Path Validation in Constrained Motion with Uncertainty*

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Abstract

The performance of tasks with robots in environments with low clearances (e.g. robotized assembly) is usually difficult, due mainly to geometric uncertainty and tolerances. Following a pragmatical approach to planar constrained-motion planning, this paper proposes a method for checking the feasibility of paths generated by gross-motion planning algorithms, taking into account the uncertainties affecting the task and the use of a compliant control mode. The method enables the extension of gross-motion planning techniques to constrained-motion planning problems, ensuring the feasibility of the task despite the uncertainties.

1 Introduction

In spite of the research effort done in constrained-motion planning, not many practical results have been yet obtained. On the other hand, a lot of work has been done in the field of gross-motion planning with results already being used in industrial applications. A pragmatical approach to constrained-motion planning is to use a gross-motion planning algorithm to generate a nominal free path and then, taking into account the uncertainties affecting the task and the use of a compliant control mode, check for its actual feasibility, i.e. verify if the reachability of the goal is guaranteed despite possible contacts during task execution. Checking the path feasibility fills the gap between gross-motion and constrained-motion, by broadening the ability to successfully follow, in a constrained environment, a gross-motion computed path.

2 Problem Statement

The configuration space ($\mathcal{C}$-space) of a manipulated object is the set of all the possible configurations of the object, a configuration being given by its position and orientation (SE(2) in planar tasks). Let assume a nominal path in $\mathcal{C}$-space generated by a gross-motion planner for executing a planar motion in a cluttered environment. The objective is to determine if this path can be successfully executed in spite of the uncertainties affecting the task. A path will be feasible iff one of the following conditions holds:

- Contacts are not possible along the path.
- Contacts might occur but the robot can comply at them and proceed towards the goal.
- Contacts blocking the task can occur but a recovery path can be planned (i.e. the contact situation where the task is blocked is known with certainty).

Therefore, in order to determine the path feasibility two problems have to be tackled:

- Given a configuration of the path, determine if it can become a contact configuration due to uncertainties. For this purpose, the set of contact situations that can take place at the given configuration must be computed.
- Given a possible contact configuration, determine the direction of the possible contact motion when the nominal command is applied.

The uncertainty sources affecting the task and the compliant control assumed are the following:

- **Uncertainty sources:** manufacturing, manipulation and sensing uncertainties must be taken into account: a) manufacturing tolerances of the objects shape and size; b) imprecision in the position and orientation of the static objects; c) imprecision in the position and orientation of the robot gripper; d) imprecision in the position and orientation of the object in the robot gripper; e) imprecision in the measurement of the force/torque sensor.
- **Compliant control:** a generalized damping model is assumed: $\mathbf{f}(t) = B(\mathbf{v}_c(t) - \mathbf{v}_o(t))$, where $\mathbf{f}(t)$ is the actual reaction force exerted by the static objects on the mobile one at time

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t, B is a diagonal damping matrix, $\mathbf{v}_o(t)$ the actual velocity of the mobile object, and $\mathbf{v}_c(t)$ the commanded velocity. $B\,\mathbf{v}_c(t)$ is called the commanded or applied force.

3 Proposed Approach

This section explains the proposed method to validate in the presence of uncertainty a path generated by a gross-motion planner. It is organized as follows. First, Section 3.1 presents a classification of configurations and paths in order to formalize the feasibility conditions introduced in the previous section. Then, Section 3.2 introduces the tools needed to make this classification, and finally Section 3.3 presents the algorithm for performing the path validation.

3.1 Path Analysis

A configuration is said to be compatible with a contact situation C_S if due to uncertainty C_S can take place. A configuration compatible with several contact situations is called distinguishable if all of them can be certainly identified by the sensed configuration and force. A configuration is called motion-feasible for a given commanded velocity $\mathbf{v}$ if $\mathbf{v}$ gives rise to the same kind of motion at all basic contacts with which it is compatible, and the motion is error-corrective (Section 3.3).

