Feeding sequence selection in a manufacturing cell with four parallel machines ☆

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Abstract

This paper deals with the problem of finding the optimum feeding sequence in manufacturing cells with machines fed by robots. The particular real case of a cell with four parallel identical machines working alternately on two pallets each one, fed by one robot and with random assistance requirements, is introduced and analyzed. The cell has been modelled and simulation results for different feeding sequences are presented. A general discriminant function to select the best feeding sequence between a fixed and a variable sequence was determined using simulation results for different working and loading times and pattern recognition techniques.

Keywords: Manufacturing; Robotics; Scheduling; Sequences; Simulation

1. Introduction

The paper deals with the problem of finding the optimum feeding sequence in manufacturing cells with machines fed by robots. In particular, the work was developed for a real cell that produces parts for the car manufacturing industry at the plant of Metaldyne International Spain SL (formerly R.J. Simpson International SL) located in Gavà, close to Barcelona (Spain), with the aim of optimizing its production. The cell, described in detail below, has one robot that feeds four identical machines that work alternately on two pallets and have random assistance requirements.

Each machine in the cell has some unproductive time when it is waiting for the robot to load the parts on it. This unproductive time depends on the machine working time on each pallet and on the sequence that the robot follows to feed the machines. Differences in the unproductive time can arrive up to two orders of magnitude for different feeding sequences; for instance, in a particular real case, the unproductive times of the machines for two different feeding strategies were 0.045% and 5.72%, respectively.

The working times of the machines on each pallet as well as the loading time of the parts in the machines depend on the type of part to be processed. Then, it is necessary to determine: "In which way should the robot feed the machines in order to optimize the productivity of the cell for a given part?" The objective of this work was the search of a function that, given the values of the variable times in the cell, returns the best robot loading sequence.

The determination of optimal feeding sequences for different types of cell under various conditions has been extensively studied, but the authors are not aware of any published general solution for the particular features of this cell. Most of the previous work in the field deal with the flowshop scheduling problem trying to determine exact solutions for deterministic problems. In the literature several variations of the feeding sequence and scheduling problems were addressed, considering

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for instance: (a) same/different types of parts; (b) existence/non-existence of storage buffers with a predetermined capacity; (c) fixed/variable working times of the machines on each type of part; (d) robots (or feeding elements) with one/several grippers; (e) machines have to execute a fixed sequence of operations (even on different parts) or can perform any work at any time; (f) loading and unloading times dependant/non-dependant on the type of part; (g) travelling time of the robot dependant/non-dependant on the robot load; (h) transitory conditions are considered/not-considered (i.e. the solution is dependant/non-dependant on the initial conditions); (i) some type of machine assistance (normally deterministic assistance) is needed/not-needed; (j) existence/non-existence of parallel machines; and, of course, (k) different number of machines and robots working in the cell.

Representative work in this area is as follows. Kise et al. [1] dealt with the problem of optimizing the movements of an Automated Guided Vehicle (AGV) that serves two machines without buffer storage, and provided algorithms for the minimization of the makespan for n parts; King et al. [2] dealt also with a two machine problem but considering unlimited buffers for the queue of each machine in a robotized cell and using a branch and bound approach. Crama and van de Klundert [3] considered the flowshop problem with one robot and parts of one type to demonstrate that the shortest cyclic scheduled for the robot can be solved in polynomial time in the number of machines. Afterwards, they proved that the sequence of activities whose execution produces exactly one part has optimal production rates for the case of three machine flowshop with one robot [4]. More recently, Crama et al. [5] presented a good survey of the specific problems and existing solution approaches to the called robotic flowshop scheduling problems. Hall et al. [6] also addressed the problem of bufferless robotized cells for identical parts, introducing a classification schema and dealing with cells with two and three machines; Kamoun et al. [7] dealt with the same problem for a three machine cell, and proposed a heuristic to find a solution that minimizes the average steady-state cycle time for the repetitive production of Minimal Part Sets (MPS). Sethi et al. [8] dealt with the problem of scheduling robot movements in dual-gripper robotic cells, they considered a single part problem without buffers between the machines and included a comparison of the dual and single gripper case for a cell with n machines. The problem of processing times dependant on the task state (i.e. dependant on the adopted solution instead being constant) was addressed by Wagneur and Sriskandarajah [9]. Although all these papers cover a wide range of manufacturing systems, none of them can be tailored to our particular problem due to the random assistance requirements.

When the presence of stochastic variations or the complexity of the manufacturing system preclude the existence of an analytical solution, discrete-event simulation has become an accepted successful tool for the performance improvement of manufacturing systems [10]. For instance, William and Narayanaswamy [11] studied the correct mix and sequencing of row materials and the reduction of material-handling costs in an AGV operated system, Duwayri et al. [12] reported a simulation study to evaluate the performance of different heuristics for scheduling setup changes in a semiconductor manufacturing system, and Korhonen et al. [13] studied the effect of queuing rules, buffer policies and lot sizes on custom service and cost efficiency of a printed circuit wiring board manufacturing. There are several software tools for the modelling and simulation of discrete-event systems like, for instance, Quest, Automod or Arena. In this work we use the Arena product family [14], a commercial tool that offers a comprehensive modelling capability, application-focused modelling templates and the ability to be integrated with databases or spreadsheets.

The analysis of the simulation results is complex. Metamodels or factorial designs are used when comparing a set of alternative models. Pattern recognition techniques were also used to classify simulation results and find a way to select the best alternative as a function of the system parameters in a wide variety of problems. Good bibliography on the use of pattern recognition, including the principles of the theory used in this work, was provided by Meisel [15], Fukunaga [16], Tou and González [17], Young and Calvert [18], and more recently by Schalkoff [19]. Other approaches, like neural networks, were given by Bishop [20], and their application to jobshop scheduling problems was treated by Alifantis and Robinson [21].

As previous works by the authors, Suárez and Rosell [22] and Suárez et al. [23], respectively presented a first analysis of the cell considered in this work and introduced the application of pattern recognition techniques to look for simple solutions.

