_y) the vector from the contact point to the mobile object reference point.

For any contact configuration, the directions of movement that instantaneously maintain the contact are those that belong to the tangent plane. The following directions belonging to the tangent plane Π_t will be of interest (figures 1 and 2) :

a) *Direction* $\vec{t}_r$: Direction of pure rotation about the contact point:

$$\vec{t}_r = (-r_y, r_x, \rho) \tag{7}$$

A positive motion along $\vec{t}_r$ corresponds to a rotation that increases the orientation ϕ of the moving object.

b) *Direction* $\vec{t}_p$: Direction perpendicular to $\vec{t}_r$, being $[\vec{t}_r, \vec{t}_p, \vec{n}]$ a right-handed frame.

c) *Direction* $\vec{t}_s$: Direction of pure sliding:

$$\vec{t}_s = (n_y, -n_x, 0) \tag{8}$$

d) *Direction* $\vec{t}_q$: Direction perpendicular to $\vec{t}_s$. A positive motion along $\vec{t}_q$ corresponds to a rotation that increases ϕ.

The sense of $\vec{t}_s$ is such that the frame $[\vec{t}_q, \vec{t}_s, \vec{n}]$ is a right-handed frame

Definition 7: The *contact reference frame* is the orthogonal reference frame $[\vec{t}_r, \vec{t}_p, \vec{n}]$ with its origin at the contact configuration.

The contact reference frame allows the vectorial decomposition of an applied force in order to analyze its effect on the movement of the mobile object [2]. Figures 1 and 2 show the contact reference frame for type-A and type-B basic contacts, respectively, corresponding to different contact configurations. In each contact configuration, the tangent plane Π_t is drawn together with the directions $\vec{t}_r$, $\vec{t}_p$, $\vec{t}_s$ and $\vec{t}_q$.

The effect of friction for planar assembly tasks in the tridimensional Configuration Space $\mathcal{C}$ has been studied in depth by Erdamnn [2], who introduces the generalized friction cone.

Definition 8: The *generalized friction cone* is the range of possible generalized reaction force directions arising from a basic contact in a given contact configuration.

The generalized friction cone is a bidimensional cone in $\mathcal{C}$, determined by $\vec{n} \pm \mu\vec{v}_f$, $\vec{n}$ being the direction normal to the $\mathcal{C}$-face defined in equation (6), $\vec{v}_f$ the generalized friction vector, and μ the friction coefficient:

$$\vec{v}_f = (n_y, -n_x, v_q/\rho) \tag{9}$$

with $v_q = n_x r_x + n_y r_y$. The unitary vectors in the directions defined by $\vec{n}+\mu\vec{v}_f$ and $\vec{n}-\mu\vec{v}_f$ will be noted by $\vec{e}^{\,+}$ and $\vec{e}^{\,-}$, respectively.

Definition 9: The *friction plane* Π_f is the plane that contains the generalized friction cone.

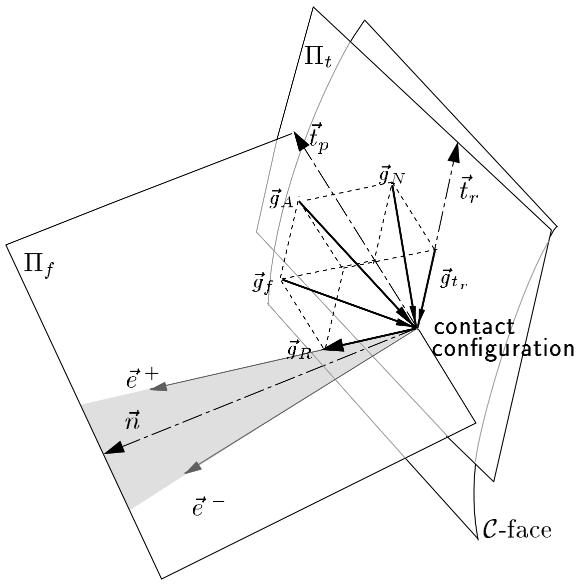

Figure 4: **Force decomposition.**

The direction normal to Π_f is the direction of pure rotation $\vec{t}_r$.

The effect of an applied force when the mobile object is in a one-point contact with the environment can be analyzed by decomposing that force, making use of the contact reference frame. As a result, a net force in the movement direction and a reaction force are obtained.

Let $\vec{g}_A$ be the applied generalized force that points into the $\mathcal{C}$-face associated to the basic contact. $\vec{g}_A$ can be decomposed in the following way

$$\vec{g}_A = \vec{g}_f + \vec{g}_{t_r} \tag{10}$$

$\vec{g}_f$ being the component on the plane Π_f and $\vec{g}_{t_r}$ the component along the direction $\vec{t}_r$, perpendicular to Π_f.