Then, a configuration of a path is classified as:

CONFIG. CLASS	Configuration Properties		
	Compatible	Motion-feas.	Distinguish.
Free	N	-	-
Compliant	Y	Y	-
Guarded	Y	N	Y
Ambiguous	Y	N	N

Finally, a path is classified into free, compliant,guarded or ambiguous depending on the type of configurations it contains:

PATH CLASS	Path configurations			
	Free	Compliant	Guarded	Ambiguous
Free	Y	N	N	N
Compliant	-	Y	N	N
Guarded	-	-	Y	N
Ambiguous	-	-	-	Y

The path is feasible if it is either free, compliant or guarded. If the path is free it can be followed as a gross-motion path. If the path is compliant it can be followed complying at contacts when they occur. In both cases the path can be followed with the guarantee that the task will be successfully completed. If the path is guarded a recovery motion must be planned to move away from the sticking contact situation. Finally, if the path is ambiguous it is not feasible, since it cannot be followed with the guarantee that the task will be successfully completed and it is not possible to plan sure recovery strategies.

3.2 Basic Tools

Two basic tools are needed to classify a path:

- The Contact Identification Tool: determines the set of contact situations with which a a given configuration is compatible.
- The Contact Motion Analysis Tool: determines the direction of the contact motion given a contact configuration and a velocity command.

Both tools make use of the dual representation of forces, which is a graphical method useful for the analysis of planar contact problems [2]. This representation maps the supporting line of a force into a point (that represents the line direction) and a sign (that expresses the force sense).

Using the dual representation, the reaction force $\mathbf{f} = (f_x, f_y)$ and torque τ with respect to a reference origin O, produced at a contact situation during a planar assembly task, are mapped into the point $F' = (f_y/|\tau|, -f_x/|\tau|)$ and the sign of τ. Geometrically, F' lies on the normal to the force line through the reference origin O and at a distance $1/d$ from O, d being the distance between the force line and O (Appendix A, Figure 4a).

3.2.1 Contact Identification Tool

Contact identification is a complex issue in the presence of uncertainties because the sensed information may correspond to several contact situations. In [1] and [8], the authors presented a contact identification algorithm based on configuration and force information with the following main characteristics:

- The procedure uses the nominal Configuration Space (i.e. without uncertainty).
- For the measured configuration $\mathbf{c}$ and for each contact situation C_S, configuration domains $\mathcal{C}(\mathbf{c}, C_S)$ and force domains $\mathcal{F}(\mathbf{c}, C_S)$ are defined taking into account the effect of all the uncertainties:
  - $\mathcal{C}(\mathbf{c}, C_S)$ is the set of configurations associated to $\mathbf{c}$ such that its intersection with the

set of configurations compatible with C_S without uncertainty is non-empty when C_S can occur for some given values of the deviations due to uncertainty.

  - $\mathcal{F}(\mathbf{c}, C_S)$ is the set of the generalized reaction forces that may arise when C_S takes place at configuration $\mathbf{c}$. They are represented in the dual force space where the uncertainty and the linear combinations of forces are more easily handled.

- A measured configuration $\mathbf{c}$ and force $\mathbf{f}$ are compatible with a contact situation C_S if the following two conditions are met:
  - $\mathcal{C}(\mathbf{c}, C_S)$ intersects the set of configurations compatible with C_S without uncertainty.
  - $\mathcal{F}(\mathbf{c}, C_S)$ intersects the set of possible actual forces compatible with $\mathbf{f}$ due to sensor uncertainty.

3.2.2 Contact Motion Analysis Tool

The problem of determining the robot commands to perform a desired motion of the manipulated object in contact with the environment has been studied by several authors. Some representative approaches are the algebraic analysis of Rajan et al. [6] for planar assemblies considering friction; the method of polyhedral convex cones (PCC) described by Hirai and Asada [4] for the analysis of manipulation in 6 d.o.f; and the application by Paul and Ikeuchi [5] of the dual representation and of the PCC method for partitioning the contact space in a finite number of states in planar assembly tasks.