2. Description of the cell

Fig. 1 shows the layout of the manufacturing cell. The cell is composed of four machines m_i, $i = 1, \dots, 4$, in a row, all of them of the same type. Each machine operates alternatively over two different pallets, A and B, whose positions are interchanged by a pallet shuttle in a time t_t. The robot loads a part into the pallet in the outer position of the shuttle, either type A or type B, while the machine is working on the pallet in the inner position.

Each part to be manufactured must first be loaded into pallet A of any machine, where a set of operations

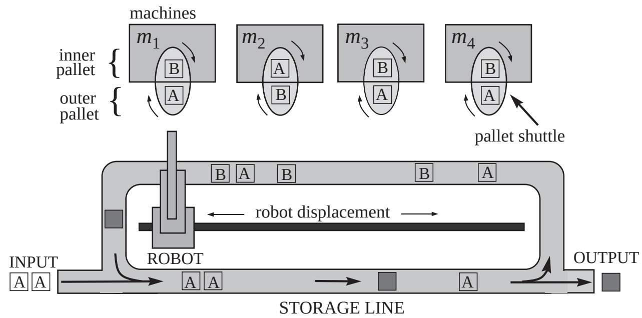

Fig. 1. Layout of the manufacturing cell.

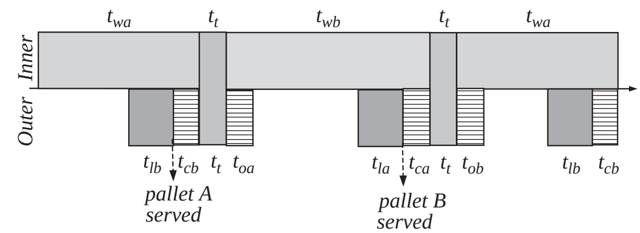

Fig. 2. Example of a machine cycle distinguishing between inner and outer pallet.

leaves it in a medium-processed state. Then, the part must be removed from that pallet and loaded into pallet B of any machine (it can be the same machine), where another set of operations leaves it in the final state (unprocessed and medium-processed parts are called type A and type B parts since they always have to be loaded into pallets A and B, respectively). It is necessary to remove the parts from pallets A of the machines and then reload them in pallets B because the parts are positioned in a different orientation in each pallet so that all the tools of the machines can work on the proper side of the parts. Pallets A and B have different sets of bridles in order to fix the parts in the required positions.

The parts are loaded into the pallets and unloaded from them by a 6 d.o.f. robot that uses a rail to move from one machine to another, requiring times t_{r1}, t_{r2} or t_{r3} depending on the distance between the machines. During regular activity, any load operation implies a previous unload operation, therefore the times t_{la} and t_{lb} considered for loading pallets A and B include the corresponding unload action. Once a part has been loaded into a pallet, a set of bridles must be closed to fix the part (requiring times t_{ca} and t_{cb}) before allowing the shuttle rotation to interchange the pallets when the work on the current inner pallet is done. After the turning of the pallet shuttle, the robot cannot recover the part from the outer pallet until the bridles are opened (requiring times t_{oa} and t_{ob}).

There is a storage line where the robot puts the parts unloaded from the machines and recovers them when necessary. Completely unprocessed parts are automatically supplied to this storage line, and therefore the robot always has direct access to either unprocessed parts as well as to medium-processed parts. The availability of parts is not a constraint in the system.

Each machine uses about 25 different tools for the operations on both pallets. These have to be replaced (or adjusted) after a number of operations. Since the lifespan of each tool is different, the result is that the machine needs assistance from a human operator after a period of time t_s that randomly varies within a given range. In this situation the machine stops working until a human operator replaces/adjusts the tools in need of attention and puts it on-line again. This is equivalent to consider that between intervals of accumulated working time t_s the machine has a longer working cycle of duration $t_{\mathrm{wa}} + t_{\mathrm{a1}}$ or $t_{\mathrm{wb}} + t_{\mathrm{a1}}$, t_{a1} being a fixed assistance time that in normal activity allows the human operator to replace the tools. During the next cycle on the same pallet, after assistance for tool replacement, the operator has to check the performance of machine with the new tools, and this produces a checking cycle that again lasts more than the regular working cycle, having a duration $t_{\mathrm{wa}} + t_{\mathrm{a2}}$ or $t_{\mathrm{wb}} + t_{\mathrm{a2}}$, where t_{a2} is also a fixed time. Only one operator is available for this purpose, thus, if one machine requests assistance while some other is already being assisted it will wait off-line for the operator assistance.

In the real cell, actual fixed times are: $t_{r1} = 6.5''$, $t_{r2} = 10,25''$, $t_{r3} = 14''$, $t_{\mathrm{ca}} = 28''$, $t_{\mathrm{cb}} = 30''$, $t_{\mathrm{oa}} = 12''$, $t_{\mathrm{ob}} = 12''$, $t_t = 14''$, $t_{\mathrm{a1}} = 120''$, $t_{\mathrm{a2}} = 60''$. t_s varies with a uniform distribution in the range $30' \leqslant t_s \leqslant 150'$. The times that may vary from one product to another are: the working times, $200'' \leqslant t_{\mathrm{wa}} \leqslant 260''$ and $140'' \leqslant t_{\mathrm{wb}} \leqslant 220''$, and the loading times that in some cases, for simple parts, can be reduced up to a 25% from the nominal values $t_{\mathrm{la}} = 47''$ and $t_{\mathrm{lb}} = 55''$. Fig. 2 shows an example of a machine working cycle.

3. Feeding strategies analysis

Only fixed sequences that include all the machines only once were considered, otherwise some machines would double the production of some others producing undesired effects like, for instance, different maintenance routines. Only variable strategies that require binary causal signals (i.e. binary signals available at the moment of the decision) were considered. With these constraints, the following strategies were analyzed following the directions of the personnel responsible for the cell.

Fixed machine and fixed pallet (*FMFP*). The machine sequence is m_1–m_2–m_3–m_4 and the robot feeds first pallets A of all the machines and then, in the next passing, all pallets B. A machine is skipped if it is being assisted by the operator.

Fixed machine and free pallet (*FM-I*). The machine sequence is m_1–m_2–m_3–m_4 and the robot feeds the pallet, either A or B, that each machine needs at the loading time. A machine is skipped if it is being assisted by the operator.

