Searching force-closure optimal grasps of articulated 2D objects with n links

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Abstract: This paper proposes a method that finds a locally optimal grasp of an articulated 2D object with n links considering frictionless contacts. The surface of each link of the object is represented by a finite set of points, thus it may have any shape. The proposed approach finds, first, an initial force-closure grasp and from it starts an iterative search of a local optimum grasp. The quality measure considered in this work is the largest perturbation wrench that a grasp can resist with independence of the direction of the perturbation. The approach has been implemented and some illustrative examples are included in the article.

Keywords: Force closure grasps, grasp quality measure, optimal grasp.

1. INTRODUCTION

The objects that can be manipulated, grasped and fixed by the great majority of robots used in different fields of application such as industry, home and school can be of different shapes and sizes. Such objects may be single rigid bodies such as a pencil, a cup, or a screwdriver, but can also be articulated objects, i.e. constituted by rigid links connected by some type of joints or hinges, such as scissors, staplers, pliers, truck toy, laptop computers, and some cell phones. The goal of this work is to fully constrain all the degrees of freedom of these articulated objects, despite the possible existence of some external force disturbances.

Typically, a grasp must satisfy one of the following properties: force-closure (hereafter FC, the forces applied by the fingers ensure the object immobility) or form-closure (the position of the fingers ensures the object immobility) (Bicchi, 1995). Both properties can be characterized in the object configuration space, that for a 2D rigid body has dimension $m = 3$, and any 2D object without rotational symmetries can be immobilized with $m + 1 = 4$ frictionless fingers (Markenscoff and Papadimitriou, 1990; Mishra et al., 1987).

The majority of the work developed in the area of object grasping with robotic hands are focused on the grasp of a single object with different number of fingers (a grasping survey is presented by Prattichizzo and Trinkle (2008)). These works, deals with FC grasps of 2D objects, either polygonal object with frictional (Park and Starr, 1990; Ponce et al., 1995) and frictionless (Cornellà and Suárez, 2009) contacts, or non polygonal objects with frictional (Niparnan and Sudsang, 2006) and frictionless contacts (Cornellà and Suárez, 2005), as well as with FC grasps of 3D objects, either polyhedral with frictional contacts (Prado and Suárez, 2008), or objects of any shape with frictionless (Roa and Suárez, 2009) and frictional (Dini and Failli, 2000; El-Khoury and Sahbani, 2010; Daoud et al., 2011) contacts. However, there are less works dealing with the grasp and manipulation of articulated objects.

Relevant works regarding the immobilization of a 2D serial chain with n polygons using frictionless contacts determined that for polygons without parallel edges $n + 2$ contacts are necessary if $n \neq 3$ and n+4 contacts otherwise (Cheong et al., 2002); and for polygons with parallel edges $n + 2$ contacts are necessary if n is even and $n + 3$ if n is odd (Cheong et al., 2007). These works also presented a procedure for robust immobilization (i.e. any contact can be perturbed slightly without destroying the immobilization) for 2D serial chains with n polygons. More recently, Rimon and van der Stappen (2012) presented a systematic procedure to find a set of contact points in order to achieve the immobilization of a 2D serial chain with n polygons using also frictionless contacts and based on second order effects; they demonstrated that the lower bound for the number of contact points needed to immobilize any chain of $n \neq 3$ hinged polygons without parallel edges is $n + 2$, and in any case, $n + 3$ frictionless points are enough to immobilize a chain of n polygons of any type. Other works deal with articulated objects using different approaches such as interactive perception algorithms (Katz and Brock, 2008) or an occlusion aware reconstruction system (Huang et al., 2012) to acquire a model of the articulated object to be manipulated by a robot.

Another active area in robotic grasping is to find the optimal grasp configuration based on one of the two properties mentioned (force or form closure), which means having to choose a quality criterion to evaluate the grasp. Several criteria have been proposed for the evaluation of the grasp quality (Roa et al., 2008). One of most used criterion is the maximum wrench that the grasp can resist in any direction (known as the criterion of the maximum

[⋆] This work was partially supported by the Spanish Government through the projects DPI2010-15446, DPI2011-22471 and DPI2013-40882-P. Contact email: raul.suarez@upc.edu

ball) (Ferrari and Canny, 1992). Based on this criterion several works were proposed to evaluate grasps for 2D objects using frictionless (Cornellà and Suárez, 2009) and frictional (Zhu et al., 2001) contacts, and for 3D objects, either polyhedral or not polyhedral (Roa and Suárez, 2009; Zheng, 2013). Another criterion used as grasp quality measure is the Q distance, used to find optimal grasps for 2D and 3D objects (Zhu and Wang, 2003; Phoka et al., 2006). There also task-oriented quality measures that take into consideration a particular condition of a task (Xue et al., 2011).

