Towards a Standardized Cost Measure of Assembly Operations *

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Abstract

A statistical measure of the cost of assembling two mating features is presented. It is based on how well the geometry of the task compensates, during the assembly, the uncertainties associated with the mating features: manufacturing tolerances and the pose uncertainty. The latter is due either to the accumulation of tolerances and clearances between mating features that are previously assembled or to the positioning errors of an assembly robot. The proposed measure allows a systematic evaluation of the assembly cost for a given assembly sequence, thus providing a method for evaluating assembly sequences and selecting an optimal one from the assembly point of view. The influence of different sources of uncertainties and the assembly procedure can also be evaluated with the proposed cost index. The paper includes an example to illustrate the use of the proposed cost measure.

1 Introduction

A product should be designed not only for the functionality but also for the manufacturability and assemblability. These factors are often reflected in the specification of tolerances and clearances involved in the mating features of parts. Although functionality, manufacturability, and assemblability are of prime necessity, it is also very important to consider, in the design, the difficulty of assembly with a direct connection to assembly cost. A product may meet the specifications of tolerances and clearances that ensure its assemblability, but the difficulty of its assembly may be too high to be acceptable.

The relationship between the difficulty of product assembly and the specification of tolerances and clearances associated with mating features is not as obvious as one might imagine. For instance, in the assembly of a chain of parts and subassemblies, the clearance that makes a part-mating easier may increase the difficulty for the next assembly (figure 1). Suppose that object 2 is first assembled with object 1, and then object 3 is added to the resulting subassembly. A large clearance between objects 1 and 2 will make the first step easier but will generate larger uncertainty in the position of object 2, i.e., the mating feature for the second step, as shown in figure 1b. This makes the final step of the assembly more difficult, and the assembly cost of object 3 can be large, since it becomes necessary to use fixtures [1], a specific sensor measurement or a fine-motion strategy [2], in order to reduce the uncertainties for the second step of the assembly.

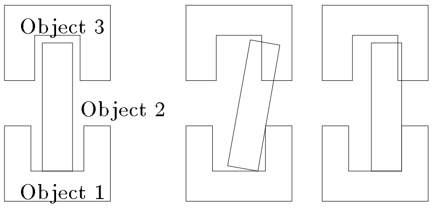

Figure 1: *Advantages and disadvantages of the clearance in a three object assembly.*

In assembly planning, it is necessary to have a viable method for automatically selecting optimal assembly sequences in terms of assembly cost since there is a large number of feasible assembly sequences involved in product assembly [3][4]. However, the development of such a method has been hampered by the difficulty of establishing good performance criteria that can be directly linked to assembly cost. For instance, a measure of selecting preferred assembly sequence is the degree of uncertainties in part positions during assembly defined in terms of entropy [5][6]. Yet, this measure is concerned with a high level decision on part mating poses in terms of part geometries;

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this does not address the uncertainties associated with mating operations both due to both tolerances and clearances of mating features and tool positioning errors. The latter is more directly connected to assembly cost. Other criteria for selecting optimal assembly sequences have been proposed, including stability, manipulability, directionality, etc; they are based on the number of required fixtures as well as on the number of required reorientations, and can be used for the estimation of assembly cost [7]. For instance, the stability of subassembly, either based on geometric form closure [8] or on force balance [9][10][11], may be an important factor for determining the required number of fixtures and reorientations during the assembly. However, selecting an assembly sequence based on the above criteria, including stability, is necessary but not sufficient: the cost associated with mating operations (in terms of the uncertainties of mating poses resulted from tolerances and clearances) should ultimately be incorporated into the optimization process.

In this paper, we present a statistical measure of assembly difficulty that provides a cost index for mating operations. The main purpose is to obtain the systematical evaluation of the cost associated with different assembly sequences of a product. The measure can also be used to evaluate the influence of different uncertainties, the assembly direction, and the shape of the objects in the assembly cost of the product.