Fixed machine and free pallet (*FM-II*). The machine sequence is m_1–m_2–m_3–m_4 and the robot feeds the pallet, either A or B, that each machine needs at the loading time. A machine is skipped if it is not ready at the time of its turn in the sequence, either due to the operator assistance or just because it is still working on another part and the outer pallet has already been fed.

First in first out (*FIFO*). Each time a machine has finished the work on a part, the pallet shuttle has turned, and the bridles have been opened, the machine is added to a queue. The robot feeds always the first machine in that queue.

A cell simulator was implemented using Arena 3.51 from Rockwell Software Inc. The output data of the cell simulator is downloaded to a Microsoft Excel worksheet, where an output statistical analysis is automatically performed. On the run command, the simulator reads an input file with a set of pairs of working times t_{wa} and t_{wb} and performs a simulation of the cell working full day during a month. Ten replications for each pair of working times are performed in order to guarantee a desired precision in the statistical results: 95% confidence interval for the given least significant digit of each mean value (shown in percentage of the absolute time). The unproductive times of the machines for each strategy were initially computed for working times $t_{wa} = 251''$ and $t_{wb} = 204''$. The results were:

Strategy	Waiting for robot	Waiting for operator	Operator assistance
FMFP (%)	5.72	0.57	5.53
FM-I (%)	0.91	0.60	5.64
FM-II (%)	0.17	0.55	5.66
FIFO (%)	0.045	0.56	5.66

It can be seen that the time that the machines wait for the robot is highly dependent on the feeding strategy, while the time that the machines wait for the operator and the time of the operator assistance are independent of the feeding strategy (nevertheless, there is a small correlation because if a strategy makes the machines to wait a lot for the robot the machine tools last for more time and the average of assistance by the operator is slightly smaller). Running the same simulations for different machine working times, the following time percentages of the waiting time for the robot were obtained:

Strategy	$t_{wa} = 251$, $t_{wb} = 204$	$t_{wa} = 239$, $t_{wb} = 207$	$t_{wa} = 240$, $t_{wb} = 180$	$t_{wa} = 210$, $t_{wb} = 150$
FMFP (%)	5.72	5.72	5.98	6.43
FM-I (%)	0.91	0.86	0.91	2.11
FM-II (%)	0.17	0.20	1.45	12.16
FIFO (%)	0.045	0.067	1.93	13.37

These results show that the best strategy varies with the working times. The machine waste time while waiting for the robot of the less flexible strategy (FMFP) has the smallest influence of the machine working times. At the other extreme, the most flexible strategy (FIFO) varies a lot for different machine working times, going from two orders of magnitude better than FMFP to a worse performance. Thus, the selection of the best strategy for a given pair of working times is a relevant problem.

4. Cell analysis

Let us define:

Machine Activity (*MA*): the time that a machine needs, after being fed by the robot, to be ready for a new load.

Robot Activity (*RA*): the time that the robot needs, after feeding a machine m_i, to feed all the other available machines (i.e. those that are not under assistance) and be ready to feed again the machine m_i.

Robot Moves (*RM*): the time due to the robot moves (displacements) from a machine to another during *RA*.

Robot Loads (*RL*): the time dedicated to unload and load machines during *RA*.

In order to avoid waste time of the machines during production, the activities must satisfy

$$MA \geqslant RA = RM + RL, \quad (1)$$

i.e. after feeding a machine m_i the robot must be able to visit all the other machines, feed them and return to m_i before m_i is ready for a new load. In order to optimize productivity, the left-hand side of inequality (1) should be minimized (reduce *MA*) up to the limit imposed by *RA*. If $MA > RA$ then the robot has to wait for a machine to become ready to be fed (typically the robot waits in front of the next machine to be loaded in a fixed sequence, or in front of the last loaded machine in a variable sequence). Let us briefly review the conditions for each activity.

4.1. Fixed sequences

Machine Activity (*MA*). *MA* is the maximum time that the machine gives to the robot to reload it before it stops working and introduces waste time, and after

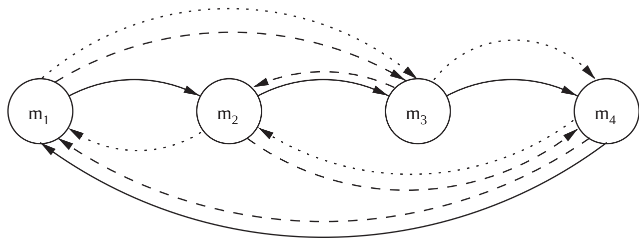

Fig. 3. Possible types of machine feeding sequences: (a) m_1–m_2–m_3–m_4 (continuous line); (b) m_1–m_3–m_4–m_2 (dotted line); (c) m_1–m_3–m_2–m_4 (dashed line).

loading a part type A or B it is $MA = MA_{\mathrm{a}} = t_{\mathrm{ca}} + t_t + t_{\mathrm{wa}} - t_{\mathrm{cb}} - t_{\mathrm{lb}}$ or $MA = MA_{\mathrm{b}} = t_{\mathrm{cb}} + t_t + t_{\mathrm{wb}} - t_{\mathrm{ca}} - t_{\mathrm{la}}$, respectively. These values are considered as the constraint in inequality (1). The example in Fig. 2 shows a machine cycle with the maximum MA that ensures no wasted time.

Robot Moves (*RM*) *in Robot Activity* (*RA*). *RM* depends on the sequence the robot follows to feed the machines. Since the robot must feed all the machines before feeding twice one of them, there are only three different possible types of sequences (Fig. 3):

(a) Sequence m_1–m_2–m_3–m_4: then $RM = RM_1 = 3t_{r1} + t_{r3}$.
(b) Sequence m_1–m_3–m_4–m_2: then $RM = RM_2 = 2t_{r1} + 2t_{r2}$.
(c) Sequence m_1–m_3–m_2–m_4: then $RM = RM_3 = t_{r1} + 2t_{r2} + t_{r3}$.