In addition to the above mentioned quality criteria, there are other criteria that have been proposed to evaluate grasps and fixtures of 2D and 3D objects, one of these criteria determines optimum gripping points by minimizing the average value of the normal forces applied on the object in order to balance any external perturbation (Mangialardi et al., 1996); on the other hand a fixturing can be evaluated using two indexes, one that minimizes the sum of all normal contact forces and the other minimizes the maximum normal contact force (Xiong et al., 2002); other proposals for assessing the quality of a grasp is the distance between the wrench origin and two sets of generalized wrenches (the union or the Minkowski sum of primitive contact wrenches) (Zheng and Qian, 2009) and the distance between the convex hulls of the absolute grasp wrench space and the object wrench space (Jeong and Cheong, 2010). Although different approaches have been proposed using different quality criteria for evaluating a grasp or fixture, these are focussed on a single object, either 2D or 3D. Therefore the aim of this paper is the proposal of a procedure to find optimal grasps for 2D articulated objects using a specific quality criterion.

In this work we consider general grasps of 2D serial articulated objects with n links, and $m = n + 2$ degrees of freedom (dof), considering the minimum number of frictionless contacts $k = m+1 = n+3$. First, we compute an FC grasp and, then, we optimize it. The first problem is addressed with the algorithm developed by Alvarado and Suárez (2013). The second problem is solved considering a particular quality measure. The algorithms developed in this work are based on the work done by Roa and Suárez (2009), extending it to the case of articulated objects, i.e. to the case of wrenches with dimensions other than 3 and 6 for 2D and 3D objects respectively.

The contribution of this work is a systematic procedure to find a local optimal FC grasp of articulated objects using a generalized wrench space. This work does not take into account the particular device used for the grasp as it is done for rigid objects by Roa et al. (2011); Gilart and Suárez (2012), so it may happen that the contact points of the optimal grasp procedure were nor reachable by a particular robotic hand. Nevertheless the approach is always useful for object fixturing in industrial applications.

The rest of the paper is structured as follows. Section II presents the problem statement and the main assumptions. Section III presents a procedure to find the elements of the vector of generalized wrenches for an articulated object with n links. Section IV describes the algorithm for the optimization process. Section V shows some examples of

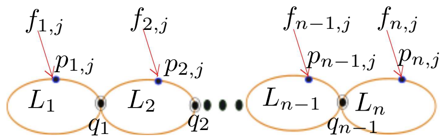

Fig. 1. Articulated object with n links.

the proposed approach. Finally, Section VI presents some conclusions and the future work.

2. PROBLEM STATEMENT AND ASSUMPTIONS

Consider a 2D serial articulated object with n links and rotational joints, as illustrated in Fig. 1. The problem to be addressed is the following:

- Find a local optimal grasp starting from initial FC grasp. The optimization is performed using as quality measure the largest perturbation wrench that the grasp can resist in any direction (Ferrari and Canny, 1992), (Roa et al., 2008).

The following assumptions are considered in this work:

- The links are connected by rotational joints.
- The links can overlap each other, this does not generate any problem.
- The objects can be of any shape (either polygonal or non-polygonal).
- The boundary of each link is represented with a (large enough) set Ω of points described by position vectors $\boldsymbol{p}_{i,j}$.
- The normal direction $\hat{\boldsymbol{n}}_{i,j}$ pointing towards the interior of the object at each point $\boldsymbol{p}_{i,j}$ is known.
- The contact points between the fingers and the object are frictionless. This assures a worst case grasp, since the existence of friction in real cases will increase the robustness of the grasp.

3. GENERALIZED WRENCHES FOR ARTICULATED OBJECTS

3.1 Generalized wrenches for a serial articulated object

For a single solid object and considering frictionless contact points, the grasp forces $\boldsymbol{f}_i$ can only be applied in the direction $\hat{\boldsymbol{n}}_i$ normal to the object boundary at the point $\boldsymbol{p}_i$. A force $\boldsymbol{f}_i$ generates a torque $\boldsymbol{\tau}_i = \boldsymbol{p}_i \times \boldsymbol{f}_i$ with respect to the center of mass of the object, $\boldsymbol{f}_i$ and $\boldsymbol{\tau}_i$ are grouped in a wrench vector $\boldsymbol{w}_i = [\boldsymbol{f}_i \boldsymbol{\tau}_i]^T$.

