Geometrical Approach for Grasp Synthesis on Discretized 3D Objects*

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Abstract—Grasp synthesis on real 3D objects is a critical problem in grasp and manipulation planning. This paper presents a geometrical approach to compute force closure (FC) grasps, with or without friction and with any number of fingers. The object's surface is discretized in a cloud of points, so the algorithm is applicable to objects of any arbitrary shape. One or more FC grasps are obtained with a geometrical approach, which embeds the FC test in the algorithm to simplify achieving the force-closure property. This initial FC grasp may be improved with a complementary optimization algorithm. The grasp quality is measured considering the largest perturbation wrench that the grasp can resist with independence of the perturbation direction. The efficiency of both algorithms is illustrated through numerical examples.

Index Terms— Grasp planning, force-closure grasps.

I. INTRODUCTION

Grasps capable of ensuring the immobility of the object under external disturbances satisfy one of the following properties: form-closure, when it is the positions of the fingers that guarantee the object immobility, or force-closure, when it is the forces applied by the fingers that guarantee the object immobility [1]. Based on any of these properties, the grasp planners calculate the position of the fingers on the object surface. The property to be used largely depends on the field of application: form-closure is used when the task requires a robust grasp not relying on friction, e.g. the fixture of objects to be manufactured or inspected, while force-closure is specially used in grasping and manipulation of objects with a lower number of frictional contacts using for instance mechanical grippers or hands. Several algorithms have been developed to determine grasps with different number of fingers and satisfying the form or force-closure condition in 2D polygonal [2] or non-polygonal [3] objects, 3D polyhedral objects [4] or objects with smooth curved surfaces [5]. However, the development of algorithms to efficiently synthesize grasps in 3D complex real-world objects is still an open research problem.

A widely used technique to represent an arbitrary object is the approximation of the object surface with a triangular mesh with a high number of faces, or equivalently, with a set of surface points and their corresponding normal direction [6], provided that the number of triangles or points is large enough to accurately represent the surface. The application of current algorithms developed for grasp synthesis of polyhedral objects to this kind of object representations would have a large computational cost, so new algorithms are being developed. A strategy based on random generation of grasps was shown to be quick and efficient to generate good grasps on arbitrary objects [7]; the complexity of such grasp planner depends on the geometric form of the object (not on its number of faces), but the generated grasps are not optimal. Wang [8] proposes an algorithm for fixture synthesis on discrete objects minimizing the workpiece positioning errors due to uncertainties in the position of the locators and in the geometry of the workpiece. Ding et al. [9] propose an algorithm to generate a form-closure grasp with seven frictionless contact points; however, it can be trapped in local minima. Liu et al. [10] extend the previous algorithm to find one force-closure grasp with frictional or frictionless contact points, and Niparnan and Sudsang [11] generate several 4-finger concurrent force-closure grasps; their contribution and the comparison with the algorithms presented in this paper will be discussed later.

This paper deals with the problem of finding a force-closure (FC) grasp with frictional or frictionless contact points, and with any number n of contacts, provided that $n \geq 3$ for frictional grasps and $n \geq 7$ for frictionless grasps. The proposed approach comprises two algorithms. The first algorithm finds at least one FC grasp with a geometrical procedure that avoids a costly FC test in each iteration. The second algorithm optimizes an initial FC grasp to get a locally optimum grasp; the optimization procedure looks for the grasp with the largest resisted perturbation wrench, with independence of the perturbation direction [12].

The rest of the paper is structured as follows. Section II provides an overview of the problem, including the main assumptions in the object and contact models, and discusses the used FC test and grasp quality measure. Section III presents the algorithm to compute at least one FC grasp. Section III provides an additional algorithm to compute locally optimum FC grasps with any number of fingers. Both algorithms have been implemented and Section V shows the results of their application to several objects. Finally, Section VI presents the conclusions of the work.

II. PROBLEM OVERVIEW

A. Problem definition

The problem to be tackled is the search of a FC grasp in a set of points representing the surface of an arbitrary 3D object. The work relies on the following assumptions:

- The object surface is represented with a large set Ω of points, described by position vectors p_i measured with respect to a reference system located in the object's center of mass (CM). Each point has an associated normal direction $\hat{n}_i$ pointing towards the interior of the object.
- The number of points in Ω is large enough to accurately represent the surface of the object.