2 Measure of the assembly cost

2.1 Problem Statement

Consider an assembly operation that, according to the nominal model of the objects and their nominal positions, is able to be solved with a movement along a given degree of freedom. The difficulty of such an assembly operation is a function of the uncertainties that produce geometric deviations in the degrees of freedom orthogonal to the assembly movement (figure 2). The greater the effect of these uncertainties, the more difficult the assembly is. Since the deviations due to these uncertainties are not exactly known, their effect has to be considered statistically.

In this context, the problem addressed in this work is the statistical quantification of the cost of an assembly operation according to how well, during the assembly, the geometry of the objects can decrease the effect of the uncertainties orthogonal to the mating direction.

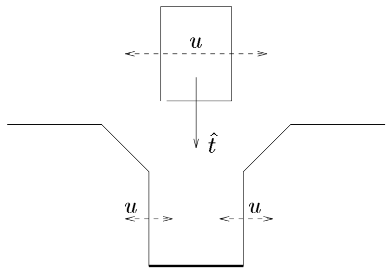

Figure 2: *Assembly direction ($\hat{t}$) and uncertanties increasing the difficulty of the task (u).*

2.2 Proposed approach

In the proposed approach, the cost of the assembly of two mating features is statistically computed. It is done by considering the cost of the assembly from each possible initial configuration of the objects (compatible with the nominal one), weighted by the probability of the occurrence of that configuration.

For any initial configuration, the cost is evaluated as the summation of a local cost at each point of the path from the initial configuration until a goal configuration.

The local cost can be defined according to different criteria, but it is basically a function that locally indicates how much one of the objects constrains the movement of the other in the assembly direction; thus, any non-contact configuration has zero local cost; a contact configuration that does not allow the movement in the assembly direction has a maximum local cost; and in any other contact configuration, the local cost will be a function of the relation between the assembly direction and the unitary normal to the C-surface associated in Configuration Space with the contact situation.

Assuming that the mating movement direction of one object with respect to the other has already been determined by any procedure (e.g. any assembly planner), the following items are directly considered in the approach:

a) The uncertainty affecting the relative position between the features to be assembled. This uncertainty is due to the accumulation of uncertainty from previous assembled objects (i.e. tolerances and the effect of clearances as in figure 1) plus the uncertainty of the assembly device itself (e.g. the robot).

b) The geometry of the features to be assembled. The shape of the objects affects the way in which the assembly is performed when there is uncertainty (e.g. use of chamfers). In this sense, one characteristic of the proposed approach is that the cost (difficulty) of the assembly operation is not just a function of the "clearance", but a function of the "variation of the clearance" during the evolution of the assembly.

c) The uncertainty affecting the geometry of the features to be assembled. The effect of tolerances of the object manufacturing must also be statistically considered in the cost evaluation.

Formal definition of the approach

Given two mating features F_i and F_j, the assembly direction $\hat{t}$, and being $\vec{p}_j$ the relative initial pose of F_j with respect to F_i, then (refer to figure 3):

Definition 1. The Play C_j of feature F_j with respect to feature F_i is the set of possible poses of F_j relative to F_i, once they have been assembled (C_j can be considered to be the goal of the assembly).$\diamond$

Definition 2. The Direct-Assembly region R_j of F_j is the set of poses swept by C_j along -$\hat{t}$ (R_j is a particular case of the back-projections defined in [13]).$\diamond$

Definition 3. The Mate-Adjustment $M_j(\vec{p}_j)$ of feature F_j is the path which, in order to perform the task, the assembly device will follow to displace F_j from $\vec{p}_j$ to $\vec{p}_c \in R_j$ (Note that $M_j(\vec{p}_j)$ may depend both on the device and the strategy used to solve the task).$\diamond$

