EMBEDING ROTATIONS IN TRANSLATIONAL CONFIGURATION SPACE*

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Abstract: This paper presents the graphical embeding of the rotational degree of freedom into the translational Configuration Space for planar assembly tasks. The resulting representation helps the visualization and understanding of motions in a simple and easy graphical way. The aim is the developing of motion planning algorithms in a two-dimensional parametrized space for planar assembly tasks (3 d.o.f.) and in a three-dimensional parametrized space for assembly tasks in space (6 d.o.f.). The method has been successfully applied to the problem of contact identification in the presence of uncertainty for planar assembly tasks. The extension to six degrees of freedom is under development.

1 Introduction

Motion planning is a key subject in robotics and it is a part of the more global objective of giving the robot the capability of planning its own motions from a task level description of the goal. It is usually divided in gross-motion planning and fine-motion planning, i.e. motions that must avoid collisions with the objects in the workspace, and motions where there may exist contact with the environment, respectively.

Motion planning algorithms make use of the Configuration Space ($\mathcal{C}$-space) [3][6]. The $\mathcal{C}$-space makes explicit the geometrical constraints on the motions by representing the manipulated object as a point, and modifying the physical obstacles accordingly. The configuration of the object is described as the position and orientation of its reference frame with respect to a reference frame attached to the workspace.

Although the $\mathcal{C}$-space is a powerful tool for motion planning, most of the implemented algorithms in the scope of fine-motion planning are restricted to degrees of freedom of translation [1], thus avoiding the complexity of rotations [4] [2].

The aim of this work is to define a methodology to tackle rotations in a simple and easy way. The basic idea is to build the $\mathcal{C}$-space for the degrees of freedom of translation, and to include in it the effect of the degrees of freedom of orientation. This paper presents the approach for planar assembly tasks (two degrees of freedom of translation and one of rotation).

A contact situation between rigid polyhedral objects can be defined as the simultaneous occurrence of a set of *basic contacts* taking place between the objects. In the case of polygonal objects moving in a plane with three d.o.f., the basic contacts are reduced to only two types: a vertex of the manipulated object against an edge of a static object (type-1) and vice versa (type-2). It must be noted that a single basic contact already represents a contact situation.

2 One Basic Contact

Let $\{W\}$ and $\{T\}$ be the reference frames attached to the workspace and to the mobile object, respectively. $\{T\}$ has the origin at the mobile object reference point, and an orientation ϕ with respect to $\{W\}$.

2.1 Contact Configurations

For movements in the plane with two degrees of freedom of translation and one of rotation, the $\mathcal{C}$-space is $\mathbf{R}^2 \times S^1_\rho$, where S^1_ρ is the circle of radius ρ, the gyration radius of the moving object. Then any configuration is described by three generalized coordinates (x, y, q), with $q = \rho\phi$, all having units of length.

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The $\mathcal{C}$-face $\mathcal{F}_i$ represents the contact configurations at which a basic contact i takes place, only considering the constraints imposed by the edges and vertices involved in the basic contact (i.e. the contact edge and the edges ending at the contact vertex). $\mathcal{F}_i$ is a ruled surface whose ruling segments are parallel to the xy-plane, of the same length of the contact edge and have their extremes on two helices over two cylinders, as it is shown below.

Let (x_m, y_m) be the coordinates of the vertex m of the fixed object measured in $\{W\}$. Let $\vec{h}_i$ be the vector defining the vertex i of the mobile object with respect to $\{T\}$, h_i its module and γ_i its orientation.

If the mobile object is translated keeping fixed its orientation and maintaining a given basic contact, the reference point describes a straight segment parallel to the contact edge and of its same length, i.e. it is a segment parallel to the xy-plane in $\mathcal{C}$-space . Each extreme of this segment correspond to the contact between the contact vertex and one vertex of the contact edge. For this situation, if the mobile object is rotated around the contact point, the reference point describes the following helix in $\mathcal{C}$-space:

$$\begin{aligned} x &= x_m + h_i \cos(\pi + \phi + \gamma_i) \\ y &= y_m + h_i \sin(\pi + \phi + \gamma_i) \\ q &= \rho\phi \end{aligned} \tag{1}$$

This helix is supported by a cylinder C_m^i of radius h_i and axis parallel to the q-axis and passing through the point $(x_m, y_m, 0)$.