This paper proposes a geometric approach to synthesize appropriate FC grasps, with two main algorithms:

1) An algorithm to search the set of points Ω for one or several FC grasps, regardless of its quality.
2) An optimization algorithm that searches for a local optimum grasp, measured with the largest perturbation wrench that the grasp can resist, and starting from one FC grasp obtained, for instance, with the previous algorithm.

B. Frictionless grasps

Seven frictionless contacts are necessary and sufficient to hold a 3D object with a FC grasp, provided that the object has no rotational symmetries [13]. With frictionless contact points, the grasp forces can only be applied in the direction normal to the object surface. A force $f_i = \alpha_i \hat{n}_i$ applied on the object at the point p_i generates a torque $\tau_i = p_i \times f_i$ with respect to CM, with α_i being a nonnegative value representing the magnitude of the grasping force. The force and the torque are grouped together in a wrench vector (also known as generalized force vector) given by

$$\tilde{\omega}_i = \begin{pmatrix} f_i \\ \tau_i \end{pmatrix} = \alpha_i \begin{pmatrix} \hat{n}_i \\ p_i \times \hat{n}_i \end{pmatrix} \qquad (1)$$

The wrenches applied through the contact points on the object can be grouped in a wrench set $W = \{\omega_1, \omega_2, \ldots, \omega_7\}$, where each ω_i, $i = 1, \ldots, 7$, is called a primitive contact wrench when $\alpha_i = 1$ in equation (1). Each physical point p_i in the set Ω has a corresponding wrench ω_i in the generalized force space; both of them will be used to indicate a grasp point.

C. Frictional grasps

Coulomb's friction model is used in this work, stating that there is no slipping at the contact point if $f_i^t \leq \mu f_i^n$, with μ being the friction coefficient. In the three-dimensional physical space this is a nonlinear model, defining a friction cone that includes all the possible grasp forces. To simplify the model, the cone is linearized with a m-side polyhedral convex cone (the more sides the better the approximation, but the greater the computational cost to deal with the linearized cone).

The grasping force at the contact point is given by

$$f_i = \sum_{j=1}^{m} \alpha_{ij} s_{ij} , \quad \alpha_{ij} \geq 0 \qquad (2)$$

with s_{ij} representing the normalized vector of the j-th edge of the convex cone. The wrench produced by the force f_i is

$$\tilde{\omega}_i = \sum_{j=1}^{m} \alpha_{ij} \omega_{ij} , \quad \omega_{ij} = \begin{pmatrix} s_{ij} \\ p_i \times s_{ij} \end{pmatrix} \qquad (3)$$

where ω_{ij} are the primitive contact wrenches in the frictional case. Therefore, each contact point has m associated points in the wrench space, one for each side s_{ij} of the pyramid. Let ω_i be the "normal contact wrench" for the force f_i, i.e. the primitive contact wrench in case of a frictionless contact point. The following relation can be established between the normal contact wrench ω_i and the primitive contact wrenches ω_{ij} for the linearized friction cone in a particular contact point:

$$\omega_i = \frac{1}{m} \sum_{j=1}^{m} \omega_{ij} \qquad (4)$$

For a given grasp $G = \{p_1, \ldots, p_n\}$ (or equivalently, $G = \{\omega_1, \ldots, \omega_n\}$), the corresponding wrench set is $W = \{\omega_{11}, \ldots, \omega_{1m}, \ldots, \omega_{n1}, \ldots, \omega_{nm}\}$.

D. Force-closure test

Several criteria have been proposed to test the force-closure property in a particular grasp. A necessary and sufficient condition for the existence of a FC grasp is that the origin of the wrench space lies strictly inside the convex hull (CH) of the primitive contact wrenches [14]. This condition can be solved as a linear programming problem using a ray-shooting technique [15], or as a particular application of an algorithm to find the distance between a point and a polytope [16]. The FC test used in this work is based on the following lemma.

