Determining independent contacts regions to immobilize 2D articulated objects

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Abstract—This paper deals with the problem of determining independent contacts regions (ICRs) on 2D articulated objects, such that a finger contact in each region guarantees a force-closure (FC) immobilization, independently of the exact position of the finger. These regions allow a robust finger or fixture placement on the links of the articulated object, despite of possible errors in the position of the contacts. The proposal defines a generalized wrench space for articulated objects and then computes the ICRs starting from an initial FC grasp, considering frictional contacts. The approach has been implemented, and some illustrative examples are provided.

Index Terms—Fixturing, force-closure grasp, grasping, independent contact regions.

I. INTRODUCTION

Immobilizing an object by using fingers or fixtures to constrain its degrees of freedom despite the possible existence of external perturbations has been an active research topic that still presents open problems [1]. The objects that can be grasped, manipulated or fixed by a robotic hand are of different shapes and sizes, and they can be either rigid or articulated. Articulated objects are formed by rigid links connected by some type of joint or hinges, such as a truck toy, staplers, or scissors (Fig. 1).

Fig. 1. Examples of articulated objects.

There are two properties commonly required for a grasp: force-closure or form-closure [2]. Both properties can be characterized in the object configuration space, that for a 2D rigid body has dimension \( d = 3 \). Any 2D rigid object can be always immobilized with \( d + 1 = 4 \) frictionless contacts or with 3 frictional contacts [3], [4].

Several algorithms have been proposed to obtain precision grasps that satisfy the properties mentioned above, either for 2D or 3D objects. In 2D, grasp planning approaches have been proposed for polygonal objects with frictionless [5] and frictional contacts [6], [7], and for non polygonal objects with frictionless [8], [9] and frictional contacts [10], [11]. There are also works dealing with the grasp of 3D objects, either polyhedral [12], [13] or non polyhedral with frictionless and frictional contacts [14], [15], [16], [17].

In a real application, the actual grasp may differ from the expected one due to finger positioning errors. To deal with these errors and provide robustness to the grasp, the concept of independent contact regions (ICRs) was introduced [18], such that the fingers can be independently positioned inside their corresponding regions while ensuring a FC grasp, regardless of the exact position of the fingers. The computation of ICRs has been done in 2D for either polygonal [7], [19] or irregular objects [20] with frictional and frictionless contacts; and for the case of 3D objects, several works have dealt with polyhedrons [21] or objects with any shape considering any number of contacts [22], [23].

The work done in the area of robotic grasping has focused mainly on the search of FC grasps in both 2D and 3D single objects with different types and number of contacts. However, few works have dealt with articulated objects, but nevertheless there are some relevant ones using different approaches, such as interactive perception [24], occlusion-aware systems [25], or even the modeling and static analysis of an articulated object with three-rigid links [26] for achieving a non-prehensile manipulation. Another relevant work [27] presents a systematic procedure to find a set of frictionless contact points that immobilizes a 2D serial chain with \( n \) polygons, based on second order effects. The lower bound of the number of contact points necessary to immobilize any chain of \( n \neq 3 \) hinged polygons without parallel edges was demonstrated to be \( n + 2 \), while for \( n = 3 \) using 5 contacts allows only the immobilization of some chains with particular shapes of the polygons. In the general case, \( n + 3 \) frictionless points are enough to immobilize any chain of \( n \) polygons.

Although these different approaches deal with finding precision FC grasps, we are not aware of any work dealing with the computation of ICRs on articulated objects. Note that for the case of articulated objects the ICRs must guarantee the immobilization of all the object internal degrees of freedom and the spatial immobilization of the object as a whole. This means that traditional procedures for ICRs computation cannot be directly applied to the case of articulated objects.
The reachability of the contact points for a particular link is represented on each link $i$, and the number of links of the articulated object (for a 2D serial articulated object) whose dimension will depend on the number of links. The rest of the paper is structured as follows. Section II provides an overview of the problem, including the main assumptions. Section III presents a procedure to find the elements of the vector of generalized wrenches for an articulated object with $n$ links. Section IV summarizes the algorithm to compute ICRs. Section V presents the procedure to compute ICRs. Section VI shows illustrative examples of the proposed approach. Finally, Section VII presents some conclusions and future work.