A grasp defined by a set of k frictionless contacts, $G = \{\boldsymbol{p}_1, ..., \boldsymbol{p}_k\}$, is able to apply k wrenches $\boldsymbol{w}_i$ on the object, which can be grouped in a wrench set $W = \{\boldsymbol{w}_1, ..., \boldsymbol{w}_k\}$. The information in W is enough to analyze whether G allows a FC grasp or not.

For a planar object, 4 frictionless contacts are sufficient to assure the FC condition, i.e. a set of points $G = \{\boldsymbol{p}_1, ..., \boldsymbol{p}_4\}$ allowing a set of wrenches $W = \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_4\}$.

The generalized wrench for a serial articulated object described in this section is the generalization to n links of the procedure developed for 2 and 3 links by Alvarado and Suárez (2013). The procedure considers a virtual robot of $n + 2$ joints (see Fig 2) wherein the first and second joint

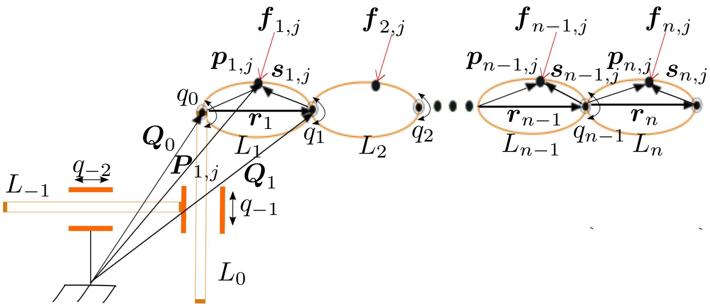

Fig. 2. General diagram of the virtual robot, where links $L_{-1} \dots L_n$ represent the total links of the virtual robot, and $L_1 \dots L_n$ are the articulated object's links.

are virtual ones and the rest of them are equivalent to the articulated object to be grasped.The following basic nomenclature is used in the procedure:

L_i**:** Link i of the virtual robot, $i = -1, \dots, n$, L_{-1} and L_0 are virtual ones, while L_1 to L_n are the real ones.

q_i**:** Joint i of the virtual robot, $i = -2, \dots, n-1$, q_{-2} to q_0 are virtual ones, and q_1 to q_{n-1} are the real ones.

$\boldsymbol{Q}_i$**:** Position of the joint q_i, $i = 0, \dots, n-1$, and position of the final end of the link $L_n (i = n)$ respect to the frame base.

$\boldsymbol{P}_{i,j}$**:** Contact point j on link L_i respect to the base frame.

$\boldsymbol{p}_{i,j}$**:** Contact point j on link L_i represented with respect to joint q_{i-1} (i.e. $\boldsymbol{p}_{i,j} = \boldsymbol{P}_{i,j} - \boldsymbol{Q}_{i-1}$), $i = 1, \dots, n$, $j = 1, \dots, k_i$, where k_i is the number of contact points on link L_i. The total number of contacts is $k = \sum_i k_i$.

$\boldsymbol{r}_i$**:** Position $\boldsymbol{Q}_i$ respect $\boldsymbol{Q}_{i-1}$ (i.e. $\boldsymbol{r}_i = \boldsymbol{Q}_i - \boldsymbol{Q}_{i-1}$).

$\boldsymbol{s}_{i,j}$**:** Contact point j on link L_i respect to $\boldsymbol{Q}_i$ (i.e. $\boldsymbol{s}_{i,j} = \boldsymbol{P}_{i,j} - \boldsymbol{Q}_i = \boldsymbol{p}_{i,j} - \boldsymbol{r}_i$).

$\boldsymbol{f}_{i,j}$**:** Force j applied to link L_i at contact point $\boldsymbol{p}_{i,j}$.

For each link L_i the Jacobian J_i is computed in order to relate the external forces and moments applied to each link L_i with the torques and forces in the joints necessary for an equilibrium condition. Considering the Jacobian J_i, the torque τ_k in joints due all forces $\boldsymbol{f}_{i,j}$ applied to all the links L_i is obtained by :

$$\boldsymbol{\tau} = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} \boldsymbol{\tau}_{i,j} = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} J_i^T \boldsymbol{w}_{i,j} = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} J_i^T \begin{bmatrix} f_{x_{i,j}} \\ f_{y_{i,j}} \\ M_{s_{i,j}} \end{bmatrix} \quad (1)$$

where, $\boldsymbol{w}_{i,j}$ is a vector that contains $\boldsymbol{f}_{i,j}$ and the moment with respect $\boldsymbol{Q}_i$ (i.e, $M_{s_{i,j}} = \boldsymbol{s}_{i,j} \times \boldsymbol{f}_{i,j}$).