Lemma 1: Let G be a grasp with a set W of primitive contact wrenches, $\mathcal{I}$ the set of strictly interior points of $CH(W)$, and H a supporting hyperplane of $CH(W)$ (i.e. a hyperplane containing one of the facets of $CH(W)$). The origin of the wrench space $O \in \mathcal{I}$ if and only if any $P \in \mathcal{I}$ and O are in the same half-space for every H of $CH(W)$.

From *Lemma 1*, checking whether a given point $P \in \mathcal{I}$ and the origin O lie in the same half-space defined by every supporting hyperplane H of $CH(W)$ is enough to prove whether O lies inside $CH(W)$, i.e. to prove the FC property for the grasp G. P is chosen as the centroid of the primitive contact wrenches, which is always an interior point of $CH(W)$. Then, the FC test verifies if the centroid P and the origin O lie on the same side for all the supporting hyperplanes of $CH(W)$; Fig. 1 illustrates the concept with a FC grasp and a non FC grasp in a hypothetical 2D wrench space (the actual wrench space is 6-dimensional).

E. Grasp quality

A quality measure is required to quantify the quality of a grasp; it also provides an optimization criterion for the algorithm presented later in this paper. Several grasp quality measures have been proposed in the literature [17]; this work uses as a quality measure the largest perturbation wrench that the grasp can resist, with independence of the
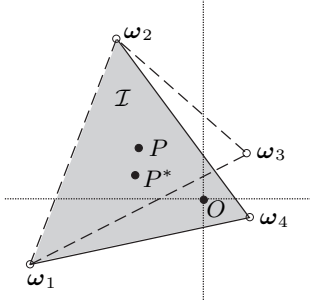
Fig. 1. Force closure test. The grasp with wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3\}$ (with CH represented in discontinuous lines) is non-FC. The grasp with wrench set $W^* = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_4\}$ (with CH represented in continuous lines) is a FC grasp.

Fig. 2. Synthesis of a FC grasp. The convex hull of the wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \mathbf{0}\}$ (in continuous lines) defines the supporting hyperplanes H_1 and H_2 that contain the origin. The convex set $\mathcal{C}_2$ contains 4 points (depicted as white squares), thus the algorithm provides 4 FC grasps, one of them illustrated with the convex hull in discontinuous lines.

perturbation direction [12]. Geometrically, that quality is the radius of the largest ball centered at the origin of the wrench space and fully contained in $CH(W)$, i.e. it is the distance from the origin of the wrench space to the closest facet of $CH(W)$. This is one of the most popular grasp quality measures, and will be referred hereafter as the largest ball criterion.

III. Synthesis of a force-closure grasp

A. The algorithm

The synthesis of a FC grasp is based on geometric reasoning, avoiding the inclusion of an explicit FC test in the synthesis algorithm. A set S^1 of $n-1$ random points ($n = 7$ for a frictionless grasp) is selected from Ω, and the convex hull $CH(W)$ of the selected points plus the origin O of the wrench space is computed; the missing contact required to get a FC grasp must be found (in this sense, the approach uses the necessary and sufficient condition provided in [4] for the FC grasp when one or more contacts are missing). Let H be a supporting hyperplane of $CH(W)$ containing the origin, and let H^+ be the half-space defined by H that contains $CH(W)$. The intersection of all of the half-spaces H^+ is the union of two convex sets, $\mathcal{C}_1$ and $\mathcal{C}_2$, as illustrated in Fig. 2. Let $\mathcal{C}_1$ be the convex set containing $CH(W)$, then, if there is at least one wrench in $\mathcal{C}_2$, it will provide a FC grasp when added to the set S^1, according to *Lemma 1*. If $\mathcal{C}_2$ is empty, the algorithm iteratively replaces one of the wrenches in S^1 and performs another search of points in the new $\mathcal{C}_2$, until it contains at least one point, i.e. until it finds at least one FC grasp. The steps in the algorithm are:

Algorithm 1: Search of a FC grasp

1) Generate a random set S^k with the wrenches corresponding to $n - 1$ contact points, $S^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_{n-1}\}$, $k = 1$.
2) Build the wrench set W^k adding the origin O of the wrench space to the set S^k.
3) Compute the convex hull $CH(W^k)$.
4) Use the supporting hyperplanes of $CH(W^k)$ containing the origin to look for the points lying in the sets $\mathcal{C}_1$ and $\mathcal{C}_2$.
5) If there is at least one point in $\mathcal{C}_2$, then the algorithm finishes and returns at least one FC grasp G. If $\mathcal{C}_2$ is
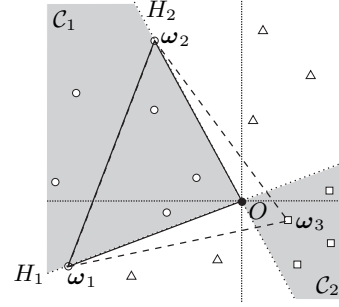
empty, choose one of the wrenches in $\Omega - \mathcal{C}_1 - \mathcal{C}_2$, and use it to replace the closer wrench in the set S^k, thus generating S^{k+1}. Go to Step 2.

To progressively explore the search space and assure the completeness of the algorithm, the wrenches lying in $\mathcal{C}_1$ are labeled as explored wrenches. When $\mathcal{C}_2$ is empty, any combination of $n - 1$ wrenches in $\mathcal{C}_1$ (including the wrenches in S^k) will not yield a FC grasp, thus all of these possible combinations are left out for subsequent searches. The algorithm keeps running and finishes when at least one FC grasp is obtained; the algorithm provides as many FC grasps as points lie in $\mathcal{C}_2$.

The algorithm was extended to frictional grasps. The primitive contact wrenches are used to compute the convex hull $CH(W)$, and any primitive wrench lying in $\mathcal{C}_2$ will provide a FC grasp when the wrenches of the corresponding grasp point are added to the set S^k. A more conservative approach may require that all of the primitive contact wrenches for a frictional grasp point lie inside $\mathcal{C}_2$ to consider it as valid grasp point; this would lead only to the more robust FC grasps, but may miss some other solutions.

B. Comparison with previous works

There is relatively little work concerning the synthesis of a FC grasp on discretized 3D objects. Niparnan and Sudsang [11] generate a number of 4-finger concurrent FC grasps to provide the user with a large set of grasps, so the user can choose an optimum one according to a quality measure appropriate for the particular task. The algorithm is based on the localization of regions in the 3D space where the axes of the friction cones seem to intersect. For each region, subsets of four grasp points are tested for the FC condition, by choosing an arbitrary point in the region and test if it is included in the four friction cones. The algorithm is capable of computing hundreds of FC grasps; the running times are below 3 seconds to get one FC grasp in objects described with 2000 surface points. However, they do not take into account any measure of grasp quality.

In a work closer to ours, Liu et al. [10] look for a force-closure grasp with frictional or frictionless contact points. First, an initial random grasp is chosen, and the algorithm iteratively moves the fingers to decrease the distance between the convex hull and the origin. When
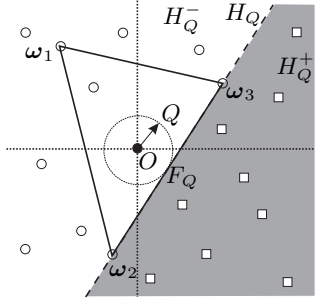
Fig. 3. Selection of the subset Ω_C^k of candidate points (depicted as white squares in the gray area) that may improve the grasp quality; in this example, $F_Q = \overline{\omega_2 \omega_3}$.
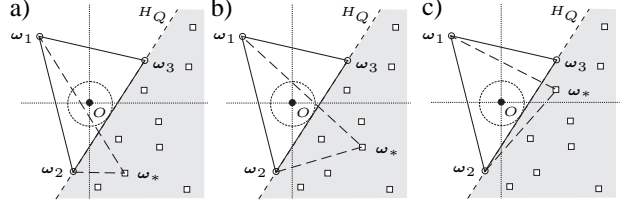

Fig. 4. Possible cases for a candidate grasp in the optimization procedure: a) Non-feasible candidate grasp, b) Discarded candidate grasp, c) Feasible candidate grasp.

the procedure is trapped in a local minimum, the point set is divided into subsets by using a separating hyperplane in the wrench space, and the FC search is decomposed into subproblems, based on existence conditions for the FC grasp. The algorithm uses a FC test in each iteration; the FC test implies the solution of a linear programming problem based on the ray-shooting technique [15]. The algorithm is complete, in the sense that it finds a FC grasp if it exists in the discrete sampling of the surface, but it does not assure any optimality. The number of iterations in several examples is provided, but no information is reported on computational times.