II. PROBLEM STATEMENT AND ASSUMPTIONS

Consider a 2D serial articulated object with $n$ links and with rotational joints, as illustrated in Fig. 2. The problems to be addressed are the following:

1) Search a set of contact points on the surface of the links that allows a FC grasp.
2) Compute the ICRs for a FC grasp on the surface of the links.

The following assumptions are considered in this work:

- The links are connected by rotational joints.
- The links can overlap each other when they rotate (i.e., the problem could be of dimension $2n$, treated as 2-dimensional for simplicity).
- The boundary of each link is represented with a large enough set of points $\Omega$ (i.e., the links can be of any shape, either polygonal or non-polygonal).
- The normal direction pointing towards the interior of the object at each boundary point is known.
- The contacts between the fingers and the object are frictional, and Coulomb’s friction law is considered.
- The reachability of the contact points for a particular device is not considered, although the approach in this paper can be integrated in other algorithms that include such analysis [29], [30].

III. GENERALIZED WRENCHES FOR ARTICULATED OBJECTS

For a single 2D solid object and considering frictional contact points, the grasp force $f_i$ applied at a contact point $p_i$ can be decomposed in two components $f_i,n$ and $f_i,t$ which are respectively normal and tangent to the object boundary. To avoid slippage of the finger, Coulomb’s law must be satisfied: $f_i,t \leq \mu f_i,n$, where $\mu$ is the friction coefficient. This implies that the force applied by the finger must lie inside a friction cone centered on the direction normal to the object boundary and limited by the so-called primitive forces, $f_i^r$ and $f_i^t$, $f_i$ is a positive combination of $f_i^r$ and $f_i^t$, $f_i$ is associated with each contact point $Q_{i,j}$ with respect to the base frame.

This section describes the generalization to $n$ links of the procedure developed in [28] to obtain generalized wrenches for a serial articulated object. The method considers a virtual robot of $n+2$ joints (Fig. 3) wherein the first and second joints are virtual ones and the other joints correspond to the real ones. The virtual robot is associated with the sets $W = \{w_1, ..., w_k\}$ and $W_p = \{w_1, w_2, ..., w_{n_1}, w_{n_1+1}, ..., w_p\}$. The virtual links and corresponding joints are done to represent the degrees of freedom of the real link. The following basic nomenclature will be used:

- $L_i$: Links of the virtual robot, $i = -1, ..., n$. $L_{-1}$ and $L_0$ are virtual ones, and $L_1$ to $L_n$ correspond to the real ones.
- $q_i$: Generalized joint coordinates for the virtual robot, $i = -2, ..., n-1$. Joints $-2$ to $0$ are virtual ones, and joints $1$ to $n-1$ correspond to the real ones.
- $Q_0$: Position of the joint $q_i$ for $i = 0, ..., n-1$. For $i = n$, $Q_n$ is the position of the final end of the link with respect to the base frame.
- $P_{i,j}$: Contact point $j$ on link $L_i$ with respect to the base frame.
- $p_{i,j}$: Position vector (in the base frame) of contact point $j$ on link $L_i$ as measured from $Q_{i-1}$ (i.e., $p_{i,j} = P_{i,j} - Q_{i-1}$), $i = 1, ..., n$, $j = 1, ..., k_i$, where $k_i$ is the number of contact points on link $L_i$. The total number of contacts is $k = \sum_i k_i$.
- $r_i$: Position vector of $Q_i$ measured from $Q_{i-1}$ (i.e., $r_i = Q_i - Q_{i-1}$).
- $s_{i,j}$: Position vector of contact point $j$ on link $L_i$ measured from $Q_i$ (i.e., $s_{i,j} = p_{i,j} - Q_i$).
- $f_{i,j}$: Force $j$ applied to link $L_i$ at contact point $p_{i,j}$.

A. Determination of the generalized wrenches

The Jacobian $J_i$ for each link $L_i$ ($i = -1, ..., n$) is computed to relate the external forces applied to each link $L_i$ with the forces or torques required in each joint for
an equilibrium condition. The total forces or torques $\tau_k$ ($k = -2, ..., n-1$) to be applied at joints $q_k$ are the components of a vector $\tau$ given by:

$$\tau = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} \tau_{i,j} = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} f_{i,j}^T w_{i,j} = \sum_{i=-1}^{n} \sum_{j=1}^{k_i} f_{i,j}^T$$

where $M_{s_{i,j}} = s_{i,j} \times f_{i,j}$.