Therefore, the total torques τ_k ($k = -2, ..., n-1$) from eq(1) are :

$$\begin{aligned}
\tau_{-2} &= \sum_j f_{x_{1,j}} + \sum_j f_{x_{2,j}} + \dots + \sum_j f_{x_{n-1,j}} + \sum_j f_{x_{n,j}} && = 0 \\
\tau_{-1} &= \sum_j f_{y_{1,j}} + \sum_j f_{y_{2,j}} + \dots + \sum_j f_{y_{n-1,j}} + \sum_j f_{y_{n,j}} && = 0 \\
\tau_{0} &= \sum_j \boldsymbol{p}_{1,j} \times \boldsymbol{f}_{1,j} + \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{2,j} + \dots + \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{n-1,j} + \\
&\quad \sum_j \boldsymbol{r}_1 \times \boldsymbol{f}_{n,j} && = 0 \\
\tau_{1} &= \sum_j \boldsymbol{p}_{2,j} \times \boldsymbol{f}_{2,j} + \dots + \sum_j \boldsymbol{r}_2 \times \boldsymbol{f}_{n-1,j} + \sum_j \boldsymbol{r}_2 \times \boldsymbol{f}_{n,j} && = 0 \\
&\vdots \\
\tau_{n-2} &= \sum_j \boldsymbol{p}_{n-1,j} \times \boldsymbol{f}_{n-1,j} + \sum_j \boldsymbol{r}_{n-1} \times \boldsymbol{f}_{n,j} && = 0 \\
\tau_{n-1} &= \sum_j \boldsymbol{p}_{n,j} \times \boldsymbol{f}_{n,j} && = 0
\end{aligned} \quad (2)$$

Now, it is possible to consider a generalized wrench space $\mathcal{W}$ defined by the base $\{\hat{\boldsymbol{\tau}}_{-2}, \hat{\boldsymbol{\tau}}_{-1}, \hat{\boldsymbol{\tau}}_0, \hat{\boldsymbol{\tau}}_1, \dots, \hat{\boldsymbol{\tau}}_{n-2}, \hat{\boldsymbol{\tau}}_{n-1}\}$, such that the generalized wrenches $\boldsymbol{W}_{i,j}$ generated respectively by forces $\boldsymbol{f}_{i,j}$ are

$$\boldsymbol{W}_{1,j} = \begin{bmatrix} f_{x_{1,j}} \\ f_{y_{1,j}} \\ \boldsymbol{p}_{1,j} \times \boldsymbol{f}_{1,j} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{W}_{2,j} = \begin{bmatrix} f_{x_{2,j}} \\ f_{y_{2,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{2,j} \\ \boldsymbol{p}_{2,j} \times \boldsymbol{f}_{2,j} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

$$\boldsymbol{W}_{n-1,j} = \begin{bmatrix} f_{x_{n-1,j}} \\ f_{y_{n-1,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{n-1,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}_{n-1,j} \\ \vdots \\ \boldsymbol{p}_{n-1,j} \times \boldsymbol{f}_{n-1,j} \\ 0 \end{bmatrix} \quad \boldsymbol{W}_{n,j} = \begin{bmatrix} f_{x_{n,j}} \\ f_{y_{n,j}} \\ \boldsymbol{r}_1 \times \boldsymbol{f}_{n,j} \\ \boldsymbol{r}_2 \times \boldsymbol{f}_{n,j} \\ \vdots \\ \boldsymbol{r}_{n-1} \times \boldsymbol{f}_{n,j} \\ \boldsymbol{p}_{n,j} \times \boldsymbol{f}_{n,j} \end{bmatrix}$$

The dimension of $\mathcal{W}$ is $n+2$ and a generalized wrench $\boldsymbol{W}_{i,j}$ has therefore $m = n+2$ components.

3.2 Force-closure Test

Considering the set $G = \{\boldsymbol{p}_{i,j}, i = 1, ...n, j = 1, ..., k_i\}$ of $k = \sum_i k_i$ contact points (where k_i is the number of contact points on link L_i) and a force $\boldsymbol{f}_{i,j}$ applied at each $\boldsymbol{p}_{i,j}$ in L_i, a set $W = \{\boldsymbol{W}_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ is obtained. The necessary and sufficient condition for the existence of an FC grasp is that the origin of the generalized wrench space lies inside the convex hull of the contact wrenches W, hereafter $CH(W)$ (Mishra et al., 1987; Murray et al., 1994). The test used in this work to verify this condition is derived from (Roa and Suárez, 2009) for the case of a single rigid object and extended by Alvarado and Suárez (2013) for an articulated 2D object (it can also be done using linear programming techniques (Liu et al., 2004; Asada and Kitagawa, 1989)). Let P be the centroid of W, O the origin of the wrench space and H a boundary hyperplane of $CH(W)$, a grasp is FC if P and O lie on the same side of each hyperplane H.