Proposition 3: *The reaction force $\vec{g}_R$ produced in a basic contact is $\vec{g}_R = -\vec{g}_f$ if $\vec{g}_f$ is inside the generalized friction cone, or the negated projection of $\vec{g}_f$ along $\vec{t}_p$ onto the edge of the friction cone, otherwise* [2].

Proposition 4: *The net force $\vec{g}_N$ that defines the movement direction is the projection of $\vec{g}_A$ along the direction determined by $\vec{g}_R$ into the plane Π_t* [2].

3.3 Motion commands that maintain one basic contact

In order to cope with uncertainty, fine-motions use the geometric constraints to guide the motion of the

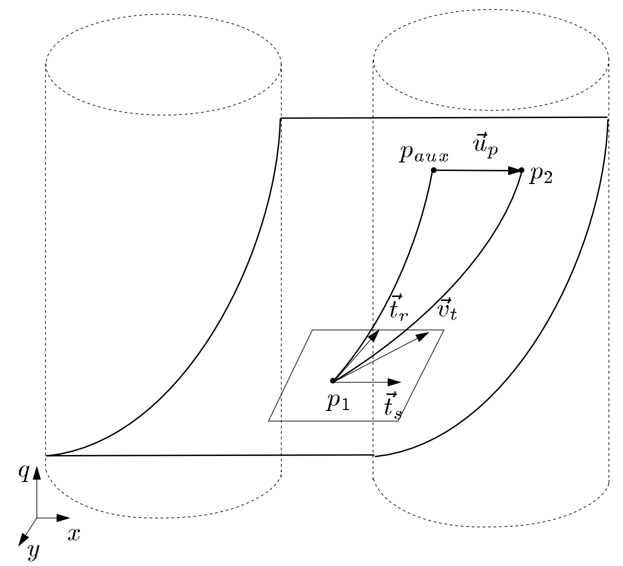

Figure 5: **Definition of motion directions over a type-B $\mathcal{C}$-face.**

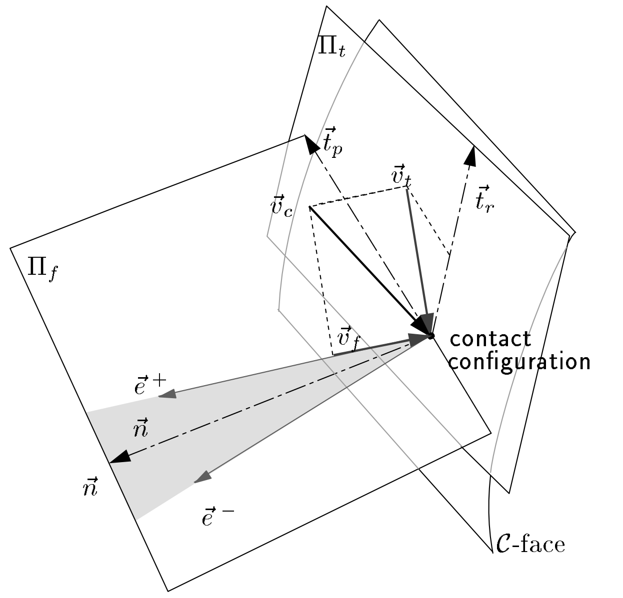

Figure 6: **Commanded velocity decomposition.**

mobile object towards its goal, i.e. the commanded velocity $\vec{v}_c$ must allow the motion of the mobile object while maintaining contact with the fixed objects:

$$\vec{v}_c = \vec{v}_t + \vec{v}_f \tag{11}$$

where

- $\vec{v}_t$ is the component that allows the motion towards the goal, moving the mobile object reference point tangentially over the $\mathcal{C}$-face; it is on the desired motion direction over the tangent plane.
- $\vec{v}_f$ is the component that allows the contact maintenance, producing a reaction force; it is in the opposite direction of the edge of the generalized friction cone which is determined by the motion direction.

The magnitudes v_t and v_f depend on the desired velocity and on the desired reaction force, respectively.

Let us define (figure 5):

- p_1 the start configuration (x_1, y_1, q_1).
- p_2 the desired stop configuration (x_2, y_2, q_2).
- p_{aux} the configuration obtained by rotating the object an angle $\alpha_r = (q_2 - q_1)/\rho$ around the contact point at configuration p_1:

$$\begin{aligned} x_{aux} &= x_1 + r_x \cos\alpha_r - r_y \sin\alpha_r \\ y_{aux} &= y_1 + r_x \sin\alpha_r + r_y \cos\alpha_r \\ q_{aux} &= q_2 \end{aligned} \tag{12}$$

- $\vec{u}_p$ the vector from p_{aux} to p_2:

$$\vec{u}_p = (x_2 - x_{aux}, y_2 - y_{aux}, 0) \tag{13}$$

Proposition 5: *For one basic contact:*

$$\vec{v}_t = \frac{\alpha_r \vec{t}_r + \alpha_s \vec{t}_s}{\mid \alpha_r \vec{t}_r + \alpha_s \vec{t}_s \mid} v_t \tag{14}$$

where $\alpha_s = (\vec{u}_p \cdot \vec{t}_s)$ *and* $\alpha_r = (q_2 - q_1)/\rho$.