Therefore, expanding eq. (1) the components $\tau_k$ ($k = -2, ..., n-1$) are:

$$\tau_{-2} = \sum_{j=1}^{k_{-2}} f_{-2,j} + \sum_{j=1}^{k_{2,j}} f_{2,j} + ... + \sum_{j=1}^{k_{n-1,j}} f_{n-1,j} + \sum_{j=1}^{k_{n,j}} f_{n,j} = 0$$
$$\tau_{-1} = \sum_{j=1}^{k_{-1,j}} f_{-1,j} + \sum_{j=1}^{k_{1,j}} f_{1,j} + ... + \sum_{j=1}^{k_{n-1,j}} f_{n-1,j} + \sum_{j=1}^{k_{n,j}} f_{n,j} = 0$$
$$\tau_0 = \sum_{j=1}^{k_{0,j}} f_{0,j} + \sum_{j=1}^{k_{2,j}} f_{2,j} + ... + \sum_{j=1}^{k_{n-1,j}} f_{n-1,j} + \sum_{j=1}^{k_{n,j}} f_{n,j} = 0$$
$$\tau_1 = 0 + \sum_{j=1}^{k_{1,j}} f_{1,j} + ... + \sum_{j=1}^{k_{2,j}} f_{2,j} + ... + \sum_{j=1}^{k_{n-1,j}} f_{n-1,j} + \sum_{j=1}^{k_{n,j}} f_{n,j} = 0$$

Now, it is possible to consider a generalized wrench space $W$ defined by the base $\{\tau_{-2}, \tau_{-1}, \tau_{0}, \tau_{1}, ..., \tau_{n-1}\}$ for the articular object, where the generalized wrenches $W_{i,j}$ generated respectively by forces $f_{i,j}$ are

$$W_{1,j} = \begin{bmatrix} f_{x_{1,j}} \\ f_{y_{1,j}} \\ f_{z_{1,j}} \\ p_{1,j} \times f_{1,j} \\ 0 \\ 0 \end{bmatrix}, \quad W_{2,j} = \begin{bmatrix} f_{x_{2,j}} \\ f_{y_{2,j}} \\ f_{z_{2,j}} \\ r_{1,j} \times f_{2,j} \\ p_{2,j} \times f_{2,j} \\ 0 \end{bmatrix}$$
$$W_{n-1,j} = \begin{bmatrix} f_{x_{n-1,j}} \\ f_{y_{n-1,j}} \\ f_{z_{n-1,j}} \\ r_{n-1,j} \times f_{n-1,j} \\ 0 \end{bmatrix}, \quad W_{n,j} = \begin{bmatrix} f_{x_{n,j}} \\ f_{y_{n,j}} \\ f_{z_{n,j}} \\ r_{n-1,j} \times f_{n,j} \\ p_{n,j} \times f_{n,j} \\ 0 \end{bmatrix}$$

The dimension of $W$ is $n + 2$, and therefore the generalized wrenches $W_{i,j}$ and $W_{n,j}$ have $n + 2$ components. Note that this is different from the traditional wrench space for a rigid object, whose dimension is 3 for 2D objects and 6 for 3D objects. Moreover, note also that each generalized wrench $W_{i,j}$ or $W_{n,j}$ has only three independent components, which come from the two independent parameters defining $f_{x_{i,j}}$ or $f_{x_{n,j}}$ and $f_{y_{i,j}}$ or $f_{y_{n,j}}$, and a third parameter defining the contact point $p_{i,j}$ or $p_{n,j}$ on the object boundary.

From the representation of generalized wrenches in eq. (4), since the last component depends only on the forces $f_{n,j}$ applied on the last link $L_n$, it is straightforward that the forces in the link $n$ must be able to produce positive and negative torques in order to counterbalance any perturbation. This in turn means that for frictionless contacts it must be $k_n \geq 2$, i.e. there must be at least two applied forces $f_{n,1}$ and $f_{n,2}$ on the last link in order to expand the whole space of $\tau_{n-1}$, while for frictional contacts only one contact could be enough if the forces in the friction cone allow both positive and negative torques (which is not always allowed by the link shape). Since the virtual links can be added to any extreme of the articular object, the same reasoning applies for the first link $L_1$.