4. FINDING OPTIMAL GRASP

In this section we present an algorithm to optimize an initial FC grasp, which could be obtained using the Grasp Synthesis (GS) algorithm presented by Alvarado and Suárez (2013). The main idea of the algorithm described here is a generalization derived from that presented by Roa and Suárez (2009), but in this case applied to an articulated object with n links and $n+2$ *dof*, producing therefore a generalized wrench space of dimension $n+2$ and not only of dimension 3 and 6 for 2D and 3D rigid objects, respectively. The quality of the grasp is measured by the criterion of the "maximum ball" (Ferrari and Canny, 1992;

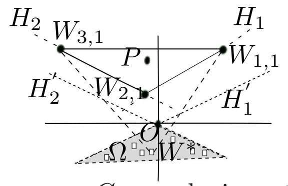

Fig. 3. The grasp G producing the wrench set $W = \{W_{1,1}, W_{2,1}, W_{3,1}\}$ is non FC. H_1 and H_2 are the hyperplanes that fail the force closure test, and then $W_{2,1}$ indicates the point to be replaced. The replacement point lie in region bounded by the separating hyperplanes H_1' and H_2', passing through the origin and parallel to H_1 and H_2, respectively. Choosing the contact point associate to the wrench W^* in this region, the new obtained grasp is FC.

Roa et al., 2008), which indicates the largest perturbation wrench that the grasp can resist with independence of its direction. Geometrically, the quality is equivalent to the radius of the largest ball centred at the origin of the wrench space and entirely contained in $CH(W)$. This measure of quality is equal to the distance from the origin of the wrench space to the nearest face of $CH(W)$ (Ferrari and Canny, 1992). Algorithm 1 formally describes the optimization procedure, whose main steps are the following.

Step (1) searches an initial FC grasp G^1, in our implementation it is done using the algorithm GS mentioned above. A brief description of the algorithm GS is as follows. It starts generating a random grasp G; if G is not FC then a iterative replacement of contact point in G is executed. The contact points to be replaced are those whose wrenches define boundary hyperplanes H of $CH(W)$ that do not satisfy the condition of the force-closure test describe in the previous section. The points for the replacement are those whose wrenches belong to the region of the wrench space bounded by separating hyperplanes H' containing the origin O and parallel to the hyperplanes H. Figure 3 illustrates a replacement for an hypothetical 2D wrench space.

Step (3) simply forms the corresponding wrench set W^l for the current considered grasp G^l.

In Step (4) the algorithm searches the facet of $CH(W^l)$ closest to the origin, hereinafter F_Q, and calculates the distance from F_Q to the origin, i.e. the quality of the current grasp Q^l.

In Step (5) a subset Ω_C^l with candidate points to replace one of the points that define F_Q is built. This subset is determined employing separating hyperplanes. Let H_Q be the hyperplane that contains the face F_Q. The subset Ω_C^l is formed with the points lying in the open half-space defined by H_Q that does not contain the origin O of the wrench space. Figure 4 shows a hypothetical 2-dimensional wrench space with the subset Ω_C^l containing all the points in half space H_Q^+.

Step (14) generates $k-1$ (where k is the number of contact points) candidate grasps G_i^*, $i = 1, ..., k-1$, using a candidate point W_j^* from Ω_C^l to replace each of the $k-1$ vertices of the face F_Q. The verification of the FC property is performed for each candidate grasp G_i^*.

Algorithm 1 Search of a local optimum FC grasp

Ensure: : Optimal FC grasp G^l.
1: Find an initial FC grasp G^l, $l = 1$.
2: **repeat**
3: Form the wrench set W^l.
4: Determine the facet of $CH(W^l)$ closest to the origin, hereinafter F_Q. The distance from the origin to FQ is the current quality Q^l.
5: Build the set Ω_C^l (set with m points) with candidate points that may produce an improvement in the grasp.
6: $i = 1$, number of the candidate grasps G_i^*.
7: $j = 1$, number of the candidate point W_j^*.
{k is the number of contact points}
8: **repeat**
9: **repeat**
10: **if** $i = k$ **then**
11: $j = j + 1$
12: $i = 1$
13: **end if**
14: Generate candidate grasps G_i^*. G_i^* is formed using a candidate point $W_j^* \in \Omega_C^l$ replacing each of the vertices that define the face F_Q.
15: $i = i + 1$
16: **until** (G_i^* is a FC grasp **or** $j = m$).
17: Compute the quality Q^* for the grasp G_i^*.
18: **until** ($Q^* > Q^l$ **or** $j = m$)
19: Update $l = l + 1$ and $G^l = G_i^*$
20: **until** ($j = m$)
21: **return** G^l