The Contact Motion Analysis Tool proposed here uses a new method, detailed in Section 4, which is easily applied to planar tasks with uncertainty. It is based on the concept of motion region defined as follows.

Definition 1 *A* motion region *associated to a basic contact (pair vertex-edge) is the set of commanded velocities that produce movements of the manipulated object in contact with the environment in a given sense of sliding and a given sense of rotation around the contact point.*

Once a configuration is classified as a possible contact configuration by the Contact Identification Tool, the Contact Motion Analysis Tool labels the commanded velocity into one of the following motion regions, provided that it neither produces the break of the contact nor produces sticking at it:

Region 1: $S^+ \wedge R^-$	Region 2: $S^+ \wedge R^+$
Region 3: $S^- \wedge R^+$	Region 4: $S^- \wedge R^-$
Region 5: $S^0 \wedge R^-$	Region 6: $S^0 \wedge R^+$

where:

- R^+ and R^- represent, respectively, the positive and negative rotation around the contact point. The no rotation motions are included in R^+.
- S^+ and S^- represent, respectively, the positive and negative sliding of the contact point.
- S^0 represents no sliding motion (it is the border between S^+ and S^- and is due to the effect of friction).

3.3 Path Evaluation Algorithm

The evaluation of a path is done by classifying first a uniform sample of its configurations using the algorithm Configuration-Evaluation($\mathbf{c}$,$\mathbf{v}$) shown below. The algorithm uses the following functions that implement the tools presented in the previous section:

- (S,d) = Contact-Identification($\mathbf{c}$): Given a configuration $\mathbf{c}$ the function returns the set S of basic contacts with which it is compatible and a flag d indicating if it is a distinguishable configuration.
- m = Motion-Region($\mathbf{c}$,s_i,$\mathbf{v}$): Given a configuration $\mathbf{c}$ compatible with a basic contact $s_i \in S$ and a velocity command $\mathbf{v}$, the function returns the label of the motion region of s_i containing $\mathbf{v}$. When $\mathbf{v}$ does not belong to any motion region the function returns 0 if $\mathbf{v}$ produces a break of contact or -1 if it cannot be determined due to uncertainty.
- e = Error-Corrective($\mathbf{c}$,s_i,$\mathbf{v}$): Given a configuration $\mathbf{c}$ compatible with a basic contact $s_i \in S$ and a velocity command $\mathbf{v}$ that belongs to a motion region, the function returns TRUE if the expected contact motion is error-corrective, and FALSE otherwise. The function performs the following steps:
  - Chooses any contact configuration $\mathbf{c}_A \in \mathcal{C}(\mathbf{c}, s_i)$ (since $\mathbf{v}$ belongs to a motion region, the result is independent of the chosen configuration $\mathbf{c}_A \in \mathcal{C}(\mathbf{c}, s_i)$).
  - Computes the resulting compliant velocity $\mathbf{v}_A$ for $\mathbf{c}_A$.
  - Computes $\mathbf{c}'$ and $\mathbf{c}'_A$ as: $\mathbf{c}' = \mathbf{c} + T\,\mathbf{v}$ and $\mathbf{c}'_A = \mathbf{c}_A + T\,\mathbf{v}_A$, where T is a time interval.
  - If $distance(\mathbf{c}', \mathbf{c}'_A) \leq distance(\mathbf{c}, \mathbf{c}_A)$ returns TRUE, and FALSE otherwise.

Configuration-Evaluation(c, v)
(S,d) := Contact-Identification($\mathbf{c}$)
k :=cardinality(S)
IF $k = 0$ THEN RETURN FREE
m := Motion-Region($\mathbf{c}$,s_1,$\mathbf{v}$)
IF $k = 1$ AND $m = 0$ THEN RETURN FREE
IF $m > 0$ THEN e:=Error-Corrective($\mathbf{c}$,s_1,$\mathbf{v}$)
ELSE e:=FALSE
IF $k = 1$ AND $m > 0$ AND e =TRUE THEN
  RETURN COMPLIANT
IF $k > 1$ AND $0<m<5$ AND e =TRUE THEN
  DO
    $i := 1$
    $n := 0$
    WHILE ($i \leq k$ AND ($n = 0$ OR ($n=m$ AND $e=\epsilon$))
      $i := i + 1$
      n := Motion-Region($\mathbf{c}$,s_i,$\mathbf{v}$)
      IF $0<n<5$ THEN ϵ:=Error-Corrective($\mathbf{c}$,s_i,$\mathbf{v}$)
      ELSE ϵ:=FALSE
    END WHILE
    IF $i > k$ THEN RETURN COMPLIANT
  END DO
IF d=TRUE THEN RETURN GUARDED
RETURN AMBIGUOUS