Since $t_{r3} \geqslant t_{r2} \geqslant t_{r1}$ it is evident that RM_3 is always worse than RM_1 and RM_2 so sequence (c) produces the worst RM. On the other hand, the convenience of RM_1 or RM_2 depends on the ratios between t_{r1}, t_{r2} and t_{r3}. For the typical trapezoidal velocity profile of a robot move (constant acceleration period, maximum velocity period, and constant deceleration period) $t_{r1} + t_{r3} \leqslant 2t_{r2}$ then $RM_1 \leqslant RM_2$, and therefore sequence (a) is preferred to sequence (b). As a conclusion, sequence (a) produces the shortest RM. Obviously, if a machine is being assisted, and therefore skipped by the robot, RM is smaller than in a regular situation.

Robot Loads (*RL*) *in Robot Activity* (*RA*). RL depends on the sequence of pallet types loaded by the robot in each machine (with independence of the position of the machine). Since there are two types of parts (A and B) and four machines, there are, in principle, 16 different combinations; nevertheless, since each machine has to process different type of parts in two consecutive cycles, the possible different sequences are reduced to eight, namely (subscripts p, q, r and s take values from 1 to 4 with $p \neq q \neq r \neq s$):

Type of the inner pallet in machine:

	m_p	m_q	m_r	m_s	m_p	m_q	m_r	m_s
(a)	A	A	A	A	B	B	B	B
(b)	A	A	A	B	B	B	B	A
(c)	A	A	B	A	B	B	A	B
(d)	A	A	B	B	B	B	A	A
(e)	A	B	A	A	B	A	B	B
(f)	A	B	A	B	B	A	B	A
(g)	A	B	B	A	B	A	A	B
(h)	A	B	B	B	B	A	A	A

During regular activity the robot has to feed three machines before repeating one of them, and the following situations are possible:

- In all sequences (a)–(h), at some point the robot has to feed consecutively one part A and two parts B or two parts A and one part B, RL being, respectively, $RL = RL_1 = t_{\mathrm{la}} + 2t_{\mathrm{lb}}$ or $RL = RL_2 = 2t_{\mathrm{la}} + t_{\mathrm{lb}}$.
- In the sequences (a), (b), (d) and (h), at some point the robot has to feed consecutively three parts A and three parts B, RL being, respectively, $RL = RL_3 = 3t_{\mathrm{la}}$ or $RL = RL_4 = 3t_{\mathrm{lb}}$.

Thus, if $t_{\mathrm{la}} = t_{\mathrm{lb}}$ any sequence from (a) to (h) implies the same RL, but if $t_{\mathrm{la}} \neq t_{\mathrm{lb}}$ then sequences (a), (b), (d) and (h) impose two additional constraints, being one of them the worst case constraint (i.e. highest value of RL), RL_3 if $t_{\mathrm{la}} > t_{\mathrm{lb}}$ and RL_4 otherwise. As a conclusion, any of the sequences (c), (e), (f) or (g) is preferred.

4.2. Variable sequences

Variable sequences, like FIFO, are determined by a heuristic that, based on the available information at the time of the decision making, chooses the next machine to be served by the robot, and therefore the analysis for fixed sequences presented above is no longer applicable. These heuristic-based strategies allows the robot to “adapt” the sequence of machines when the regular current activity is altered by any reason (like the machine assistance), giving more flexibility and increasing the capability of dealing with random perturbations.

On the other hand, as it was shown in the experimental results in Section 3, the performance of FIFO (more flexible) strategy has large variations depending on the machine working times. This is because the FIFO strategy may increase the robot travelling time by carrying the robot from one machine to another with the possibility of passing in front of a machine ready to be loaded without loading it. This effect is more significant when the machines are fast enough compared with the robot activity RA.

In order to model the constraints of the FIFO strategy, consider that at time t^k a machine m^k becomes ready for a new load and at that moment it is included in the queue; also consider $margin_{\mathrm{a}} = t_{\mathrm{wa}} - t_{\mathrm{ob}} - t_{\mathrm{lb}} - t_{\mathrm{cb}}$ and $margin_{\mathrm{b}} = t_{\mathrm{wb}} - t_{\mathrm{oa}} - t_{\mathrm{la}} - t_{\mathrm{ca}}$. Then, in order to avoid machine m^k to wait for the robot it has to be fed at time t_1^k satisfying

$$t_1^k = t^k + \lambda^k \begin{Bmatrix} margin_{\mathrm{a}} \\ margin_{\mathrm{b}} \end{Bmatrix} \quad \text{with } 0 < \lambda^k < 1, \qquad (2)$$

where any of the values between braces may be considered (in this case it depends on the pallet concerned). The same can be stated for the next machine in the queue, m^{k+1}, if it is ready for a new load at t^{k+1} it has to be loaded at

$$t_1^{k+1} = t^{k+1} + \lambda^{k+1} \left\{ \begin{array}{c} margin_a \\ margin_b \end{array} \right\} \quad \text{with } 0 < \lambda^{k+1} < 1. \tag{3}$$

Besides, t_1^k and t_1^{k+1} are related by the time t_{rm} that the robot needs, after arriving to m^k, to load it and go to m^{k+1}, i.e.,

$$t_1^{k+1} \geqslant t_1^k + t_{rm} = t_1^k + \left\{ \begin{array}{c} t_{lb} \\ t_{la} \end{array} \right\} + \left\{ \begin{array}{c} t_{r1} \\ t_{r2} \\ t_{r3} \end{array} \right\}. \tag{4}$$

Considering $t^{k+1} = t^k + \Delta t$ and substituting (2) and (3) in (4) the following inequality results

$$\Delta t + \lambda^{k+1} \left\{ \begin{array}{c} margin_a \\ margin_b \end{array} \right\} \geqslant \left\{ \begin{array}{c} \lambda^k margin_a + t_{lb} \\ \lambda^k margin_b + t_{la} \end{array} \right\} + \left\{ \begin{array}{c} t_{r1} \\ t_{r2} \\ t_{r3} \end{array} \right\}. \tag{5}$$

Since the machine m^{k+1} can wait up to the limit imposed by $\lambda^{k+1} = 1$ before having to wait for the robot, the worst case is produced when $\lambda^k = 1$ (machine m^k also loaded in the limit time), and $\Delta t \to 0$ (the two machines enter in the queue almost at the same time). In this case the existence of unproductive time in m^{k+1} depends on the distance between m^k and m^{k+1} and on the relation between $margin_a$ and $margin_b$ if the pallets to be loaded are of different type; but it is clear that if in some situations the inequality is satisfied, in some others, with the same probability, this would not be the case.