For a type-1 basic contact the ruling segments are parallel as shown in figure 1b, which represent the $\mathcal{C}$-face corresponding to the type-1 basic contact shown in figure 1a. For a type-2 basic contact the ruling segments are tangent to a cylinder concentric with the supporting ones, and with the radius equal to the distance in physical space from the reference point to the line containing the contact edge. Figure 1d shows the $\mathcal{C}$-face corresponding to the type-2 basic contact shown in figure 1c.

The set of orientations $R_\phi = [\phi_m, \phi_M]$ for which the contact situation is possible, only considering the constraints imposed by the edges and vertices involved in the contact, is computed as follows. Let ψ_W^{fix} and ψ_T^{mob} be the orientations of the normals to an edge of the fixed object measured in $\{W\}$, and to an edge of the mobile object measured in $\{T\}$, respectively. The condition

$$\psi_W^{fix} = \phi + \psi_T^{mob} + \pi \tag{2}$$

imposes the parallelism between both edges. Since ϕ_m and ϕ_M are the orientations for which the adjacent edges of the contact vertex are parallel to the contact edge, they can be evaluated using equation (2).

2.2 Contact Positions

If the orientation of the mobile object remains fixed at a given orientation ϕ_o, the configuration space of the object is two-dimensional since only the two degrees of freedom of translation are to be considered. This translational configuration space is the section of the three-dimensional $\mathcal{C}$-space for the orientation ϕ_o.

Let the *parametrized translational configuration space* ($\mathcal{C}'$-space) be the space union of the translational configuration spaces for each possible orientations of the mobile object. The $\mathcal{C}'$-space can be obtained by projecting the $\mathcal{C}$-space on the xy-plane ($q = 0$), and associating to the projection of each ruling segment the value of its orientation, i.e. parametrizing the projection with the orientation.

The $\mathcal{C}'$-space contains the same information as $\mathcal{C}$-space. The projection of each helix representing the extremes of the ruling segments of a $\mathcal{C}$-face is an arc over a supporting circumference. This projection can be parametrized taking the projection of the point corresponding to $q = 0$ as the reference of orientation on the supporting circumference. The angle between the radius to any point of the arc and this reference, expresses the orientation ϕ of the corresponding point in the $\mathcal{C}$-space. Therefore the projection of any ruling segment contains the information of its orientation in any of its extremes, which are points of an arc over the supporting circumference.

A $\mathcal{C}'$-face $\mathcal{F}'_i$ of a basic contact i is the mapping into the $\mathcal{C}'$-space of the corresponding $\mathcal{C}$-face $\mathcal{F}_i$. It represents the set of contact positions parametrized in orientation, and can be expressed as:

$$\mathcal{F}' = \bigcup_{\phi=\phi_m}^{\phi_M} f'(\phi) \tag{3}$$

$f'(\phi)$ being the segment representing the projection of the ruling segment of the $\mathcal{C}$-face for orientation ϕ.

For a type-1 basic contact the segments f' are parallel, as shown in figure 2a, where the contact positions parametrized in orientation correspond to the contact situation shown in figure 1a.

For a type-2 basic contact the segments f' are tangent to a circumference concentric with the supporting ones, and with the radius equal to the distance in physical space from the reference point to the line containing the contact edge. For type-2 basic contacts the orientations will be measured in this latter circumference, where the reference orientation will be attached. Figure 2b shows the contact positions parametrized in orientation corresponding to the contact situation shown in figure 1c.