Then, the algorithm Path-Evaluation($\mathcal{P}$) classifies a path $\mathcal{P}$ depending on the results of the evaluation of its configurations.

Path-Evaluation($\mathcal{P}$)
$t = 0$
$g = 0$
P = discretize $\mathcal{P}$
FOR ALL $\mathbf{c} \in P$
  r = Configuration-Evaluation($\mathbf{c}$, $\mathbf{v}$)
  IF r = AMBIGUOUS THEN RETURN AMBIGUOUS
  ELSE IF r = GUARDED THEN $g = 1$
  ELSE IF r = COMPLIANT THEN $t = 1$
END FOR
IF $g = 1$ THEN RETURN GUARDED
ELSE IF $t = 1$ THEN RETURN COMPLIANT
RETURN FREE

4 Motion Region Determination

The Contact Motion Analysis Tool classifies the commanded velocity into a motion region. This section explains the procedure to determine the motion regions. First, Section 4.1 explains the determination of the motion direction produced when a velocity command is applied at a contact configuration. The procedure decomposes the applied generalized force vector corresponding to the velocity into a reference frame associated to the contact configuration. Section 4.2 presents the algorithm that computes the motion regions using the dual representation of the contact reference frame. Section 4.3 tackles the effect of uncertainty: motion regions shrink when uncertainty is considered because some commands may produce different contact motions.

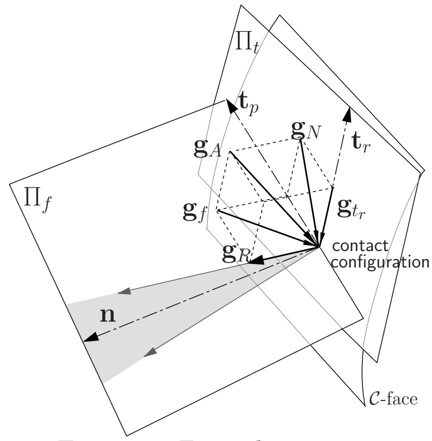

Figure 1: *Force decomposition.*

4.1 Force decomposition

In order to obtain the direction of the motion resulting from applying a force at a contact configuration, this force will be represented by its components (Figure 1) in the contact reference frame defined below. Let the $\mathcal{C}$-face be the set of configurations in the Configuration Space where the basic contact occurs, and let the generalized friction cone be the set of possible generalized reaction force directions produced at a basic contact in a given contact configuration [3].

Definition 2 *The* contact reference frame *is the orthogonal coordinate frame* $[\mathbf{t}_r, \mathbf{t}_p, \mathbf{n}]$ *with the origin at the contact configuration* $\mathbf{c}_o$*, where:*

- $\mathbf{n}$ *is the normal to the* $\mathcal{C}$*-face at* $\mathbf{c}_o$.
- $\mathbf{t}_r$ *has the direction of pure rotation about the contact point. A positive motion along* $\mathbf{t}_r$ *corresponds to a rotation that increases the orientation angle of the manipulated object.*
- $\mathbf{t}_p$ *has the direction perpendicular to* $\mathbf{t}_r$ *and* $\mathbf{n}$*. The sense of* $\mathbf{t}_p$ *is such that* $[\mathbf{t}_r, \mathbf{t}_p, \mathbf{n}]$ *is right-handed.*

The contact reference frame defines the following planes through $\mathbf{c}_0$:

Tangent plane Π_t: Plane tangent to the $\mathcal{C}$-face at $\mathbf{c}_o$.