4.3. *Effect of the assistance to the machines*

When one of the machines requires assistance it is automatically disconnected from the manufacturing line until a human operator assists it, changes the relevant tools, and puts it on-line again. When a machine is in an assistance state the Robot Activity *RA* is reduced since both *RM* and *RL* are reduced, and therefore inequality (1) is more easily satisfied. Nevertheless, when the machine is put on-line again it may have to wait for the robot to reach it, producing an undesired waste time that is different for fixed or variable feeding sequences.

For a FM sequence it can happen that the machine gets into the line immediately after its nominal turn in the sequence, then it will have an unproductive time until the robot reaches it again. It may be even worst with a FMFP sequence when a machine gets into the line with the pallet that does not correspond to the sequence; in this case the robot will not stop at the machine in the first passing in front of it but instead the robot will load it in the next cycle, although the machine is ready to work immediately after re-entering the line. For a FIFO sequence this effect is less relevant because, if necessary, the robot can load the machine that has just entered into the line if no other one is in the queue, although this strategy cannot avoid some waste time, as it was shown in Section 4.2.

The machines request assistance randomly within a given period of time, and although experimental results and a simple analysis show that fixed sequences are more sensible to the random effect of the assistance than variable sequences, the exact influence on the different loading strategies requires an statistical analysis as a complement to the previous one. A complete model for such analysis has not yet been developed, and therefore the determination of a precise criterion for the feeding strategy selection is not straightforward.

5. Feeding strategy selection

Since there is no general theoretical solution for the mentioned type of cell under the real condition of random assistance requirements by the machines, a two-step procedure was followed. First, the simulator of the cell was used to analyze the machine waste time for different sequences with different working times. Second, pattern recognition techniques were used to identify the domain in which a given feeding sequence is better than another.

One fixed (FM-I) and one variable (FIFO) feeding strategies were considered from those described in Section 3. The selection of the FM-I strategy as representative of the fixed sequences is because the strategy FMFP is always worse than the FM-I, and when the FM-II is better that FM-I it always happens that the FIFO is better than FM-II (see experimental results in Section 3); therefore, FMFP and FM-II were ruled out.

The work was done according to the following steps:

(1) Obtention of a set of labelled samples from simulations of the FM-I and FIFO sequences with different working times t_{wa} and t_{wb}. A labelled sample is obtained by associating to a pair $[t_{wa}, t_{wb}]$ the feeding sequence with lower unproductive time (the machine unproductive time is computed as a percentage of the total absolute time).

(2) Computation of a linear discriminant function using the set of labelled samples obtained from the previous step. The function indicates the best strategy for any given working times t_{wa} and t_{wb}.

Details about each step are given in the following subsections. Once the linear discriminant function is determined, the work to be done by the cell operators in order to choose the best feeding strategy for a new set of working times on pallets A and B is just the evaluation of a simple linear function (the discriminant), and the sign of the numerical result will indicate the best feeding strategy.

5.1. Obtention of labelled samples

A number of simulations were done for the sequences FM-I and FIFO for several values of t_{wa} and t_{wb} within their usual ranges ($200'' \leqslant t_{\mathrm{wa}} \leqslant 260''$ and $140'' \leqslant t_{\mathrm{wb}} \leqslant 220''$). Considering the nominal loading times, the statistical results are shown in Tables 1 and 2.

The average unproductive times obtained with both strategies were interpolated in order to represent them as two surfaces (Fig. 4). The three-dimensional graphical representation shows the relation between the surfaces as well as an approximation of their intersection (white line in the figure), whose projection on the base plane determines the discriminant between the two regions where one strategy is better than the other.

5.2. Linear discriminant

The Linear Discriminant Function (*LDF*) is a linear function such that $LDF = 0$ divides the two-dimensional workspace defined by $(t_{\mathrm{wa}}\, t_{\mathrm{wb}})^{\mathrm{T}}$ into two regions ($LDF > 0$ and $LDF < 0$, respectively) such that in each of these regions one of the feeding strategies is better than the other [15]. The *LDF* is computed using the set of labelled samples obtained by simulations of the cell. The lost function to be minimized for the automatic determination of the *LDF* is proportional to the distance from the discriminant to the labelled samples that are misclassified. A dead zone around the discriminant is considered in order to emphasize the influence of samples close to the linear discriminant. Fig. 5 illustrates these concepts.

The linear discriminant is given by $LDF(t_{\mathrm{wa}}, t_{\mathrm{wb}}) = \gamma_{\mathrm{a}} t_{\mathrm{wa}} + \gamma_{\mathrm{b}} t_{\mathrm{wb}} + \gamma_{\mathrm{c}} = 0$ where γ_{a}, γ_{b} and γ_{c} are the parameters that define the linear discriminant (straight line) in the two-dimensional space of the working times $(t_{\mathrm{wa}}\, t_{\mathrm{wb}})^{\mathrm{T}}$.

Let us consider the following notation:

$T_W = (t_{\mathrm{wa}}, t_{\mathrm{wb}}, 1)^{\mathrm{T}}$: vector representing a sample of working times t_{wa} and t_{wb}.

T'_W: a labelled sample, i.e. a vector $(t_{\mathrm{wa}}, t_{\mathrm{wb}}, 1)^{\mathrm{T}}$ for which the best feeding strategy is known.

T'_{Wi}: a labelled sample for which the best feeding strategy is the FIFO.

T'_{Wj}: a labelled sample for which the best feeding strategy is the FM-I.

$U_i(T_W)$: unproductive time for the working times t_{wa} and t_{wb} of a sample T_W using FIFO strategy.