B. Force-closure Test

Considering the set $G = \{p_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ of $k = \sum_{i=1}^{n} k_i$ contact points (with $k_i$ is the number of contact points on link $L_i$), and a force $f_{i,j}$ applied at each $p_{i,j}$, two sets $W = \{W_{i,j}, i = 1, ..., n, j = 1, ..., k_i\}$ and $W_p = \{W_{p_{i,j}}, i = 1, ..., n, j = 1, ..., k_i, c \in \{l, r\}\}$ are obtained. The necessary and sufficient condition for the existence of a FC grasp is that the origin of the generalized wrench space lies inside the convex hull $CH(W_p)$ of the contact wrenches $W_p$ [4], [31]. This guarantees that the grasp can generate appropriate wrenches to counteract perturbation wrenches in any direction, i.e. to counterbalance any force(s) $f_{i,j}$ applied on any link $L_i$ of the articular object. Note that this test is a generalization of the traditional FC test for objects without internal degrees of freedom. The test used in this work to verify this condition is derived from [16] for the case of a single rigid object and then extended in [28] for an articulated 2D object. Let $P$ be the centroid of the primitives wrenches, $O$ the origin of the wrench space and $H_1$ a boundary hyperplane of $CH(W_p)$; in order for a grasp $G$ to be FC, $P$ and $O$ must lie on the same side of $H_1 \forall i$.

IV. FINDING AN INITIAL FC GRASP

The algorithm described in this section is the extension to frictional contacts of the algorithm presented in [28] for frictionless contacts. The procedure generates an initial grasp $G^m$, $m = 1$, by selecting $k$ random points from the set $\Omega$ that describes the object boundary, then computes the corresponding set $W^m$ when frictionless contact points are considered, and $W_p^m$ with primitives contact wrenches for frictional contacts. The next step is to check whether the points in $G^m$ lead to a FC grasp. If $G^m$ does not
provide a FC grasp, then a search of new contact points is done, based on separating hyperplanes in the wrench space that define candidate points to replace one of the current points in $G^m$ to obtain another grasp $G^{m+1}$. This is iteratively repeated until a FC grasp is found. The procedure is detailed in Algorithm 1 and explained below.

If grasp $G^m$ fails the FC-test mentioned in Section III-B, the search procedure, Steps (3) to (8), iteratively tries to improve the grasp by changing one of the points in $G^m$.

In Step (4) a subset $G^m_R$ of $G^m$ is generated with the points of the wrench space that simultaneously define all the critical hyperplanes $H$ defining the boundary of $\text{CH}(W)$ that produce a failure of the FC-test (i.e. $P$ and $O$ lie on different sides of the plane).

In Step (5) a subset $\Omega^m_C$ with candidate points to replace one point in $G^m_R$ is determined by hyperplanes $H'$ passing through the origin and parallel to the critical hyperplanes $H$. The replacement candidate points are those that simultaneously lie on the opposite side of the point $P$ with respect to all the hyperplanes $H'$. In Step (6) one of the points in $G^m_R$ is replaced by a point producing a wrench $W_*$ randomly taken from $\Omega^m_C$. $W_*$ replaces the closest point in $G^m_R$, generating an auxiliary grasp $G_{aux}$. The centroid $P'$ and the distance $|P'O|$ are computed for the wrenches of the auxiliary grasp $G_{aux}$. Let $P^m$ be the centroid of the set of wrenches $W_*$ in the iteration $m$. If the relation $|P'O| < |P^mO|$ is satisfied then the auxiliary grasp $G_{aux}$ is selected as new grasp.

In the example shown in Fig. 5 for, again, a hypothetical 2-dimensional wrench space. An initial FC grasp $G = \{p_{1,1}, p_{2,1}, p_{3,1}\}$ generating the set of wrenches $W = \{W_{1,1}, W_{2,1}, W_{3,1}\}$ and $W_p = \{W_{1,1}', W_{1,1}''', W_{3,1}', W_{3,1}''\}$ was obtained using Algorithm 1; its initial grasp quality $Q_0$ and the closest facet to the origin $F_0$ are also shown in Fig. 5. Now, in order to determine the search region associated with the point $p_{3,1}$ with primitives wrenches $W_{3,1}'$ and $W_{3,1}''$, the hyperplane $H_1$ parallel to $H_1$ at a distance $Q_0$ from $O$ is built (note that at least one $W_{3,1}'$, $c \in \{1, r\}$, must belong to $H_1$, which in this case is $W_{3,1}'$). $H_1$ and $F_0$ define the region $S_{3,1}$ in the wrench space where, in the example, the wrenches corresponding to two neighboring points of $p_{3,2}$ are located, and therefore selected for the corresponding ICR. The search zones $S_{3,1}$ for each grasp point are depicted in different color, and the wrenches associated with neighboring points within each ICR are depicted with squares (black ones representing $W_{3,1}'$). The core of Algorithm 2 are Step (6) and the loop starting in Step (9). In Step (6) the hyperplanes $H_1$ parallel to $F_0$ and at a distance $Q_0$ from $O$, as well as $H_1^+$ are computed. Step 9 is the most costly one because it is necessary to check whether the primitive wrenches $W_{3,1}'$ of an unknown number of points $p_{o,i}$ belong to each half-space $H_1^+$.