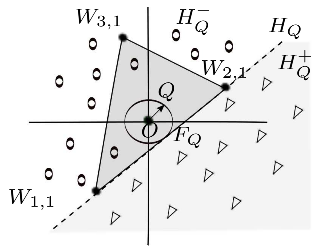

Fig. 4. Example of the subset Ω_C^k with the candidate points (triangles within the shaded area) that could improve the quality of the grasp by replacing a vertex of $F_Q = \overline{W_{1,1}W_{2,1}}$

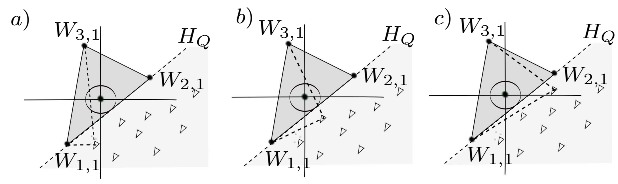

Fig. 5. Possible cases of candidate grasps: (a) non FC candidate grasp, (b) candidate grasp reducing the current quality, (c) feasible candidate grasp.

In Steps (17) and (19) the quality Q^* is calculated for each candidate grasp G_i^*; if a candidate grasp fulfills $Q^* > Q^l$ it becomes the new grasp G^l. The procedure continues until no improvement in the quality is achieved.

Figure 5 illustrates the three possible cases regarding a candidates grasp; case (a) is an unreliable grasp since it loses the FC condition; case (b) is discarded because the quality of the candidate grasp is smaller than the quality

Table 1. Result of the optimal grasp search.

	# iter. & time first FC grasp	# iter. & time optimal FC grasp	IQ	FQ	Ratio
Example 1	6 in 8s	8 in 8s	0.0451	0.1190	2.6386
Example 2	12 in 14s	6 in 9s	0.1322	0.1633	1.2352
Example 3	2 in 1s	7 in 14s	0.0267	0.0558	2.0899
Example 4	11 in 8s	5 in 11s	0.0268	0.0459	1.7127
Example 5	2 in 1s	6 in 11s	0.1051	0.1109	1.0552
Example 6	8 in 14s	7 in 13s	0.0424	0.0631	1.4882
Example 7	6 in 11s	6 in 8s	0.0131	0.0140	1.0687
Example 8	104 in 250s	3 in 7s	0.02025	0.02038	1.0064
Example 9	7 in 9s	5 in 7s	0.00856	0.02866	3.3481

of the current grasp, and case (c) is a candidate grasp that improves the quality of the current grasp, therefore it becomes the current grasp for the next iteration.

5. EXAMPLES

In this section the proposed optimization strategy is illustrated with some examples for articulated objects with 2 and 3 links. The proposed approach has been implemented using Matlab and C++ on a Intel Core2 Duo 2.0 GHz computer. Let IQ and FQ be the initial and the optimal quality of FC grasps, respectively. Table 1 shows the results of the examples, which are graphically shown in Fig. 6 to Fig. 14.

6. CONCLUSIONS

In this paper we propose an approach to obtain a locally optimal grasp for 2D articulated objects with n links considering frictionless contacts. First, an initial FC grasp is obtained (we use for it an algorithm developed in a previous work), and then, starting from this initial grasp, and optimization procedure is applied iteratively changing the contact points until arriving to a local optimum grasp, this optimization procedure is the main contribution of this work. The quality measure used in the optimization process is the largest wrench than the grasp can resist in any direction. As future work we consider obtaining of optimal grasps for 2D articulated objects with frictional contacts and the generalization of the approach for 3D articulated objects with both frictionless and frictional contacts. Considering independent contact regions (ICRs) as regions such that a contact in each of them assures an FC grasp, another line of research for future work is the computation of ICRs for articulated objects in order to deal with finger positioning errors in real applications.