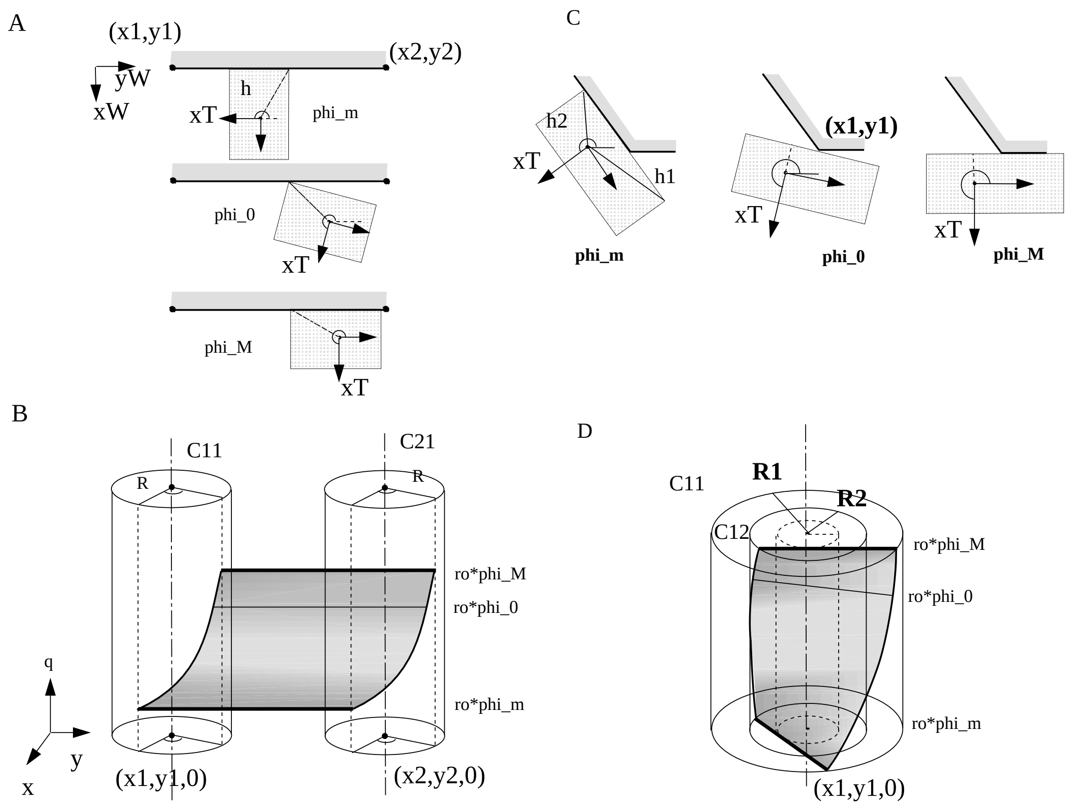

Figure 1: *Representation of a type-1 (a and b) and a type-2 (c and d) basic contact in physical space (a and b) and in configuration space (b and c).*

The supporting lines of the segments f' for the two types of basic contact have the expression

$$x\cos(\psi_W) + y\sin(\psi_W) = D \qquad (4)$$

where D is the distance from the supporting line to the origin of $\{W\}$ and ψ_W is the orientation of the normal to the contact edge measured in $\{W\}$. For a type-1 basic contact, ψ_W is constant and for a type-2 basic contact ψ_W depends on the orientation ϕ of the mobile object in the following way:

$$\psi_W = \psi_T + \phi + \pi \qquad (5)$$

ψ_T being the orientation of the normal to the contact edge measured in $\{T\}$.

D has different expressions depending on the type of basic contact. For a type-1 basic contact is given by (figure 3a):

$$D = h_i\cos(\psi_W + \pi - \gamma_i - \phi) + d_W \qquad (6)$$

and for a type-2 basic contact by (figure 3b):

$$D = x_m\cos(\psi_W) + y_m\sin(\psi_W) + d_T \qquad (7)$$

where d_W and d_T are the distances drom the straight line that contains the contact edge to the origins of $\{W\}$ and $\{T\}$, respectively.

3 Two Basic Contacts

The $\mathcal{C}$-edge $\mathcal{E}_{ij}$ corresponding to a contact situation involving two basic contacts i and j is the set of configurations in the $\mathcal{C}$-space for which the contact situation takes place, only considering the constraints imposed by the edges and vertices involved in the contacts. Then, the configurations of a $\mathcal{C}$-edge belong to the $\mathcal{C}$-faces of each involved basic contact.