V. Determining the ICRs

This section presents the algorithm to compute the ICRs, such that if a contact is located inside each region the resulting grasp is always FC. The algorithm works as follows.

For a given FC grasp, the grasp quality $Q_0$ is fixed by the facet $F_Q$ of the convex hull closest to the origin. Let $F_o$ denote a facet of CH($W$) that contains at least one primitive wrench $W_{1,1}'$ for a particular grasp point $p_{1,1}$. Several hyperplanes $H_o$ parallel to each facet $F_o$ are built at a distance $Q_o$ from the origin (i.e. tangent to the hypersphere of radius $Q_o$). The role of these hyperplanes is determining regions $S_{i,j}$ of the wrench space where new wrenches (associated with new contact points) can generate FC grasps with equal or greater quality. The regions $S_{i,j}$ are the intersection of the half-spaces $H_o^+$ that do not contain the origin $O$. The ICRs are determined by the set of neighbor points of $p_{1,1}$ such that at least one of its primitive wrenches falls into the corresponding search zone $S_{1,1}$.

The procedure to determine the ICRs is given in Algorithm 2 and illustrated in Fig. 5 for, again, a hypothetical 2-dimensional wrench space. An initial FC grasp $G = \{p_{1,1}, p_{2,1}, p_{3,1}\}$ generating the set of wrenches $W = \{W_{1,1}, W_{2,1}, W_{3,1}\}$ and $W_p = \{W_{1,1}', W_{1,1}'', W_{3,1}', W_{3,1}''\}$ was obtained using Algorithm 1; its initial grasp quality $Q_0$ and the closest facet to the origin $F_0$ are also shown in Fig. 5. Now, in order to determine the search region associated with the point $p_{3,1}$ with primitives wrenches $W_{3,1}'$ and $W_{3,1}'''$, the hyperplane $H_1$ parallel to $H_1$ at a distance $Q_0$ from $O$ is built (note that at least one $W_{3,1}'$, $c \in \{1, r\}$, must belong to $H_1$, which in this case is $W_{3,1}'$). $H_1$ and $F_0$ define the region $S_{3,1}$ in the wrench space where, in the example, the wrenches corresponding to two neighboring points of $p_{3,2}$ are located, and therefore selected for the corresponding ICR. The search zones $S_{3,1}$ for each grasp point are depicted in different color, and the wrenches associated with neighboring points within each ICR are depicted with squares (black ones representing $W_{3,1}'$). The core of Algorithm 2 are Step (6) and the loop starting in Step (9). In Step (6) the hyperplanes $H_1$ parallel to $F_0$ and at a distance $Q_0$ from $O$, as well as $H_1^+$ are computed. Step 9 is the most costly one because it is necessary to check whether the primitive wrenches $W_{3,1}'$ of an unknown number of points $p_{o,i}$ belong to each half-space $H_1^+$.

VI. Examples

In this section we present the example of the ICRs is illustrated with examples for articulated objects with 2, 3 and 4 links. The considered friction coefficient was $\mu = 0.5$. The implementation was done using Matlab and C++ on an Intel Core2 Duo 2.0 GHz computer. The library Qhull [32] was used to compute the convex hulls. The figures of the examples show: a) the initial randomly generated grasp, which in general is a non-FC grasp, and, b) the obtained FC Grasp and corresponding ICRs.
Algorithm 2 Computation of ICRs