The algorithm presented here also looks for the FC grasp in the wrench space; it is applicable to any number of frictional or frictionless fingers, and progressively covers the search space until it finds a FC grasp, or until all of the space has been covered and no FC grasp is found. It is computationally simpler than the algorithm in [10], as it does not include an explicit FC test in the algorithm; the FC condition is embedded in the search process. Moreover, the algorithm finds one or more FC grasps, depending on the number of wrenches lying in the subset $\mathcal{C}_2$; if there are several FC grasps, they can be classified according to a quality measure to pick the best candidate among the grasps provided by the algorithm. If a more robust grasp is required, the selected initial grasp may be optimized with the algorithm presented in the following section.

IV. Synthesis of a locally optimum grasp

The optimization algorithm begins with an initial FC grasp obtained, for instance, with the procedure described in the previous section. The optimization is done according to the largest ball criterion. The steps in the algorithm for the frictionless case are:

Algorithm 2: Search of a locally optimum FC grasp

1) Find an initial FC grasp, $G^k = \{\omega_1, \dots, \omega_7\}$, $k = 1$, using the algorithm presented in Section III, and form the corresponding wrench set W^k.
2) Determine F_Q, the facet of the convex hull $CH(W^k)$ closest to the origin. The distance from the origin O to F_Q is the current grasp quality Q^k.
3) Build the subset Ω_C^k with the candidate points that may produce an improvement in the grasp if they replace

one point in F_Q. Let H_Q be the hyperplane containing the facet F_Q. The subset Ω_C^k contains the points lying in the open half-space defined by H_Q that does not contain the origin O, i.e. H_Q^+, as illustrated in Fig. 3 for a hypothetical two-dimensional wrench space.
4) Generate 6 candidate grasps G_i^*, $i = 1, \dots, 6$, by picking one point ω_* from Ω_C^k and replacing each one of the vertices defining the facet F_Q. Due to the selection procedure, all the wrenches $\omega^* \in \Omega_C^k$ are external points to $CH(W)$, therefore, when replacing one vertex ω_i from the actual CH with the candidate wrench ω_*, the latter will become a vertex of the new CH. The explicit computation of the new CH is not required, as its facets are constructed from the old ones replacing ω_i with ω_*.
5) Check the candidate grasps for the FC property using *Lemma 1*. For the FC candidate grasps, the expected grasp quality Q^* is computed; if for any candidate grasp $Q^* > Q^k$, then the candidate becomes the new grasp G^{k+1}. Fig. 4 illustrates three possible cases related to the candidate grasps; case (a) is a non-feasible grasp because it loses the FC property, case (b) is discarded because the grasp has a smaller quality than the previous one, and case (c) is a FC grasp that improves the current grasp quality, thus it becomes the grasp for the next iteration cycle. If any candidate grasp improves the quality, the algorithm goes back to Step 2. If there is no improvement in Q^k once all the points in Ω_C^k have been considered, then a local minimum has been reached, the algorithm finishes and returns the current grasp G.

When frictional contacts are considered, the vertices defining the facet F_Q are primitive contact wrenches associated to different grasp points p_i (Fig. 5). Therefore, the candidate grasps in Step 4 are generated by replacing all the contact points p_i that contribute with at least one primitive contact wrench in the facet F_Q. When a grasping point is substituted by a new one in Step 5, the explicit computation of the new convex hull is required, which increases the computational complexity of the algorithm in the frictional case.