Table 1
Percentage of absolute time that the machines were unproductive because they were waiting for the robot, for a fixed machine sequence (FM-I)

		Working time on pallet B								
	FM	140	150	160	170	180	190	200	210	220
Working time on pallet A	200	16.35		12.32		8.36		4.73		1.94
	210		12.32		8.33				1.88	
	220	12.34				3.24				0.75
	230			6.53		3.44		0.95		
	240	8.51			3.65	2.25	1.11	0.79	0.78	0.80
	250		5.58			1.43	0.89	0.829	0.83	
	260	5.87		3.01	1.79	1.08		0.88		0.88

Table 2
Percentage of absolute time that the machines were unproductive because they were waiting for the robot, for a variable sequence (FIFO)

		Working time on pallet B								
	FIFO	140	150	160	170	180	190	200	210	220
Working times on pallet A	200	17.48		13.47		9.25		5.29		1.72
	210		13.37		9.32				1.71	
	220	13.35					3.42			0.15
	230			7.43		3.43		0.577		
	240	9.39			3.59	1.93	0.71	0.18	0.048	0.02
	250		5.60			0.87	0.26	0.07	0.02	
	260	5.73		2.38	1.08	0.36		0.05		0.01

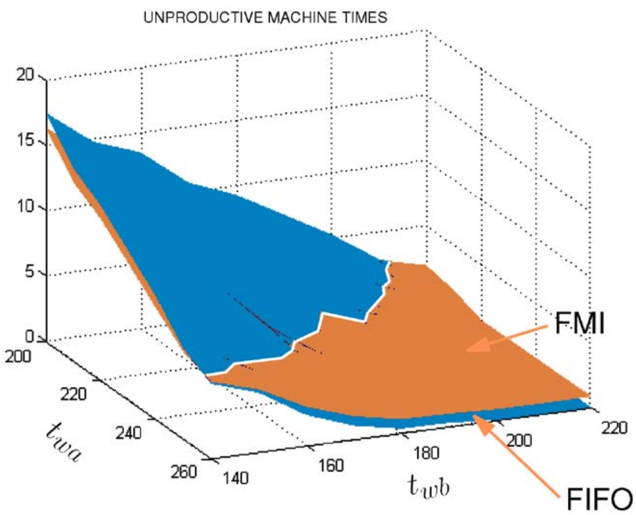

Fig. 4. Three-dimensional representation of the unproductive time as a function of t_{wa} and t_{wb}.

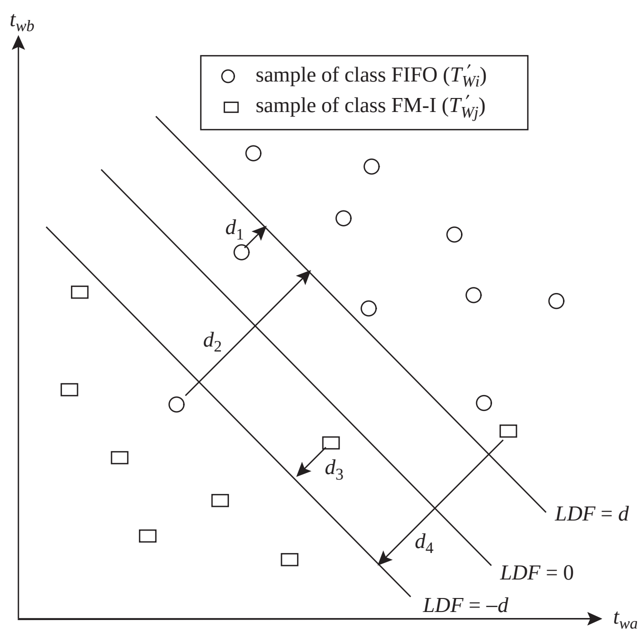

Fig. 5. Illustration of the *LDF*, the dead zone of width $2d$ and the distances used in the lost function for the misclassified samples.

$U_j(T_W)$: unproductive time for the working times t_{wa} and t_{wb} of a sample T_W using FM-I strategy.

Then, the *LDF* can be expressed as $LDF(T_W) = \Gamma \cdot T_W = 0$ where $\Gamma = (\gamma_{\mathrm{a}}, \gamma_{\mathrm{b}}, \gamma_{\mathrm{c}})^{\mathrm{T}}$ is a vector that defines de *LDF*.

By convention, the signs to identify each class of samples are assigned such that $LDF(T'_{Wi}) > 0$ and $LDF(T'_{Wj}) < 0$.

The dead zone, of width $2d$, between the two classes is defined by the linear functions given by $LDF(T_W) = \Gamma \cdot T_W = \pm d$.

The lost considered in this work when a given labelled sample is misclassified is the distance from that sample to the correct class domain multiplied by the difference in the unproductive time between both strategies. This lost is formalized with the following lost function.

The lost function computed when a sample T'_{Wi} (i.e with FIFO as best strategy) is classified as FM-I is (see for instance d_1 and d_2 in Fig. 5):

$$L_I(T'_{Wi}) = \begin{cases} 0 & \text{if } LDF(T'_{Wi}) \geqslant d, \\ [U_i(T'_{Wi}) - U_j(T'_{Wi})] & \\ \quad [d - \Gamma \cdot T'_{Wi}] & \text{if } LDF(T'_{Wi}) < d \end{cases} \tag{6}$$

and the lost function computed when a sample T'_{Wj} (i.e. with FM-I as best strategy) is classified as FIFO is (see for instance d_3 and d_4 in Fig. 5):

$$L_J(T'_{Wj}) = \begin{cases} 0 & \text{if } LDF(T'_{Wj}) \leqslant -d, \\ [U_j(T'_{Wj}) - U_i(T'_{Wj})] & \\ \quad [d + \Gamma \cdot T'_{Wj}] & \text{if } LDF(T'_{Wj}) > -d. \end{cases} \tag{7}$$

Then, the total lost function to be minimized in order to look for the optimum *LDF* is

$$R = \sum_i L_I(T'_{Wi}) + \sum_j L_J(T'_{Wj}). \tag{8}$$

The minimization of R was done using MATLAB Optimization Toolbox [24]. Considering $d = 25$ (determined empirically according to the distance between the samples), the result of the minimization process was a loss $R = 0.017$ for the discriminant parameters

$$\Gamma = \begin{pmatrix} \gamma_{\mathrm{a}} \\ \gamma_{\mathrm{b}} \\ \gamma_{\mathrm{c}} \end{pmatrix} = \begin{pmatrix} 0.27083 \\ 0.21527 \\ -100 \end{pmatrix}, \tag{9}$$

so the optimum *LDF* is

$$\begin{aligned} LDF(t_{\mathrm{wa}}, t_{\mathrm{wb}}) &= 0.27083 t_{\mathrm{wa}} + 0.21527 t_{\mathrm{wb}} - 100 \\ &= 0. \end{aligned} \tag{10}$$

Fig. 6 shows the obtained *LDF* with the corresponding samples obtained by simulation.