Ensure: Independent contact regions ICRs.
1: Find an initial FC grasp $G$ using Algorithm 1.
2: Compute the initial quality $Q_g$.
3: Compute $CH(W_p)$.
4: for $i = 1$ to $n$ do
5:   for $j = 1$ to $k_i$ (i.e., for each contact point $p_{i,j} \in G$) do
6:     For each facet $F_v$ of $CH(W_p)$ with at least one vertex $W_{i,j}$, build the hyperplane $H_v$ parallel to $F_v$ and at distance $Q_g$ from the origin $O$, leaving $O$ and $F_v$ in different half-spaces. Let $H^+_v$ be the open half-space such that $W_{i,j} \in H^+_v$. The search regions $S_{i,j}$ are determined by intersecting the half-spaces $H^+_v$, i.e., $S_{i,j} = \cap_v H^+_v$.
7:     Initialize $ICR_{i,j} = \{p_{i,j}\}$.
8:     Label $p_{i,j}$ as open
9:     while there are open points $p_{i,u} \in ICR_{i,j}$ do
10:        for all the neighboring points $p_{i,u}$ of $p_{i,v}$ do
11:           if $\exists c$ such that $w_{i,v} \in S_{i,j}$ then
12:              $ICR_{i,j} = ICR_{i,j} \cup \{p_{i,u}\}$
13:          Label $p_{i,u}$ as open
14:       end if
15:     end for
16:     Label $p_{i,u}$ as closed
17:   end while
18: end for
19: return $ICR = \{ICR_{i,j}, i = 1, \ldots, n \text{ and } j = 1, \ldots, k_i\}$

Fig. 5. Search for ICRs ensuring a minimum grasp quality. Search zones $S_{i,j}$ for each grasping point are depicted in color, and wrenches associated with neighboring points within each ICR are depicted as squares.

Example 1: Articulated object with 2 links shown in Fig. 6. The initial FC grasp was obtained after 2 iterations in 1 s, and the ICRs were obtained in 0.6 s.

Example 2: Articulated object with 3 links shown in Fig. 7. The initial FC grasp was obtained after 4 iterations in 4 s, and the ICRs were obtained in 1.3 s.

Example 3: Articulated object with 4 links shown in Fig. 8. The initial FC grasp was obtained after 14 iterations in 17 s, and the ICRs were obtained in 1.6 s.

Example 4: Articulated object with 4 links shown in Fig. 9. The initial FC grasp was obtained after 6 iterations in 11 s, and the ICRs were obtained in 2.1 s. Note that in this case there are no contacts on the third link, which does not prevent the FC grasp.
imply more degrees of freedom and therefore higher dimensional spaces of any dimension, since the proposed approach is valid for future work is the generalization of the approach for 3D articulated objects with any number of links of a 2D articulated object. Thus, the complexity of this case the analysis could be directly done using the proposed algorithms were implemented and examples for articulated objects with two, three and four links were presented.

As future work we consider the extension to the computation of ICRs for objects with both rotational and prismatic joints; in this case the analysis could be directly done using the proposed approach based on the appropriate Jacobian matrices. Another future work is the generalization of the approach for 3D articulated objects considering frictionless and frictional contacts. 3D objects imply more degrees of freedom and therefore higher dimensional spaces; however, the algorithms were already running on wrench spaces of any dimension, since the proposed approach is valid for any number of links of a 2D articulated object. Thus, the complexity of the generalization for 3D objects could be determined by the development of the proper model for the generalized wrenches. Finally, another future development is the consideration of branched articulated objects and closed kinematic chains, for both 2D and 3D articulated objects.

VII. CONCLUSIONS

In this paper we proposed an approach to obtain independent contact regions (ICRs) for 2D articulated objects with \( n \) links considering frictional contacts. The approach has two stages: the first one performs the synthesis of a FC grasp, and the second one computes the ICRs around the contact points of the FC grasp. The algorithms were implemented and examples for articulated objects with two, three and four links were presented.

As future work we consider the extension to the computation of ICRs for objects with both rotational and prismatic joints; in this case the analysis could be directly done using the proposed approach based on the appropriate Jacobian matrices. Another future work is the generalization of the approach for 3D articulated objects considering frictionless and frictional contacts. 3D objects imply more degrees of freedom and therefore higher dimensional spaces; however, the algorithms were already running on wrench spaces of any dimension, since the proposed approach is valid for any number of links of a 2D articulated object. Thus, the complexity of the generalization for 3D objects could be determined by the development of the proper model for the generalized wrenches. Finally, another future development is the consideration of branched articulated objects and closed kinematic chains, for both 2D and 3D articulated objects.

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