To the best of the authors knowledge the algorithms presented in the literature for FC grasp synthesis in discretized 3D objects focus only on getting one FC grasp, regardless of whether it is (locally) optimal or not. As a consequence, no direct comparison can be established between the present and other previously published works.
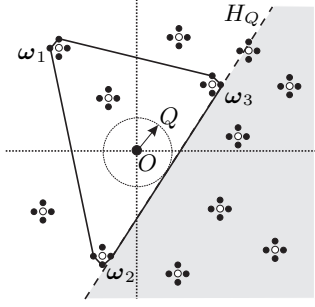
Fig. 5. Optimization procedure for frictional grasps. White circles represent normal contact wrenches, and black circles are the corresponding primitive contact wrenches. Wrenches falling in the gray zone belong to the subset Ω_C^k of candidate points that may improve the grasp quality.
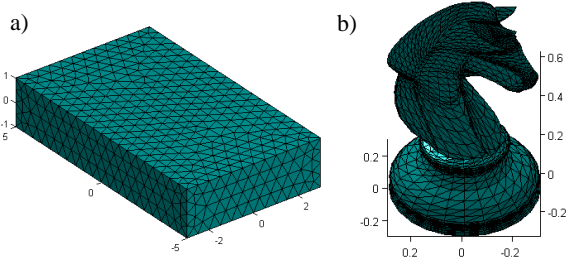

Fig. 6. Objects used in the examples: a) Parallelepiped discretized with a mesh of 1628 triangles, b) Knight discretized with 4750 triangles.

V. EXAMPLES

The proposed algorithms have been implemented using Matlab on a Pentium IV 3.2 GHz computer. The performance of the approach is illustrated using the two objects shown in Fig. 6: a parallelepiped and a chess knight. The object surfaces are represented with triangular meshes. The contact points p_i considered on the object surface are the centroids of the triangles in the mesh, and the corresponding surface normal directions are the directions normal to the triangles.

A. Example 1: frictionless grasp on a parallelepiped

The parallelepiped is described with a mesh of 1628 triangles. This simple figure makes more difficult the search of the first FC grasp, as the initial randomized grasp may place all the fingers on just one or two faces (because there are two large faces, the probability of placing a finger on those faces is greater than on the others). Fig. 7a and 7b show the initial and locally optimal FC grasps for a particular case. The time elapsed to obtain the initial FC grasp is 0.13 seconds in 2 iterations; Algorithm 1 provides other 22 possible FC grasps. The time to get the locally optimum grasp with Algorithm 2 is 16.8 seconds in 21 iterations. Fig. 7c plots the grasp quality in the optimization phase; the quality always increases monotonically until it finds the locally optimum grasp.

The obtained locally optimum grasp depends on the initial FC grasp. In the example, the initial grasp quality is $Q_I = 0.04$, and the locally optimum gasp has a quality of $Q_F = 0.212$. The improvement factor Q_F/Q_I is 5.3. However, as the improvement factor depends on the initial FC grasp, 50 locally optimum grasps were computed to
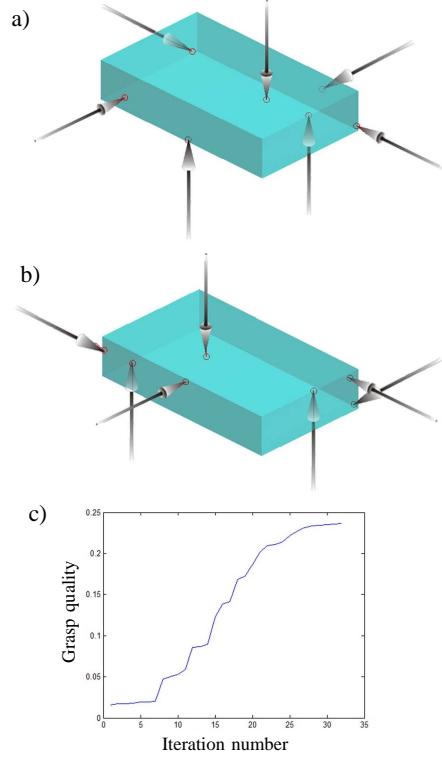

Fig. 7. FC grasp on the parallelepiped: a) Initial FC grasp, b) Locally optimum FC grasp, c) Increase in the grasp quality.
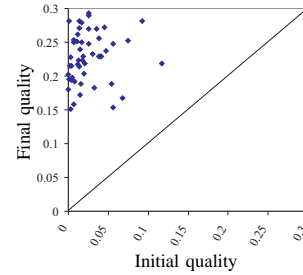

Fig. 8. Initial vs. final quality for the parallelepiped frictionless grasps.

obtain a better insight into the performance of the whole process. The correlation between the initial and final qualities is shown in Fig. 8. The average qualities give an idea of the behavior of the algorithm, they are 0.025 and 0.227 for the initial and locally optimum FC grasps, respectively; the average improvement factor is 9.