Friction plane, Π_f: Plane normal to $\mathbf{t}_r$. Π_f contains the generalized friction cone.

Rotation plane, Π_r: Plane normal to $\mathbf{t}_p$.

Then, an applied generalized force $\mathbf{g}_A$ that points into the $\mathcal{C}$-face can be decomposed as $\mathbf{g}_A = \mathbf{g}_f + \mathbf{g}_{t_r}$, with $\mathbf{g}_f$ on the plane Π_f and $\mathbf{g}_{t_r}$ along the direction $\mathbf{t}_r$, perpendicular to Π_f.

Finally, the direction of the reaction force and the direction of the motion in contact are given by the following two propositions [7].

Proposition 1 *The reaction force $\mathbf{g}_R$ produced at a basic contact is $\mathbf{g}_R = -\mathbf{g}_f$ if $\mathbf{g}_f$ is inside the generalized friction cone, or it is the projection of $-\mathbf{g}_f$ onto the edge of the friction cone along the direction of $\mathbf{t}_p$, otherwise.*

Proposition 2 *The net force $\mathbf{g}_N$ that produces the actual motion is the projection of $\mathbf{g}_A$ on the plane Π_t along the direction of $\mathbf{g}_R$* (Figure 1).

In order to classify $\mathbf{g}_A$ into a motion region, the sign of $\mathbf{g}_N\mathbf{t}_r$ discriminates between R^+ and R^-, and the sign of $\mathbf{g}_N\mathbf{t}_p$ discriminates between S^+ or S^-, provided that $\mathbf{g}_f$ lies outside the generalized friction cone (otherwise there is no sliding motion, i.e. S^0).

Since it is useful to explicitly determine the border of motion regions (also considering the effect of uncertainty), next Section computes them in the dual plane, where uncertainty is easily handled.

4.2 Motion Regions

Proposition 3 *The generalized force space F_3 is related with the dual plane as follows: a plane $\Pi \subset F_3$ and its normal direction $\mathbf{n}$ are mapped in the dual plane into a line π' and a point N', respectively, such that π' and N' maintain between them a relation of duality, i.e. N' can be obtained as the dual point of π'* (property 1, Appendix A).

Using this proposition, the planes defined by the contact reference frame (Section 4.1) are mapped into lines that, together with the dual representation of the friction cone, partition the dual plane into motion regions. The following algorithm details the steps of this procedure, which is illustrated in Figure 2 for a basic contact between a vertex of the manipulated object and an edge of an static object.

An applied generalized force $\mathbf{f}$ that do not produce a lost of contact (i.e. $\mathbf{f} \cdot \mathbf{n} < 0$) is classified into a motion region M if F' (the dual representation of $\mathbf{f}$) satisfies $F' \in M$.

Dual-plane-partition

(1) Represent the vector $\mathbf{t}_r$ by the point T'_r (which coincides with the contact point).

(2) Represent the vector $\mathbf{n}$ by the dual point N' of the line normal to the contact edge passing through the contact point (T'_r) (property 1).

(3) Represent Π_f by the dual line π'_f of the point T'_r (proposition 3).

(4) Represent Π_t by the dual line π'_t of N' (proposition 3).

(5) Represent the vector $\mathbf{t}_p$ by the intersection point T'_p of lines π'_f and π'_t (proposition 3).

(6) Represent the friction cone as the dual segment FC of the physical friction cone (property 2).

(7) Label the negative and positive linear combination of T'_r and FC (property 3) as regions 5 and 6, respectively.

(8) Label regions 1 to 4 (bounded by π'_f, π'_t and the border of regions 5 and 6), according to their characteristics.

4.3 The effects of uncertainty

When uncertainty is present, motion regions shrink because the lines and points defining the motion regions become themselves regions corresponding to applied forces that may produce different contact motions. Uncertainty is considered as shown in the following items, and then propagated through the steps of the procedure presented in the previous Section, making use of the properties of the dual representation:

- The uncertainty in the position of the contact point is considered by substituting the contact point by a segment centered at the contact point and parallel to the contact edge, such that all possible reaction forces cross this segment.
- The uncertainty in the orientation of the contact edge is considered by substituting $\mathbf{n}$ by a cone. The friction cone is enlarged accordingly.