As a simple example of the use of the discriminant consider that a given part with working times $t_{\mathrm{wa}} = 222''$ and $t_{\mathrm{wb}} = 163''$ has to be manufactured; computing the value of *LDF* for these working times results $LDF(222, 163) = 0.27083 \times 222 + 0.21527 \times 163 - 100 = -4.786730 < 0$, and therefore the best feeding sequence for this part is FM-I.

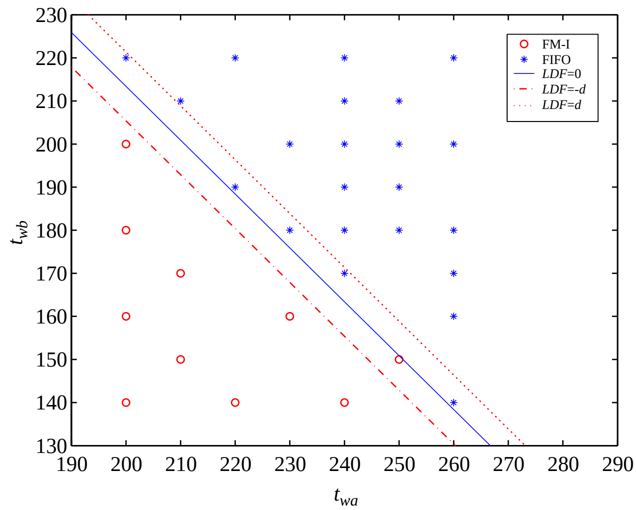

Fig. 6. Labelled samples from simulations and the obtained optimum *LDF* (the three parallel lines represent $LDF = 0$ and $LDF = \pm d$).

6. Case of variable loading times

As it was mentioned in Section 2 loading times t_{la} and t_{lb} can be reduced for simple parts that allow more simple robot movements when positioning the part in the pallet. The reduction in loading times can be up to a 25% of the nominal values $t_{la} = 47''$ and $t_{lb} = 55''$. This introduces a change in the operation that may alter the results regarding the best feeding strategy.

In order to minimize the increment of the sample space dimension, the effect of different loading times t_{la} and t_{lb} was not considered independently. Instead, the effect of a percentage reduction on both current loading times was considered, maintaining always the same range of working times for t_{wa} and t_{wb}.

Then, the problem is now reformulated as the search for a discriminant function in a three-dimensional workspace that selects the best feeding sequence depending on the working times t_{wa} and t_{wb} and on the percentage, r, of reduction of the loading times from their nominal values. The linear discriminant function was obtained in the same way as it was done for a two-dimensional problem and described in the previous section, since it is a general procedure and can be applied to any number of degrees of freedom.

Simulation results for unproductive times with $r =$ 10% and 15% are shown in Fig. 7. The discriminant functions corresponding to $r = 10\%$ and 15% are shown in Fig. 8. Finally, the *LDF* for the three-dimensional space defined by t_{wa}, t_{wb} and r is the plane (adjusted with a misclassifying lost of 0.0067) shown in Fig. 9 and corresponds to

$$4.1t_{wa} + 3.1t_{wb} - 10.6r - 1461.5 = 0. \quad (11)$$

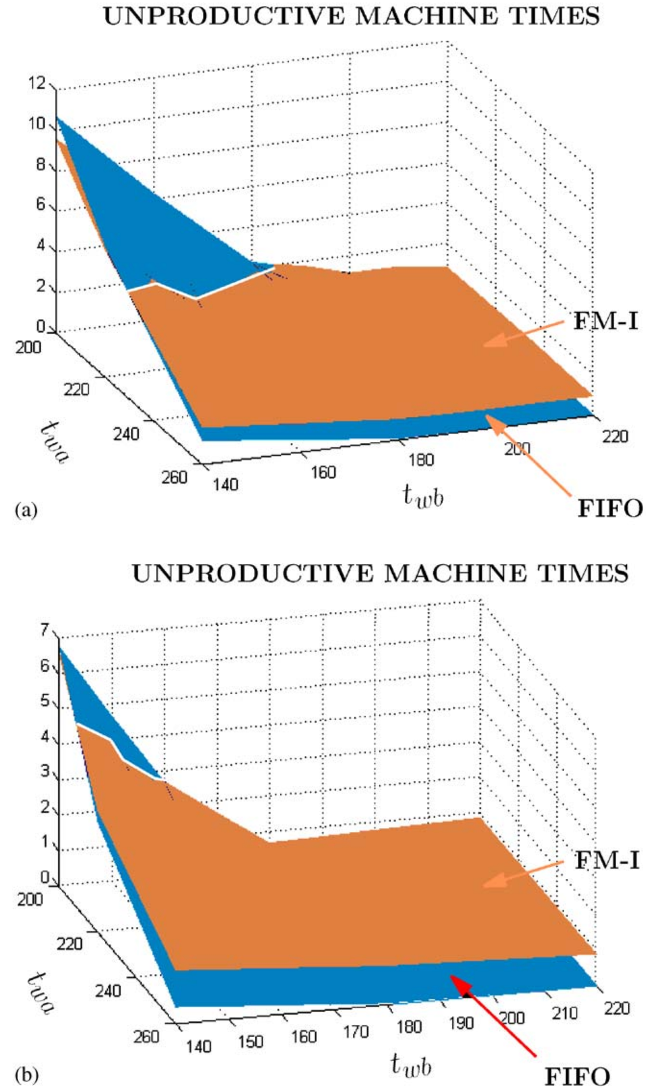

Fig. 7. Three-dimensional representation of the unproductive time as a function of t_{wa} and t_{wb} for (a) $r = 10\%$ and (b) $r = 15\%$.