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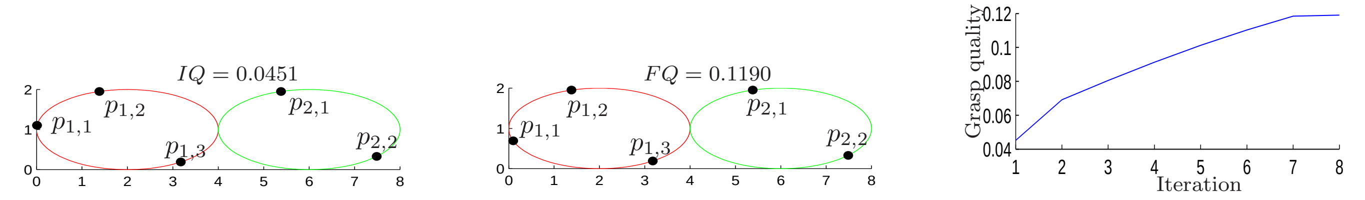
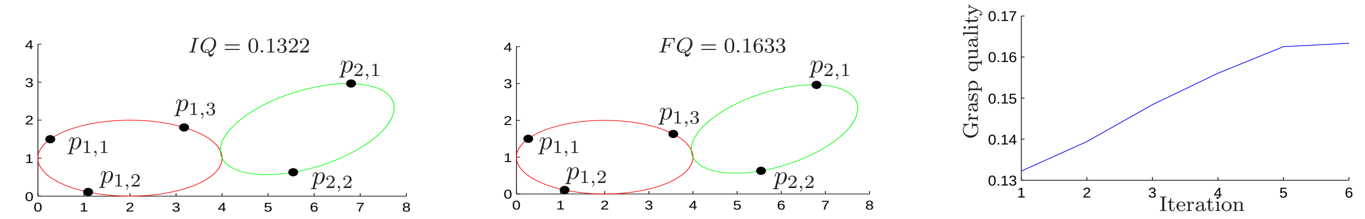


a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 6. Example 1.



a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 7. Example 2.

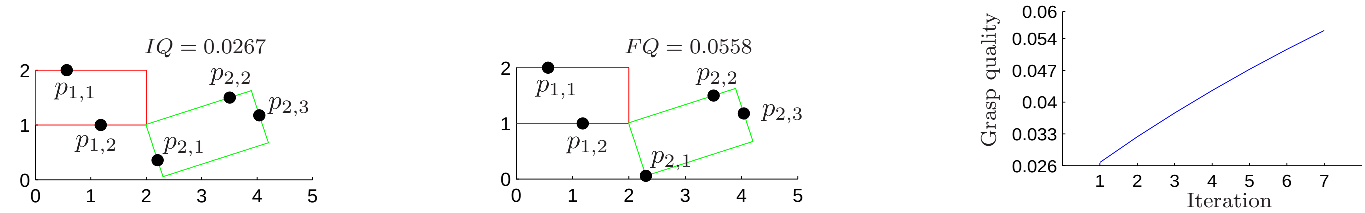


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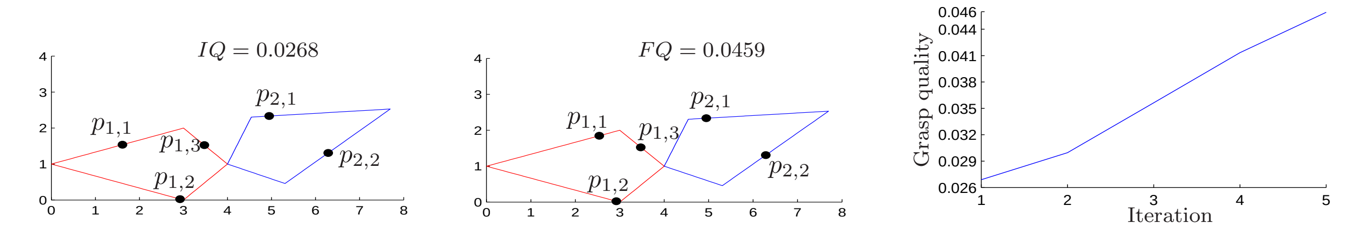
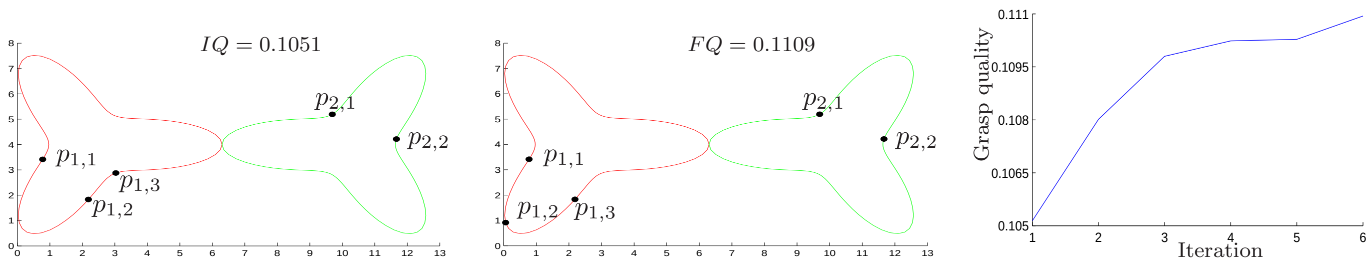
Fig. 8. Example 3.



a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 9. Example 4.



a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 10. Example 5.