Definition 4. The cost of the assembly operation of F_i and F_j from $\vec{p}_j$ is

$$f(\vec{p}_j) = \int_{M_j(\vec{p}_j)} g(\vec{p}) \quad \mathrm{d}p$$

where $g(\vec{p})$ is the local cost associated with each pose $\vec{p} \in M_j(\vec{p}_j)$.$\diamond$

Definition 5. The Statistical Cost $\mathcal{D}$ of the assembly of two mating features F_i and F_j is:

$$\mathcal{D} = \int_{domain(\vec{p}_j)} f(\vec{p}_j) \;\; d(\vec{p}_j) \quad \mathrm{d}p_j$$

where $d(\vec{p}_j)$ is the probability distribution of $\vec{p}_j$.$\diamond$

Procedure to compute $\mathcal{D}$

1. Given the nominal conditions and the assembly direction $\hat{t}$, compute the probability distribution $d(\vec{p}_j)$ of the initial pose $\vec{p}_j$ of feature F_j with respect to F_i by:
   (a) propagating the pose uncertainty due to previous subassemblies (tolerances and clearances).
   (b) adding the positioning system uncertainty.

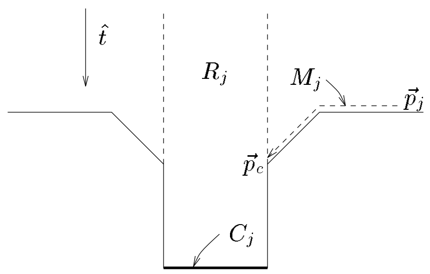

Figure 3: *Configuration space of F_j referred to F_i.*

2. For each initial pose $\vec{p}_j$ compute the cost of the adjustment in the object positions in order to perform the assembly by:
   (a) computing the path $M_j(\vec{p}_j)$ to solve the task.
   (b) computing the cost $f(\vec{p}_j)$ of the path as the summation of the local cost $g(\vec{p})$ in each point $\vec{p} \in M_j(\vec{p}_j)$.
3. Compute the statistical total cost $\mathcal{D}$ of the assembly as the summation for each initial pose $\vec{p}_j$ of the cost $f(\vec{p}_j)$ weighted with the probability density $d(\vec{p}_j)$.

2.3 On the uncertainties in the pose of mating features due to tolerances and clearances

As pointed out previously, the uncertainties in the pose of mating features come either from the accumulation of tolerances of, and clearances between, mating features already assembled or the positional errors of mating tools such as a robot. Since the positional errors of mating tools are straightforward to define, we will then focus on how the accumulation of tolerances and clearances determines the uncertainties in the pose of mating features.

In general, the pose and dimension of mating features of a part are specified in the design in terms of their nominal values and associated tolerances [12]. Then, the clearance between two mating features can be determined from the nominal clearance range defined from the nominal dimensions of two mating features and the tolerance of clearance range defined from the tolerances in the nominal dimensions of individual mating features. The uncertainties associated with mating features are then determined from the pose tolerances and clearances of individual mating features. For instance, assuming that a part A is placed on the fixture, the uncertainty in the pose of a mating feature a1 of A can be determined by the pose tolerance associated with a1. Also assume that we bring part B to assemble with part A such that the mating feature b1 of B mates with a1. Then, the uncertainty in the pose of a mating feature b2 of B can be determined by the Minkowsky sum of the pose tolerances of a1 and b2 as well as the clearance range between a1 and b1.