The representation, $\mathcal{E}'_{ij}$, of a $\mathcal{C}$-edge $\mathcal{E}_{ij}$ in the $\mathcal{C}'$-space is an arc of a curve $\mathcal{L}_{ij}(\phi)$. Each point of this curve lies at the intersection of the lines containing the projection of the ruling segments corresponding to a different value of the orientation, which is described

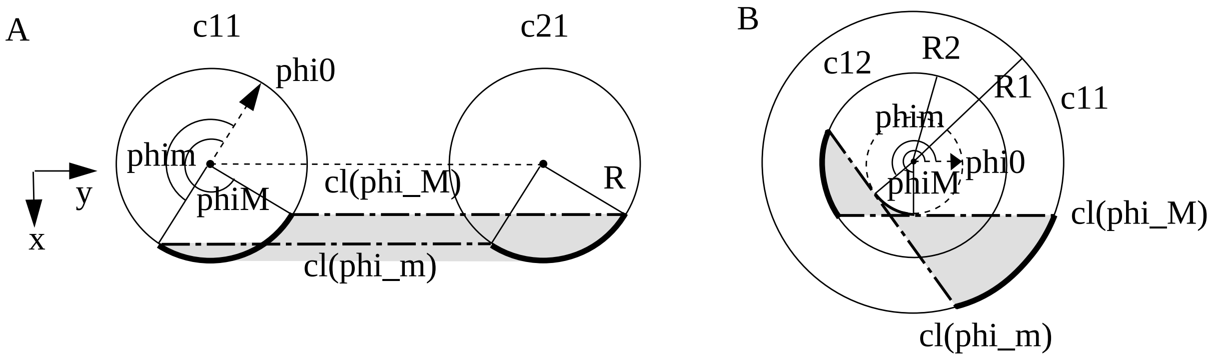

Figure 2: *Representation of type-1 (a) and type-2 (b) $\mathcal{C}$-faces in the parametrized translational configuration space.*

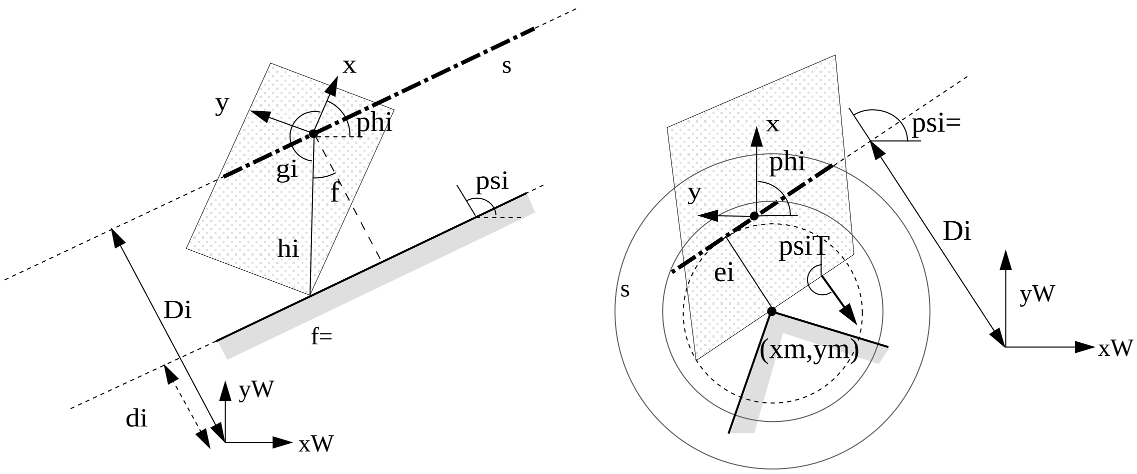

Figure 3: *Distance D to the origin of $\{W\}$ of the supporting line of the projection of the ruling segment of a type-1 and type-2 $\mathcal{C}$-faces, respectively, for some given orientations.*

by the following system of equations:

$$\begin{aligned} x\cos(\psi_W^i) + y\sin(\psi_W^i) &= D_i \\ x\cos(\psi_W^j) + y\sin(\psi_W^j) &= D_j \end{aligned} \qquad (8)$$

If $\sin(\psi_W^j - \psi_W^i) \neq 0$, solving for x and y it leads to the following expression:

$$x = \frac{D_i \sin(\psi_W^j) - D_j \sin(\psi_W^i)}{\sin(\psi_W^j - \psi_W^i)} \qquad (9)$$

$$y = -\frac{D_i \cos(\psi_W^j) - D_j \cos(\psi_W^i)}{\sin(\psi_W^j - \psi_W^i)}$$

Otherwise the solution is the straight line:

$$x\cos(\psi_W^i) + y\sin(\psi_W^i) = D_i \qquad (10)$$

D_i and D_j are described by equations (6) or (7) depending on the type of basic contact.