Figure 3 shows the effect of uncertainty in the example presented in Figure 2 (border regions are expressed as $\mathcal{U}$ with a subindex indicating the name of the corresponding point or line).

5 Conclusions

Checking for path feasibility is a useful way to use gross-motion planning results to successfully perform constrained-motion tasks when uncertainties are

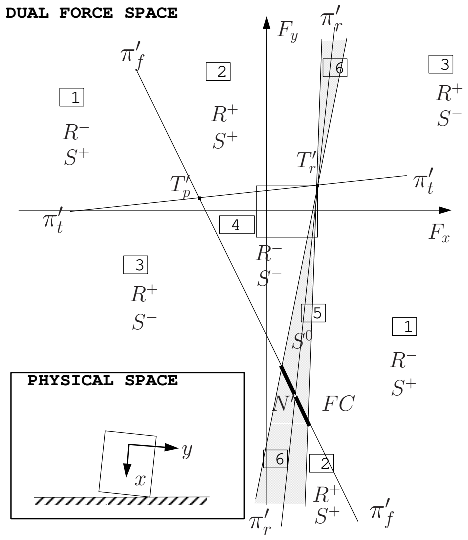

Figure 2: *Dual Space Partition*

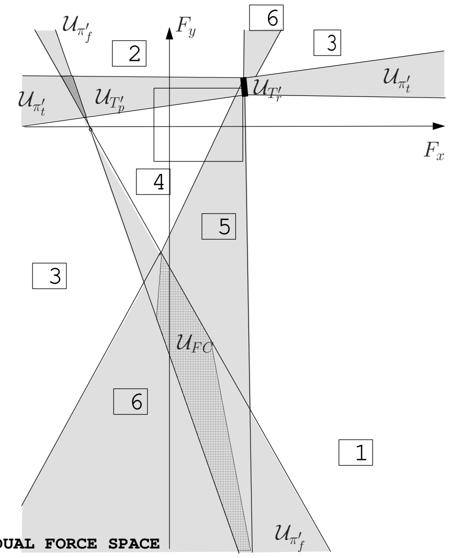

Figure 3: *Dual Space Partition taking into account the effect of uncertainty.*

present and a compliant robot control is considered. The paper has presented a method for checking path feasibility in planar tasks based on the dual representation of forces. The proposed algorithm can be embedded into a gross-motion planner in order to turn it into a constrained-motion planner, producing practical results applicable to industrial tasks.

Appendix A: Properties

The following are some of the main properties of the dual representation of forces:

Property 1: The supporting line $ax + by + c = 0$ of a force $\mathbf{f}$ maps into the point $F' = (\frac{a}{c}, \frac{b}{c})$ (Fig. 4a).

Property 2: The supporting lines of force passing through a point P and lying inside a cone $\widehat{ab}$, map into points of a segment $\overline{A'B'}$ on the dual line p' of point P, A' and B' being the dual points of lines a and b, respectively.

Property 3: The supporting lines of forces that are non-negative linear combinations of a set of forces, map into the points of the convex hull defined by the dual points of the supporting lines of the forces in the set (Figure 4b).

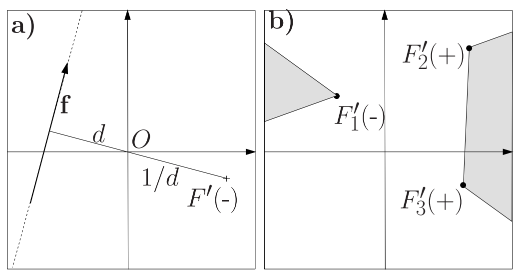

Figure 4: *a) A force* $\mathbf{f}$ *and its dual representation* F' *b) Linear combination of three dual forces, one of them with different sign from the others.*

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