Assembly involves the combination of serial and parallel chains of mating operations that is defined by a particular assembly sequence. A serial chain operation is here understood as the mating operation which does not interconnect between two parts or subassemblies placed on fixtures. In this case, a mating feature of a part, or a subassembly which is to be mated with a mating feature of another part, or a subassembly on fixtures, is free to move in space. On the other hand, a parallel chain operation involves the mating of two parts or subassemblies which are placed on fixtures. In a serial mating operation of B with A (A is fixtured) at the mating features of a1 of A and b1 of B, where A and B are either a part or a subassembly, the uncertainty in the pose of a mating feature, b2, of B is the accumulation (or the statistical Minkowski sum) of the pose uncertainties of a1 and b2 and the clearance range between a1 and b1. In a parallel mating operation of B and A (both are fixtured) at the mating features a1 of A and b1 of B, the pose uncertainties of a mating feature b2 of B is the intersection of the pose uncertainties of b1 with the pose uncertainties of a1, accumulated with the clearance range between a1 and b1. The actual computation of this accumulation and intersection of pose uncertainties can be performed, for example, by representing the tolerance and clearance volumes in the coordinate frame in the form of ellipsoids, and by applying the Monte-Carlo simulation method for the statistical computation involved in tolerance propagation, accumulation, and intersection. For more details, refer to [12].

3 Example : Determination of an optimal sequence

The example of figure 1 considering only translational degrees of freedom will be used to illustrate the selection of a better assembly sequence according to the evaluation of the total assembly cost.

Let $2h_1$ and $2h_3$ be the width of the holes H_1 and H_3 in objects 1 and 3 respectively, and $2p_2$ be the width of the peg P_2, i.e object 2 (figure 4). The clearance between H_1 and P_2 is $c_{1,2} = 2(h_1 - p_2)$ and between H_3 and P_2 is $c_{2,3} = 2(h_3 - p_2)$. In order to simplify the example, the uncertainty in the object

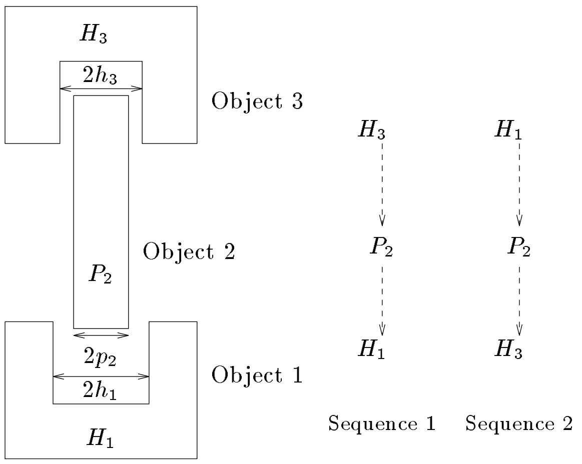

Figure 4: *The three object to be assembled.*

shape and size is neglected. Considering the assembly direction parallel to the y-axis, and since there are no chamfers, the local cost is constant for any configuration in which the objects cannot be assembled,i.e. $g(x) = n_y$.

Let us assume that the first object is precisely placed on the fixture and that the manipulator used for the assembly has a uniform deviation in the range $\pm\delta_r$, which will also be the deviation in the positioning of the second and third objects when they are manipulated. The symmetry axis of the objects are nominally aligned for the assembly. Next, the cost of the two possible assembly sequences will be analyzed.

Sequence 1: H_1 with P_2 and then H_3.

Cost $\mathcal{D}_{1,2}$ of the assembly of H_1 with P_2. Since the manipulator deviation has a uniform distribution, the statistical distribution $d_1(x)$ of the initial pose of P_2 with respect to H_1 is also uniform, as is shown in figure 5. Since the local cost is constant, $g(x) = n_y$, then $f(x) = n_y(x - c_{1,2})$, and the total cost $\mathcal{D}_{1,2}$ of this subassembly is:

if $\delta_r < c_{1,2}$ then $\mathcal{D}_{1,2} = 0$ else

$$\mathcal{D}_{1,2} = 2\int_{c_{1,2}}^{\delta_r} n_y(x - c_{1,2})\frac{1}{2\delta_r}\,\mathrm{d}x = \frac{n_y(\delta_r - c_{1,2})^2}{2\delta_r}$$