Proof: The direction of rotation about the contact point is $\vec{t}_r$ and the amount to be rotated is α_r. The direction of pure sliding is $\vec{t}_s$ and the amount to be translated in this direction is α_s, since the rotation about the contact point also translates the reference point. The proposed $\vec{v}_t$ is the composition of both motions. $\diamond$

Proposition 6: *For one basic contact:*

$$\vec{v}_f = \begin{cases} -v_f \vec{e}^{\,-} & if \quad \vec{v}_t \cdot \vec{t}_p > 0 \\ -v_f \vec{e}^{\,+} & otherwise \end{cases}$$

Proof: When the commanded velocity points towards the tangent plane, a reaction force arises and the mobile object moves along an instantaneous direction of motion over the tangent plane. If the object motion has a positive component along the direction $\vec{t}_p$, then the reaction force will have the direction $\vec{e}^{\,-}$, or $\vec{e}^{\,+}$ otherwise. The proposed $\vec{v}_f$ is in the opposite direction. $\diamond$

Figure 6 shows the decomposition of the commanded velocity $\vec{v}_c$ in its components.

3.4 Motion commands that maintain two basic contacts

For a contact situation involving two basic contacts the commanded velocity is decomposed in the same way as in the one basic contact case, i.e. $\vec{v}_c = \vec{v}_t + \vec{v}_f$.

Proposition 7: *For two basic contacts $\vec{v}_t$ is the tangent to the curve $\mathcal{L}_{ij}$ at the contact configuration.*

Proof: Since $\vec{v}_t$ must be tangent to both $\mathcal{C}$-faces, it is the direction tangent to $\mathcal{L}_{ij}$, because the intersection $\mathcal{E}_{ij}$ of both $\mathcal{C}$-faces is an arc of $\mathcal{L}_{ij}$. ⋄

As an example, $\vec{v}_t$ has the following expression for two type-B basic contacts satisfying $\sin(\psi_{Wj} - \psi_{Wi}) \neq 0$:

$$\vec{v}_t = \frac{(t_x, t_y, t_q)}{\sqrt{t_x^2 + t_y^2 + t_q^2}} v_t \tag{15}$$

with

$$t_x(\phi) = \frac{\frac{dD_i}{d\phi}\sin\psi_{Wj} - \frac{dD_j}{d\phi}\sin\psi_{Wi}}{\sin(\psi_{Wj} - \psi_{Wi})} \tag{16}$$

$$t_y(\phi) = -\frac{\frac{dD_i}{d\phi}\cos\psi_{Wj} - \frac{dD_j}{d\phi}\cos\psi_{Wi}}{\sin(\psi_{Wj} - \psi_{Wi})}$$

$$t_q(\phi) = \rho \tag{17}$$

where $\frac{dD_k}{d\phi} = h_k \sin(\psi_{Wk} + \pi - \gamma_k - \phi)$ with $k = i, j$.

Proposition 8: *For two basic contacts:*

$$\vec{v}_f = -\frac{\beta_i \vec{e}_i + \beta_j \vec{e}_j}{\mid \beta_i \vec{e}_i + \beta_j \vec{e}_j \mid} v_f \tag{18}$$

where

$$\vec{e}_k = \begin{cases} \vec{e}_k^{\,-} & if \quad \vec{v}_t \cdot \vec{t}_p^{\,k} > 0 \\ \vec{e}_k^{\,+} & otherwise \end{cases}$$

with $k = i, j$, and $\beta_i, \beta_j > 0$.

Proof: When the object moves maintaining both contacts due to a commanded velocity pointing towards the tangent planes, a reaction force results whose direction is a linear combination of the involved edges of the corresponding generalized friction cones. The proposed $\vec{v}_f$ is in the opposite direction. ⋄

4 Conclusions

The geometric constraints of an assembly task are often used to guide the robot motions, giving rise to the need of specifying compliant motions.

In this paper the task constraints of contact situations involving one or two basic contacts have been analyzed. As a consequence, it has been possible to easily determine compliant motions for assembly tasks in the plane, including the rotational degree of freedom and considering the effects of friction.

Assuming a generalized damping control mode, a commanded velocity has been proposed. This velocity has a component in the desired motion direction and another one devoted to the contact maintenance. The algorithms to compute the first component have been implemented in C and simulated with a Silicon Graphics workstation.

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