For two type-1 basic contacts, equation (9) describes an ellipse whose axes are on the bisectrices of the angles defined by the contact edges, with their lengths depending on the values of h and γ that describe the contact vertices of the mobile object.

As an example, figure 4a shows two type-2 basic contacts. Figure 4b shows the curve $\mathcal{L}_{ij}(\phi)$ corresponding to the contact situation involving both contacts. It illustrates the orientations of the mobile object which delimit the valid arc of $\mathcal{L}_{ij}(\phi)$, defining the set R_ϕ^{ij} of orientations for which the contact situation is possible.

R_ϕ^{ij} is computed in the following way. The contact situation can only ocurr for orientations in which all the involved basic contacts can simultaneously ocurr:

$$R_\phi^{ij} \subset D_\phi^{ij} = R_\phi^i \cap R_\phi^i = [\phi_m^i, \phi_M^i] \cap [\phi_m^j, \phi_M^j] \quad (11)$$

$\mathcal{L}_{ij}$ is the set of intersection points of the lines containing $f_i'(\phi)$ and $f_j'(\phi)$. The arc of $\mathcal{L}_{ij}$ for the range(s) of orientations where these intersection points belong to f_i' and f_j' is $\mathcal{E}_{ij}'$. Due to continuity, the limits of the range(s) ocurr for the extremes of f_i' and f_j'. Then, letting c be any of the supporting circumferences of the involved basic contacts, D_ϕ^{ij} can be partitioned in subintervals by the orientations ϕ_k satisfying the following conditions:

$$\begin{aligned} c(\phi_k) &= \mathcal{L}_{ij}(\phi_k) \quad 1 \leq k \leq n && (12)\\ \phi_k &\in D_\phi^{ij} && (13)\\ (x_k, y_k) &\in f_i'(\phi_k) && (14)\\ (x_k, y_k) &\in f_j'(\phi_k) && (15) \end{aligned}$$

(x_k, y_k) being the coordinates of the points satisfying equation (12).

Then, R_ϕ^{ij} is the set of orientations in the subinterval(s) of D_ϕ^{ij} whose orientations satisfy that the evaluation of $\mathcal{L}_{ij}$ belong to both $\mathcal{C}'$-faces $\mathcal{F}_i$ and $\mathcal{F}_j$.

4 Three Basic Contacts

Since planar movements have three degrees of freedom, the contact situations involving three non-redundant basic contacts take place only at a given configuration. The $\mathcal{C}'$-vertex $\mathcal{V}_{ijk}'$ representing this contact configuration in $\mathcal{C}'$-space is the point satisfying the conditions:

- $\mathcal{V}_{ijk}'$ is the intersection point of any two of the curves representing the contact situations involving two of the three basic contacts.
- $\mathcal{V}_{ijk}'$ has the same value of the parameter ϕ for both curves.

5 Considering all the constraints

In a given contact situation there can be more constraints than those imposed by the edges and vertices involved in the basic contacts of this situation, either due to concave objects or due to the existence of several static objects. The $\mathcal{C}'$-space considering these additional constraints is built by first generating the $\mathcal{C}'$-faces, the $\mathcal{C}'$-edges and the $\mathcal{C}'$-vertices in this order as descibed in the previous sections; and then by pruning these sets in the reverse order, taking into account the new imposed constraints.

6 Conclusions

Motion planning algorithms often make use of the Configuration Space where motion constraints are explicitly represented by reducing the manipulated object to a point, and modifying the obstacles accordingly. Nevertheless the dimension of this space (three for tasks in the plane and six for tasks in the space) makes it difficult the understanding of motions and thus the development of the algorithms.

This paper presented an intuitive graphical representation for planar assembly tasks, that embeds the rotational degree of freedom into the translational Configuration Space. This representation gives an insight into the motion of a mobile object in contact with static objects. It is useful for the identification of the current contact situation during the performance of an assembly task, also giving an easy and intuitive measure of its clearance.