Cost $\mathcal{D}_{2,3}$ of the assembly of H_1 and P_2 with H_3. Here, besides the manipulator deviation, the effect of the play of the previous subassembly must be analized. Considering that the position of P_2 inside H_1 has a uniform distribution, the resulting statistical distribution $d_2(x)$ of the initial pose of H_3 with respect to P_2 is shown in figure 6. Again, since the local cost is constant, $g(x) = n_y$, then $f(x) = n_y(x - c_{2,3})$, and the

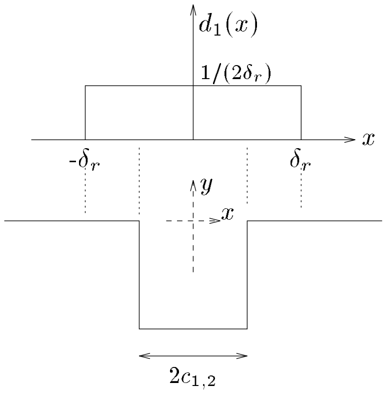

Figure 5: *Distribution of probability $d_1(x)$ of the initial configuration of P_2 with respect to H_1 and the corresponding Configuration Space.*

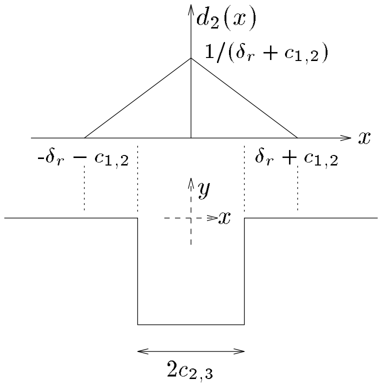

Figure 6: *Distribution of probability $d_2(x)$ of the initial configuration of H_3 with respect to P_2 and the corresponding Configuration Space.*

total cost $\mathcal{D}_{2,3}$ of this subassembly is:
if $\delta_r + c_{1,2} < c_{2,3}$ then $\mathcal{D}_{2,3} = 0$ else

$$\begin{aligned}\mathcal{D}_{2,3} &= 2\int_{c_{2,3}}^{\delta_r+c_{1,2}} n_y(x - c_{2,3})d_2(x)\ \mathrm{d}x = \\ &= \frac{n_y(\delta_r + c_{1,2} - c_{2,3})^3}{3(\delta_r + c_{1,2})^2}\end{aligned}$$

Total Cost $\mathcal{D}$ of the assembly. $\mathcal{D}$ is the summation of the costs of each assembly operation of the sequence,

$$\mathcal{D} = \mathcal{D}_{1,2} + \mathcal{D}_{2,3}$$

Sequence 2: H_3 with P_2 and then H_1.
The costs of each subassembly are determined analogously to those of sequence 1.

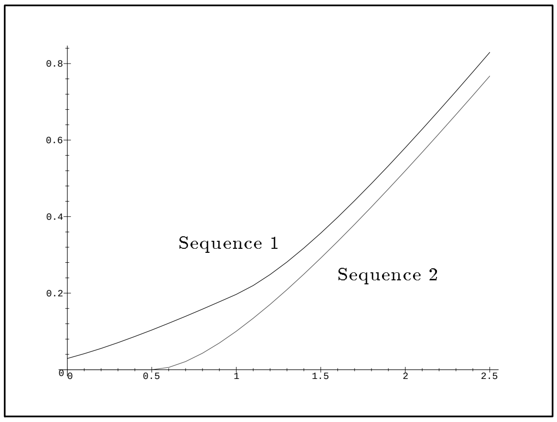

Figure 7: *Cost $\mathcal{D}(\delta_r)$ for each sequence.*

Cost of the assembly of H_3 with P_2.
if $\delta_r < c_{2,3}$ then $\mathcal{D}_{2,3} = 0$ else

$$\mathcal{D}_{2,3} = 2\int_{c_{2,3}}^{\delta_r} n_y(x - c_{2,3})\ \mathrm{d}x = \frac{n_y(c_{2,3} - \delta_r)^2}{2\delta_r}$$

Cost of the assembly of H_3 and P_2 with H_1.
if $\delta_r + c_{2,3} < c_{1,2}$ then $\mathcal{D}_{1,2} = 0$ else

$$\mathcal{D}_{1,2} = \frac{n_y(\delta_r + c_{2,3} - c_{1,2})^3}{3(\delta_r + c_{2,3})^2}$$

Total Cost $\mathcal{D}$ of the assembly. $\mathcal{D} = \mathcal{D}_{1,2} + \mathcal{D}_{2,3}$.