B. Example 2: frictionless grasp on a knight

The knight is discretized with 4750 triangles (Fig. 6b). Fig. 9 shows the results for a particular case; the initial grasp is found in 0.2 seconds with no iterations (the initial random wrenches yield a FC grasp), and 49 additional FC grasps are provided. The locally optimum grasp is obtained after 35 iterations in 31.6 seconds. The grasp qualities are 0.009 and 0.058 for the initial and locally optimum FC grasps, respectively, with an improvement factor of 6.4. Fig. 10 shows the correlation between initial and final grasp qualities in 50 cases. The average quality for the initial FC grasp is 0.0052, and 0.067 for the locally optimum grasp; the average improvement factor is 12.9.
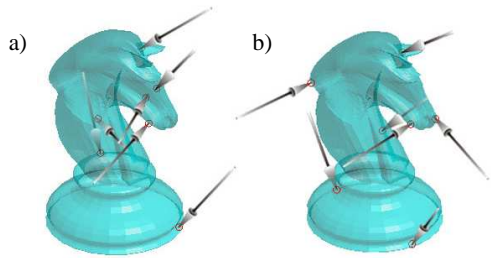
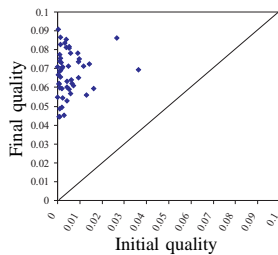Fig. 9. Frictionless grasp on the knight: a) Initial FC grasp, b) Locally optimum FC grasp.

Fig. 10. Initial vs. final quality for the knight frictionless grasps.

C. Example 3: frictional grasp on a knight

The frictional grasps on the knight are computed considering a friction coefficient of 0.2 and 4 fingers (the minimal number of fingers to guarantee a frictional FC grasp). The friction cones have been linearized with an 8-side polyhedral convex cone. Fig. 11 shows the results for a particular case; Algorithm 1 provides 16 FC grasps in 11 seconds and 1 iteration. The locally optimum grasp is obtained after 16 iterations in 13 minutes. Fig. 12 shows the correlation between initial and final grasp qualities for 50 initial and locally optimum grasps. The average quality for the initial FC grasp is 0.015, and 0.04 for the locally optimum grasp. The average improvement factor is 2.8.
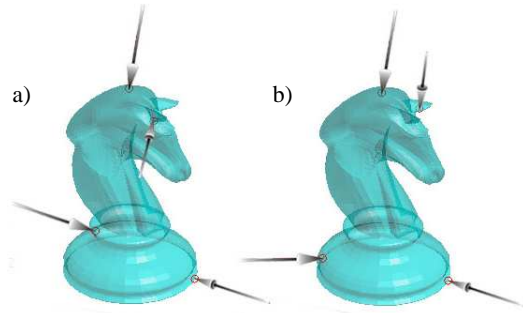

Fig. 11. Frictional grasp on the knight: a) Initial FC grasp, b) Locally optimum FC grasp.
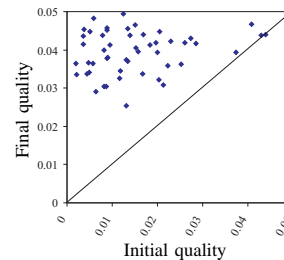
Fig. 12. Initial vs. final quality for the knight frictional grasps.

VI. CONCLUSIONS

This paper proposes a geometrical approach to obtain at least one force-closure (FC) grasp in 3D discretized objects, with frictional or frictionless contacts and with any number of fingers. The presented algorithm embeds the FC condition in the procedure to avoid an explicit FC test in each iteration. The initial FC grasp may be improved with an oriented search procedure, optimizing the grasp quality according to the largest ball criterion. The algorithms were implemented and the execution results (as the examples shown in the paper) illustrate the relevance and efficiency of the approach. Future work includes the determination of independent contact regions for 3D discretized objects, such that placing a finger in each contact region assures a force-closure grasp on the object, independently of the exact position of the contact points, and providing robustness in front of finger positioning errors in grasp and fixturing applications.

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