The representation has been implemented on a Silicon Graphics workstation, and has been successfully used in the context of a fine-motion planner for contact identification in the presence of uncertainty in planar assembly tasks [5]. The extension to assembly tasks in the space is under development.

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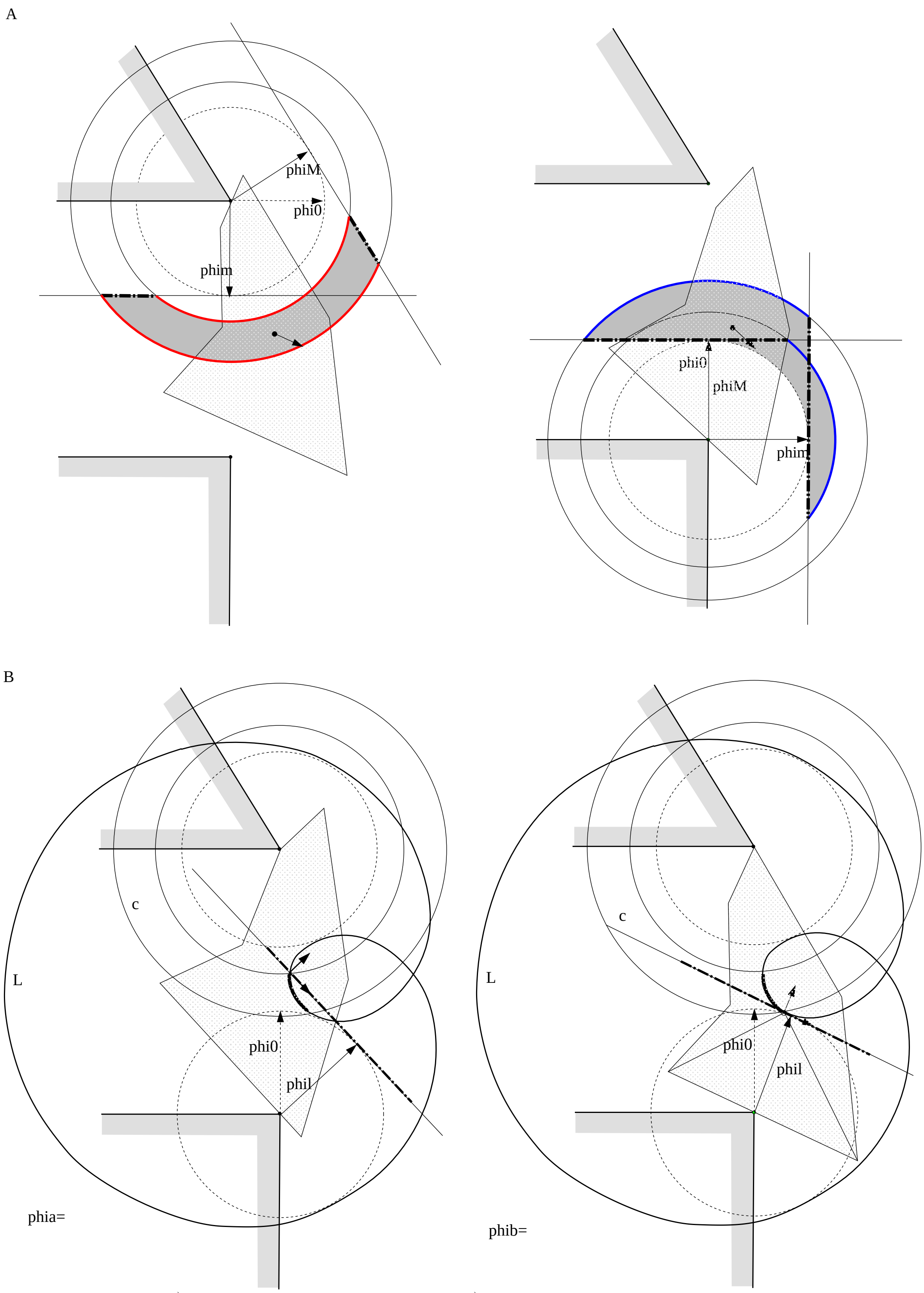

Figure 4: *a) Type-2 basic contacts i and j b) Corresponding set of contact orientations.*