It can be seen that the reduction in the loading times is favorable to the FIFO strategy; this is due to the fact that the bottleneck is not the robot activity but the rigidness of the FM-I strategy, as commented in Section 4.2.

7. Conclusions

The problem of feeding in an optimal way a manufacturing cell composed by four parallel machines was addressed using discrete event simulation and the theory of linear discriminant functions. The cell is located in a car-parts manufacturing company and has some features that make the problem a special case, and therefore there is no general solution available in the literature. For some conditions, differences in the

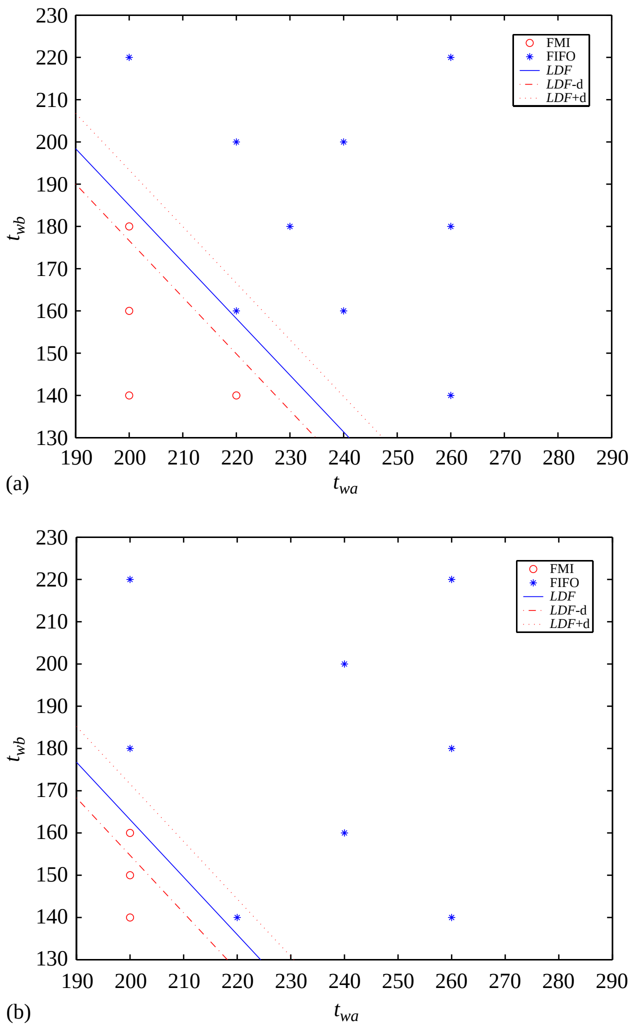

Fig. 8. Labelled samples from simulations and the obtained two-dimensional *LDF* for (a) $r = 10\%$ and (b) $r = 15\%$.

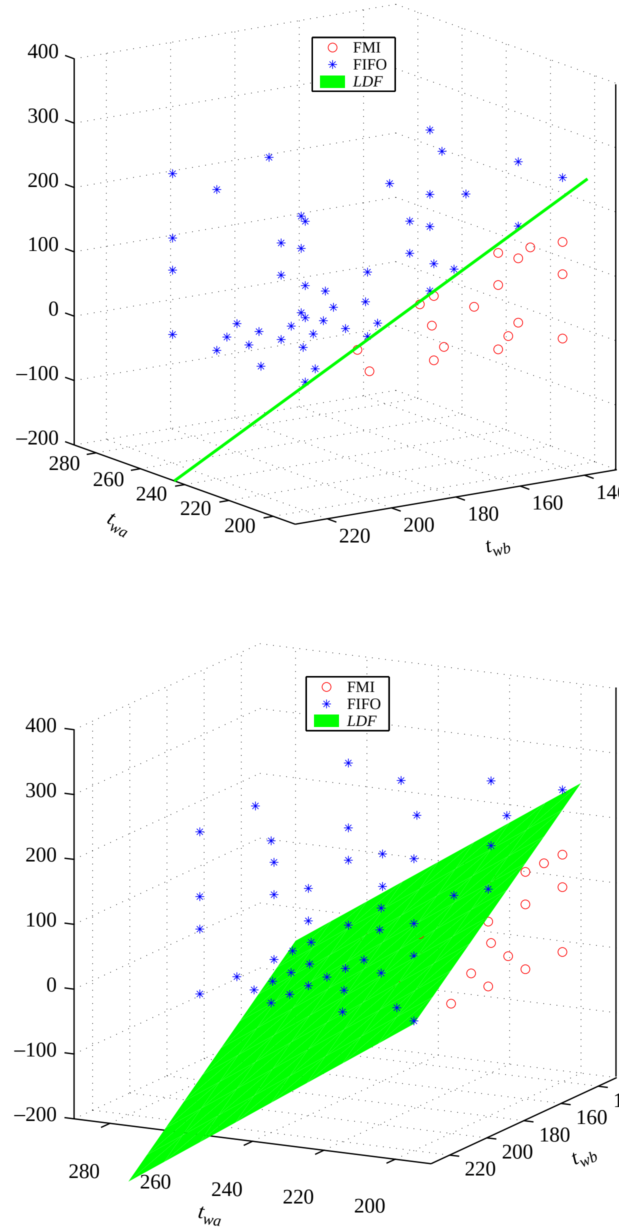

Fig. 9. Two views of the labelled samples from simulations and the obtained three-dimensional *LDF*.

unproductive time of the machines can be quite significative, so choosing the best feeding sequence is really important. The cell was modelled and a discrete event simulator of the cell was implemented. Simulation results for different feeding sequences and machine working times were then used to determine a linear discriminant function. This approach, frequently used in pattern recognition, allows the determination of the best feeding strategy in a very simple way just by evaluating a linear function. The cell operator only needs to introduce the values of the new data (working times in each pallet and reduction of the loading time) in the linear discriminant and the resulting sign indicates the best loading strategy.

The proposed approach is a simple solution to an open problem without a precise deterministic solution (new theoretical approaches are still to be developed), and therefore is a first step in the search of a practical solution for a real industrial application.

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