Numerical examples
Let us now consider some numerical examples. Let be: $n_y = 1$, $\delta_r = 1.25$, $h_1 = 10$, $h_3 = 9.5$ and $p_2 = 9$ (then $c_{1,2} = 1$ and $c_{2,3} = 0.5$). The resulting costs are:

sequence 1:	$\mathcal{D}_{1,2} = .025$	$\mathcal{D}_{2,3} = .352$	$\mathcal{D} = .377$
sequence 2:	$\mathcal{D}_{2,3} = .225$	$\mathcal{D}_{1,2} = .045$	$\mathcal{D} = .270$

thus, sequence 2 is better than sequence 1 from the assembly point of view. In fact, if the robot has precision $\delta_r < c_{2,3}$, sequence 2 always allows to solve the task without any adjustment, which is not the case in sequence 1. Figure 7 shows the cost $\mathcal{D}(\delta_r)$ for both sequences. Moreover, both $\mathcal{D}_{1,2}$ and $\mathcal{D}_{2,3}$ are proportional to n_y because the local cost is independent of x; therefore, if we add chamfers to the peg (wider than $\delta_r + c_{2,3}$ to guarantee that $g(x) = n_y$ is still valid), the cost of the assembly can be reduced according to n_y, e.g. with the sinus of the chamfer slope. If the chamfer is not wider than $\delta_r + c_{2,3}$, the cost will also be reduced but new expressions for $\mathcal{D}_{1,2}$ and $\mathcal{D}_{2,3}$ must be determined.

4 Discussion

The proposed measure of the assembly cost is simple enough to be statistically computed in real cases and, at the same time, it combines the main variables that can be manipulated in order to optimize the assembly. The proposed approach does not consider dynamic effects; nevertheless, the local cost function $g(\vec{p})$ and also $f(\vec{p})$ can capture some information related to friction and expected reaction forces. The effect of friction can be modeled and included in the local cost function $g(\vec{p})$ for simple cases and, in general, function $f(\vec{p})$ can be determined without any major problems; yet, this is not the case for complex assemblies or when rotational d.o.f are considered [13].

The proposed index of assembly cost must be always given with the used local cost function $g(\vec{p})$. In this sense, $g(\vec{p})$ can be selected according to different criteria like the execution time or the magnitude of the expected reaction forces. For instance, the ratio between the local cost outside and on a chamfer can be increased because in the first case more than one movement is necessary to solve the task. Also, $g(\vec{p})$ can be increased when the slope of the chamfer is not enough to allow the peg slide despite friction. On the other hand, $f(\vec{p})$ depends on the system and on the *strategy* used to solve the task; thus, the proposed index can also be used to evaluate the performance of an assembly strategy. An easy and systematic way to represent different strategies by $f(\vec{p})$ is also an interesting open problem.

5 Conclusion

In this paper, a statistical measure of an assembly cost index has been presented and illustrated with examples.

The measure of the assembly cost indicates how well the geometry of the objects can compensate the uncertainties affecting the mating features; these are obtained by statistical propagation of the effect of tolerances and clearances of the mating features previously assembled, and also of the positioning errors of the assembly robot.

The proposed measure provides a statistical cost index for each subassembly operation in the assembly of a product. This gives a new parameter for the evaluation of assembly sequences in order to determine the optimal one in terms of the assembly cost; it also provides a new parameter for the analysis of the influence of different sources of uncertainty and the mating movement in an assembly operation.

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