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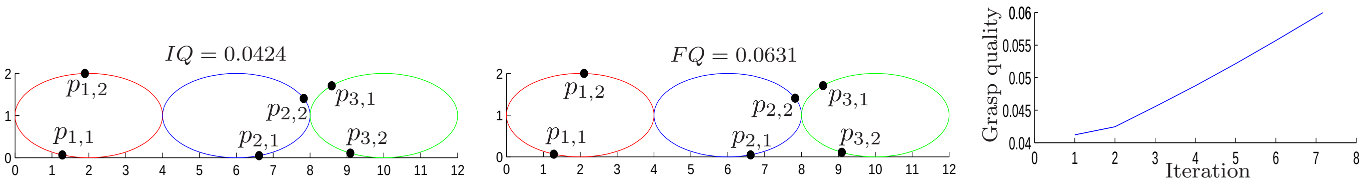
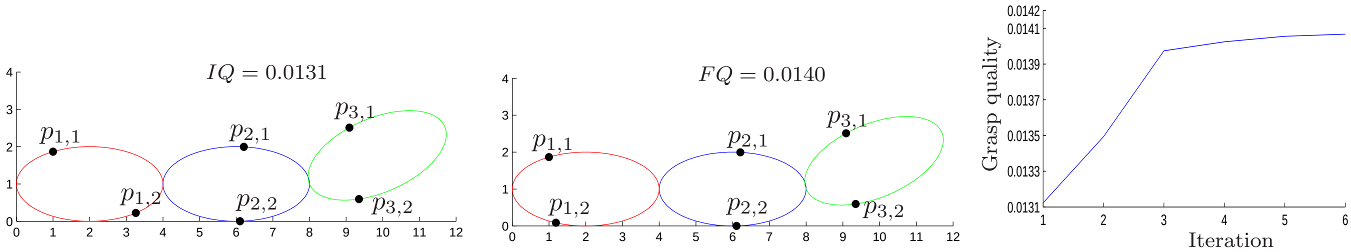


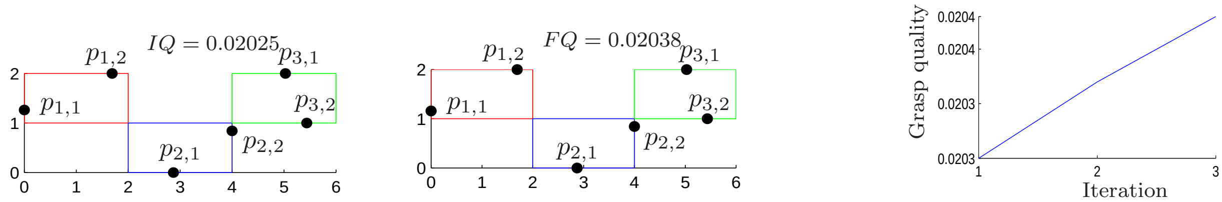
a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 11. Example 6.



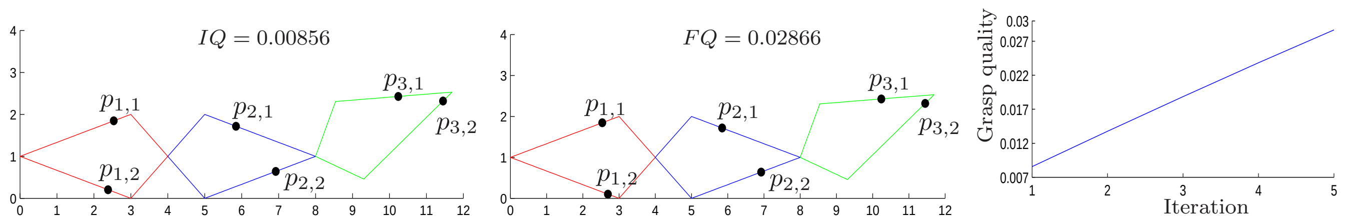
a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 12. Example 7.



a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

Fig. 13. Example 8.



a) Initial FC grasp b) Optimal FC grasp c) Grasp quality

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